CSCE 496/896 Lecture 6: Reinforcement Learning

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(Adapted from Eleanor Quint)

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Introduction

- Consider learning to choose actions, e.g.,
  - Robot learning to dock on battery charger
  - Learning to choose actions to optimize factory output
  - Learning to play Backgammon, chess, Go, etc.
- Note several problem characteristics:
  - Delayed reward (thus have problem of **temporal credit assignment**)
  - Opportunity for active exploration (versus exploitation of known good actions)
    - $\Rightarrow$ Learner has some influence over the training data it sees
  - Possibility that state only **partially observable**

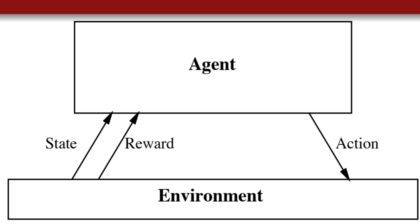
Example: TD-Gammon (Tesauro, 1995)

- Learn to play Backgammon
- Immediate Reward:
  - $+100$ if win
  - -100 if lose
  - 0 for all other states
- Trained by playing 1.5 million games against itself
- Approximately equal to best human player at that time

Outline

- Markov decision processes
- The agent's learning task
- Q learning
- Temporal difference learning
- Deep Q learning
- Example: Learning to play Atari
- Example: AlphaGo

Reinforcement Learning Problem



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \ldots$$

Goal: Learn to choose actions that maximize

$$r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots \text{, where } 0 \leq \gamma < 1$$

Markov Decision Processes

Assume

- Finite set of states S
- Set of actions A
- At each discrete time t agent observes state $s_t \in S$ and chooses action $a_t \in A$
- Then receives immediate reward r_t, and state changes to s_{t+1}
- Markov assumption: $s_{t+1} = \delta(s_t, a_t)$ and $r_t = r(s_t, a_t)$
  - I.e., r_t and s_{t+1} depend only on **current** state and action
  - Functions δ and r may be nondeterministic
  - Functions δ and r not necessarily known to agent

Agent's Learning Task

- Execute actions in environment, observe results, and
  - Learn **action policy** $\pi : S \to A$ that maximizes

$$\mathrm{E}\left[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots\right]$$

  from any starting state in S
  - Here $0 \le \gamma < 1$ is the **discount factor** for future rewards
- Note something new:
  - Target function is $\pi : S \to A$
  - But we have no training examples of form $\langle s, a \rangle$
  - Training examples are of form $\langle \langle s, a \rangle, r \rangle$
  - I.e., not told what best action is, instead told reward for executing action a in state s

Value Function

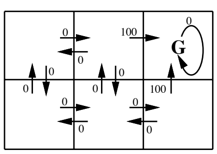
- First consider deterministic worlds
- For each possible policy π the agent might adopt, we can define **discounted cumulative reward** as

$$V^\pi(s) \equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots = \sum_{i=0}^{\infty} \gamma^i r_{t+i} \ ,$$

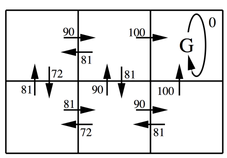
  where $r_t, r_{t+1}, \ldots$ are generated by following policy π, starting at state s
- Restated, the task is to learn an **optimal policy** π^*

$$\pi^* \equiv \operatorname*{argmax}_{\pi} \ V^\pi(s), \quad (\forall s)$$

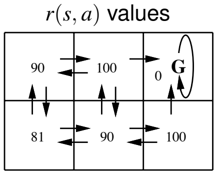
Value Function



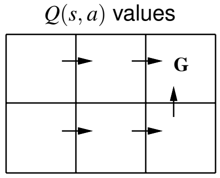
$r(s, a)$ values



$Q(s, a)$ values



$V^*(s)$ values



One optimal policy

What to Learn

- We might try to have agent learn the evaluation function V^{π^*} (which we write as V^*)
- It could then do a lookahead search to choose best action from any state s because

$$\pi^*(s) = \operatorname*{argmax}_{a} \left[r(s, a) + \gamma V^*(\delta(s, a))\right] \ ,$$

  i.e., choose action that maximized immediate reward + discounted reward if optimal strategy followed from then on
- E.g., $V^*(bot.\ ctr.) = 0 + \gamma 100 + \gamma^2 0 + \gamma^3 0 + \cdots = 90$
- A problem:
  - This works well if agent knows $\delta : S \times A \to S$, and $r : S \times A \to \mathbb{R}$
  - But when it doesn't, it can't choose actions this way

Q Function

- Define new function very similar to V^*:

$$Q(s, a) \equiv r(s, a) + \gamma V^*(\delta(s, a))$$

  i.e., $Q(s, a)$ = total discounted reward if action a taken in state s and optimal choices made from then on
- If agent learns Q, it can choose optimal action even without knowing δ

$$\begin{aligned}\pi^*(s) &= \operatorname*{argmax}_{a} \left[r(s, a) + \gamma V^*(\delta(s, a))\right] \\ &= \operatorname*{argmax}_{a} \ Q(s, a)\end{aligned}$$

- Q is the evaluation function the agent will learn

Training Rule to Learn Q

- Note Q and V^* closely related:

$$V^*(s) = \max_{a'} \ Q(s, a')$$

- Which allows us to write Q recursively as

$$\begin{aligned}Q(s_t, a_t) &= r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t))) \\ &= r(s_t, a_t) + \gamma \max_{a'} \ Q(s_{t+1}, a')\end{aligned}$$

- Let $\hat{Q}$ denote learner's current approximation to Q; consider training rule

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \ \hat{Q}(s', a') \ ,$$

  where s' is the state resulting from applying action a in state s

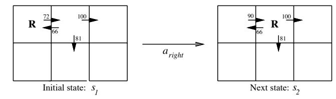
Q Learning for Deterministic Worlds

- For each s, a initialize table entry $\hat{Q}(s, a) \leftarrow 0$
- Observe current state s
- Do forever:
  - Select an action a (greedily or probabilistically) and execute it
  - Receive immediate reward r
  - Observe the new state s'
  - Update the table entry for $\hat{Q}(s, a)$ as follows:

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

  - $s \leftarrow s'$
- Note that actions not taken and states not seen don't get explicit updates (might need to generalize)

Updating $\hat{Q}$



$$\begin{aligned} \hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\ &= 0 + 0.9 \ \max\{66, 81, 100\} \\ &= 90 \end{aligned}$$

Can show via induction on n that if rewards non-negative and $\hat{Q}$s initially 0, then

$$(\forall s, a, n) \ \ \hat{Q}_{n+1}(s, a) \geq \hat{Q}_n(s, a)$$

and

$$(\forall s, a, n) \ \ 0 \leq \hat{Q}_n(s, a) \leq Q(s, a)$$

Updating $\hat{Q}$
Convergence

- **$\hat{Q}$ converges to Q:** Consider case of deterministic world where each $\langle s, a \rangle$ is visited infinitely often
- **Proof**: Define a **full interval** to be an interval during which each $\langle s, a \rangle$ is visited. Will show that during each full interval the largest error in $\hat{Q}$ table is reduced by factor of γ
- Let $\hat{Q}_n$ be table after n updates, and Δ_n be the maximum error in $\hat{Q}_n$; i.e.,

$$\Delta_n = \max_{s,a} |\hat{Q}_n(s, a) - Q(s, a)|$$

- Let $s' = \delta(s, a)$

Updating $\hat{Q}$
Convergence

- For any table entry $\hat{Q}_n(s, a)$ updated on iteration $n + 1$, error in the revised estimate $\hat{Q}_{n+1}(s, a)$ is

$$\begin{aligned} |\hat{Q}_{n+1}(s, a) - Q(s, a)| &= |(r + \gamma \max_{a'} \hat{Q}_n(s', a')) \\ &\quad -(r + \gamma \max_{a'} Q(s', a'))| \\ &= \gamma |\max_{a'} \hat{Q}_n(s', a') - \max_{a'} Q(s', a')| \\ (*) \qquad &\leq \gamma \max_{a'} |\hat{Q}_n(s', a') - Q(s', a')| \\ (**) \qquad &\leq \gamma \max_{s'', a'} |\hat{Q}_n(s'', a') - Q(s'', a')| \\ &= \gamma \Delta_n \end{aligned}$$

$(*)$ works since $|\max_a f_1(a) - \max_a f_2(a)| \leq \max_a |f_1(a) - f_2(a)|$
$(**)$ works since max will not decrease

Updating $\hat{Q}$
Convergence

- Also, $\hat{Q}_0(s, a)$ and $Q(s, a)$ are both bounded $\forall s, a$ $\Rightarrow$ Δ_0 bounded
- Thus after k full intervals, error $\leq \gamma^k \Delta_0$
- Finally, each $\langle s, a \rangle$ visited infinitely often $\Rightarrow$ number of intervals infinite, so $\Delta_n \rightarrow 0$ as $n \rightarrow \infty$

Nondeterministic Case

- What if reward and next state are non-deterministic?
- We redefine V, Q by taking expected values:

$$\begin{aligned} V^\pi(s) &\equiv \mathsf{E}\left[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots\right] \\ &= \mathsf{E}\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i}\right] \end{aligned}$$

$$\begin{aligned} Q(s, a) &\equiv \mathsf{E}\left[r(s, a) + \gamma V^*(\delta(s, a))\right] \\ &= \mathsf{E}\left[r(s, a)\right] + \gamma \mathsf{E}\left[V^*(\delta(s, a))\right] \\ &= \mathsf{E}\left[r(s, a)\right] + \gamma \sum_{s'} P(s' \mid s, a) \, V^*(s') \\ &= \mathsf{E}\left[r(s, a)\right] + \gamma \sum_{s'} P(s' \mid s, a) \, \max_{a'} Q(s', a') \end{aligned}$$

Nondeterministic Case

- Q learning generalizes to nondeterministic worlds
- Alter training rule to

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a')]$$

where

$$\alpha_n = \frac{1}{1 + visits_n(s,a)}$$

- Can still prove convergence of $\hat{Q}$ to Q, with this and other forms of α_n (Watkins and Dayan, 1992)

Temporal Difference Learning

- Q learning: reduce error between successive Q estimates
- **Q estimate using one-step time difference**:

$$Q^{(1)}(s_t,a_t) \equiv r_t + \gamma \max_a \hat{Q}(s_{t+1},a)$$

- **Why not two steps**?

$$Q^{(2)}(s_t,a_t) \equiv r_t + \gamma r_{t+1} + \gamma^2 \max_a \hat{Q}(s_{t+2},a)$$

- **Or** n?

$$Q^{(n)}(s_t,a_t) \equiv r_t + \gamma r_{t+1} + \cdots + \gamma^{(n-1)} r_{t+n-1} + \gamma^n \max_a \hat{Q}(s_{t+n},a)$$

Temporal Difference Learning

- **Blend all of these** $(0 \le \lambda \le 1)$:

$$\begin{aligned} Q^\lambda(s_t,a_t) &\equiv (1-\lambda)\left[Q^{(1)}(s_t,a_t) + \lambda Q^{(2)}(s_t,a_t) + \lambda^2 Q^{(3)}(s_t,a_t) + \cdots\right] \\ &= r_t + \gamma\left[(1-\lambda)\max_a \hat{Q}(s_{t+1},a) + \lambda\, Q^\lambda(s_{t+1},a_{t+1})\right] \end{aligned}$$

- TD(λ) algorithm uses above training rule
  - Sometimes converges faster than Q learning
  - Converges for learning V^* for any $0 \le \lambda \le 1$ (Dayan, 1992)
  - Tesauro's TD-Gammon uses this algorithm

Representing $\hat{Q}$

- Convergence proofs assume that $\hat{Q}(s,a)$ represented **exactly**
  - E.g., as an array
- How well does this scale to real problems?
- What can we do about it?

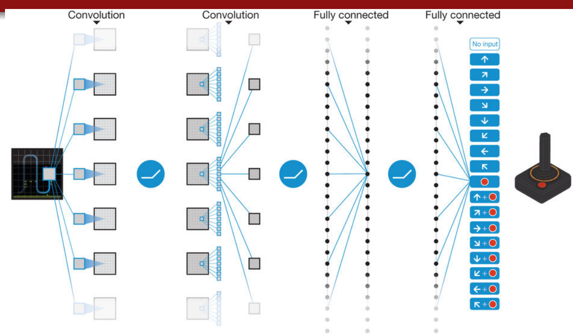
Deep Q Learning

- We already have machinery to approximate functions based on labeled samples
- Search for a **deep Q network** (DQN) to implement function Q_θ approximating Q
- Each training instance is $\langle s,a \rangle$ with label $y(s,a) = r + \gamma \max_{a'} Q_\theta(s',a')$
  - I.e., take action a in state s, get reward r and observe new state s'
  - Then use Q_θ to compute label $y(s,a)$ and update as usual
- Convergence proofs break, but get scalability to large state space

DQN Example: Playing Atari (Mnih et al., 2015)

- Applied same architecture and hyperparameters to 49 Atari 2600 games
  - System learned effective policy for each, very different, game
  - No game-specific modifications
- State description consists of raw input from emulator
- Frames rescaled to 84×84, single channel
- Each state is sequence of four most recent frames
- Rather than take s and a as inputs, network takes s and gives prediction of $Q(s,a)$ for all a as outputs
- Clipped positive rewards to $+1$ and negative to -1
- Evaluated each policy's performance against professional human tester

DQN Example: Playing Atari (Mnih et al., 2015)
Architecture



- Input: $84 \times 84 \times 4$, 3 convolutional layers, two dense
- Conv: 32 20×20, 64 9×9, 64 7×7
- 512 units in dense layers
- 18 outputs: Output i is estimate of $Q(s, a_i)$

DQN Example: Playing Atari (Mnih et al., 2015)
Training

- Reward signal at time t: $+1$ if score increased, -1 if decreased, 0 otherwise
- Action in game selected via ϵ-**greedy policy:** With probability ϵ choose action u.a.r., with probability $(1-\epsilon)$ choose $\mathrm{argmax}_a\, Q_{\boldsymbol{\theta}}(s, a)$
- Chosen action a_t run in emulator, which returns reward r_t and next frame for state s_{t+1}
- Update:

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \alpha \left[r_t + \gamma \max_{a'} Q_{\boldsymbol{\theta}_t}(s_{t+1}, a') - Q_{\boldsymbol{\theta}_t}(s_t, a_t) \right] \nabla Q_{\boldsymbol{\theta}_t}(s_t, a_t)$$

- Trained with RMSProp, mini-batch size of 32

DQN Example: Playing Atari (Mnih et al., 2015)
Modifications

Deep RL systems can be unstable or divergent, so Mnih:

1. Used **experience replay:** Rather than train on consecutive tuples, tuple (s_t, a_t, r_t, s_{t+1}) from game play added to **replay memory**
   - Replay memory sampled u.a.r. for training mini-batches
   - Independent instances in mini-batches reduces correlations in training data
   - Trained **off-policy** (policy trained is not the one choosing actions in game)
2. Used separate **target network** $\tilde{\theta}$ to generate labels:

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \alpha \left[r_t + \gamma \max_{a'} Q_{\tilde{\boldsymbol{\theta}}_t}(s_{t+1}, a') - Q_{\boldsymbol{\theta}_t}(s_t, a_t) \right] \nabla Q_{\boldsymbol{\theta}_t}(s_t, a_t)$$

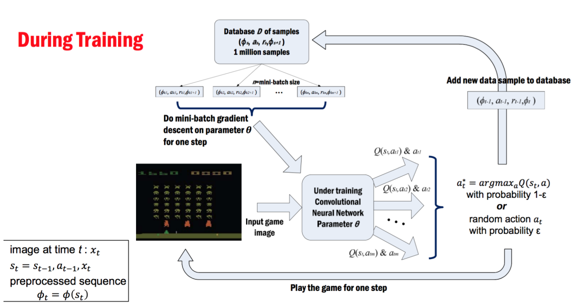
   Copied $\boldsymbol{\theta}$ into $\tilde{\boldsymbol{\theta}}$ every C updates
3. **Clipped** error term $[r_t + \cdots - Q_{\boldsymbol{\theta}_t}(s_t, a_t)]$ to $[-1, 1]$

DQN Example: Playing Atari (Mnih et al., 2015)
Pseudocode

Algorithm 1: deep Q-learning with experience replay.
Initialize replay memory D to capacity N
Initialize action-value function Q with random weights θ
Initialize target action-value function $\hat{Q}$ with weights $\theta^- = \theta$
For episode = 1, M **do**
  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  For t = 1,T **do**
    With probability ε select a random action a_t
    otherwise select $a_t = \mathrm{argmax}_a Q(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D
    Set $y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ with respect to the network parameters θ
    Every C steps reset $\hat{Q} = Q$
  End For
End For

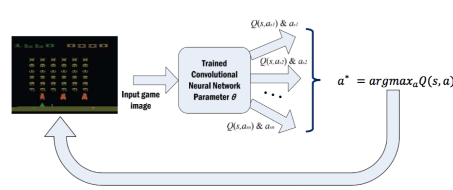
DQN Example: Playing Atari (Mnih et al., 2015)

During Training



DQN Example: Playing Atari (Mnih et al., 2015)

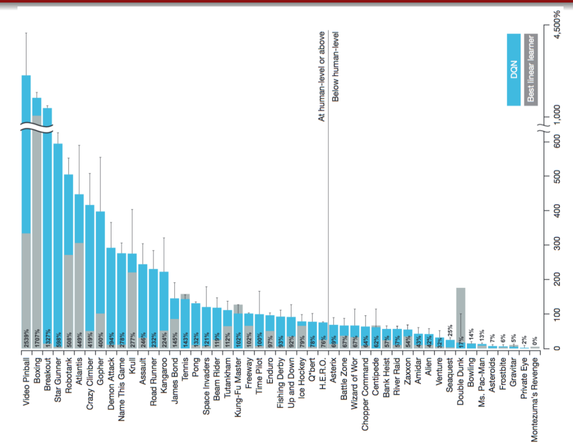
After Training



DQN Example: Playing Atari (Mnih et al., 2015)
Results

- Trained on each game for 50 million frames, no transfer learning
- Testing: Averaged final score over 30 sessions/game
- Measured performance of DQN RL and linear learner RL (with custom features) vs. human player: 100(RL−random)/(human−random)
  - I.e., human=100%, random=0%
- DQN outperformed linear learner on all but 6 games, outperformed human on 22, and comparable to human on 7
- Shortcoming: Performance poor (near random) when long-term planning required, e.g., Montezuma's revenge

DQN Example: Playing Atari (Mnih et al., 2015)
Results



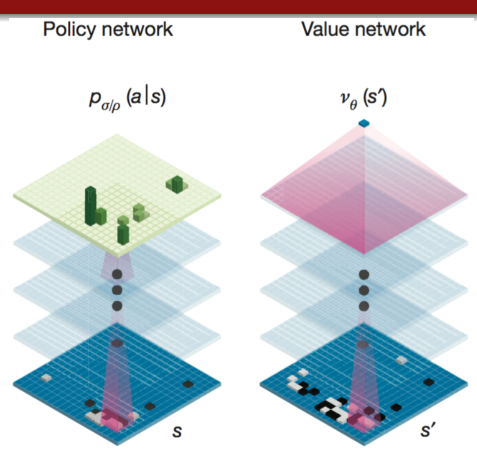
Go Example

- One of the most complex board games humans have
- Checkers has about 10^{18} distinct states, Backgammon: 10^{20}, Chess: 10^{47}, **Go:** 10^{170}
  - Number of atoms in the universe around 10^{81}
  - Another issue: Difficult to quantify goodness of a board configuration
- **AlphaGo:** Used RL and human knowledge to defeat professional player
- **AlphaGo Zero:** Improved on AlphaGo without human knowledge
- **AlphaZero:** Generalized to chess and shogi with general RL

AlphaGo (Silver et al., 2016)
Overview

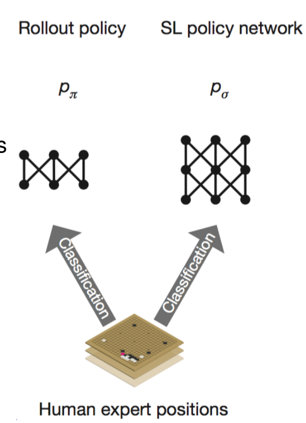
- Input: $19 \times 19 \times 48$ image stack representing player's and opponent's board positions, number of opponent's stones that could be captured there, etc.
- Training
  - **Supervised learning** (classification) of policy networks p_π and p_σ based on expert moves for states
  - **Transfer learning** from p_σ to policy network p_ρ
  - **Reinforcement learning** to refine p_ρ via **policy gradient** and **self-play**
  - **Regression** to learn value network v_θ
- Live play
  - Uses these networks in **Monte Carlo tree search** to choose actions during games
- 99.8% winning rate vs other Go programs and defeated human Go champion 5-0

AlphaGo (Silver et al., 2016)
Overview



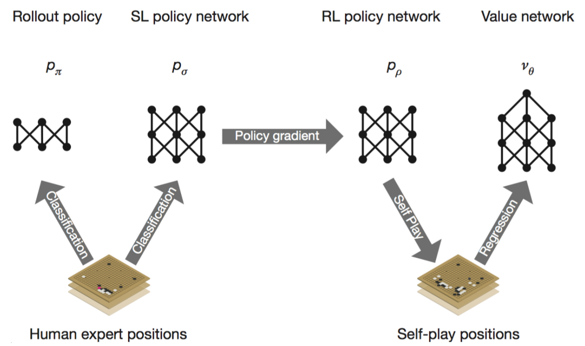
AlphaGo (Silver et al., 2016)
Supervised Learning

- **Supervised learning** of policies p_π and p_σ
- Board positions from **KGS Go Server**, labels are experts' moves
- Supervised learning of policies p_π and
- p_σ is full network (accuracy 57%, 3ms/move), p_π is simpler (accuracy 24% 2μs/move)



AlphaGo (Silver et al., 2016)
Transfer Learning

Transfer learning of p_σ to p_ρ (same arch., copy parameters)



AlphaGo (Silver et al., 2016)
Reinforcement Learning

- Trained p_ρ via play against $p_{\tilde{\rho}}$ (randomly selected earlier version of p_ρ)
- For state s_t, **terminal reward** $z_t = +1$ if game ultimately won from s_t and -1 otherwise
- Note p_ρ does not compute value of actions like Q-learning does
  - It **directly** implements a policy that outputs a_t given s_t
  - Use **policy gradient** method to train:
    - If agent chooses action a_t in state s_t and ultimately wins 90% of the time, what should happen to $p_\rho(a_t \mid s_t)$?
    - How can we make that happen?

AlphaGo (Silver et al., 2016)
Policy Gradient

- **REINFORCE**: **RE**ward **I**ncrement = **N**onnegative **F**actor times **O**ffset **R**einforcement times **C**haracteristic **E**ligibility
- Perform **gradient ascent** to increase probability of actions that on average lead to greater rewards:

$$\Delta\rho_j = \alpha(r - b_s)\frac{\partial \log p_\rho(a \mid s)}{\partial \rho_j} \quad ,$$

  α is learning rate, r is reward, a is action taken in state s, and b_s is **reinforcement baseline** (independent of a)
- b keeps expected update same but reduces variance
- E.g., if all actions from s good, b_s helps differentiate
- Common choice: $b_s = \hat{v}(s) =$ estimated value of s

AlphaGo (Silver et al., 2016)
Policy Gradient

- AlphaGo uses REINFORCE with baseline $b_s = v_\theta(s)$, $r = z_t$, and sums over all game steps $t = 1, \ldots, T$
- Average updates over games $i = 1, \ldots, n$

$$\Delta\rho = \frac{\alpha}{n}\sum_{i=1}^{n}\sum_{t=1}^{T^i}(z_t^i - v_\theta(s_t^i))\nabla_\rho \log p_\rho(a_t^i \mid s_t^i)$$

AlphaGo (Silver et al., 2016)
Value Learning

- $v_\theta(s)$ approximates $v^{p_\rho}(s) =$ value of s under policy p_ρ
- Regression problem on state-outcome pairs (s, z)
- Train with MSE
- Analogous to experience replay, mitigated overfitting by drawing each instance from a unique self-play game:
  1. Choose time step U uniformly from $\{1, \ldots, 450\}$
  2. Play moves $t = 1, \ldots, U$ from p_σ
  3. Choose move a_U uniformly
  4. Play moves $t = U + 1, \ldots, T$ from p_ρ
  5. Instance (s_{U+1}, z_{U+1}) added to train set

AlphaGo (Silver et al., 2016)
Live Play

- Now, we're ready for live play
- Rather than exclusively using p_ρ or v_θ to determine actions, will instead base action choice on a **rollout algorithm**
- Use the functions learned to simulate game play from state s forward in time ("rolling it out") and computing statistics about the outcome
- Repeat as much as time limit allows, then choose most favorable action
  - $\Rightarrow$ **Monte Carlo Tree Search** (MCTS)

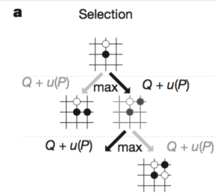
AlphaGo (Silver et al., 2016)

Monte Carlo Tree Search

- Given current state s, MCTS runs four operations:
  (a) **Selection:** Given a tree rooted at s, follow **tree policy** to traverse and select a leaf node
  (b) **Expansion:** Expand selected leaf by adding children
  (c) **Evaluation (simulation):** Perform rollout to end of game
    - Use p_π to speed up this part
  (d) **Backup:** Use rollout results to update action values of tree
- Each tree edge ((s, a) pair) has statistics:
  - **Prior probability** $P(s, a)$
  - **Action values** $W_v(s, a)$ and $W_r(s, a)$
  - **Value counts** $N_v(s, a)$ and $N_r(s, a)$
  - **Mean action value** $Q(s, a)$
- After many parallel simulations, choose action maximizing $N_v(s, a)$

AlphaGo (Silver et al., 2016)

Monte Carlo Tree Search: Selection



- Before reaching leaf state, choose action

$$a_t = \underset{a}{\mathrm{argmax}} \left(Q(s_t, a) + u(s_t, a) \right) ,$$

where

$$u(s, a) = cP(s, a) \frac{\sqrt{\sum_b N_r(s, b)}}{1 + N_r(s, a)}$$

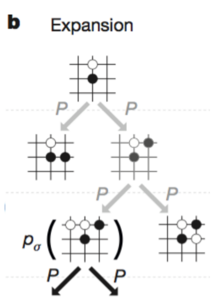
- I.e., if (s_t, a_t) has been evaluated a lot relative to other actions from s_t, $N_r(s_t, a_t)$ is large and a_t is evaluated mainly by Q
- Otherwise, exploration is encouraged
- To avoid all searches choosing same actions: When (s_t, a_t) chosen, update stats as if n_{vl} games lost

$$N_r(s_t, a_t) = N_r(s_t, a_t) + n_{vl}$$

$$W_r(s_t, a_t) = W_r(s_t, a_t) - n_{vl}$$

AlphaGo (Silver et al., 2016)

Monte Carlo Tree Search: Expansion

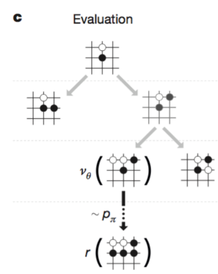


- If $N_r(s, a) > n_{thr}$, expand next state s' in tree

$$[N_v(s', a) = N_r(s'a) = 0, W_v(s'a) = W_r(s', a) = 0, P(s', a) = p_\sigma(a \mid s')]$$

AlphaGo (Silver et al., 2016)

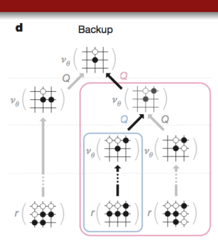
Monte Carlo Tree Search: Evaluation



- Expand from leaf s_L until game ends
- At each time $t \geq L$, **each player** chooses $a_t \sim p_\pi$
- At game's end, compute $z_t = \pm 1$ for all t

AlphaGo (Silver et al., 2016)

Monte Carlo Tree Search: Backup



At end of simulated game, update statistics for all steps $t \leq L$

1. Undo virtual loss and update z:

$$N_r(s_t, a_t) = N_r(s_t, a_t) - n_{vl} + 1$$

$$W_r(s_t, a_t) = W_r(s_t, a_t) + n_{vl} + z_t$$

2. After leaf evaluation done:

$$N_v(s_t, a_t) = N_v(s_t, a_t) + 1$$

$$W_v(s_t, a_t) = W_v(s_t, a_t) + v_\theta(s_L)$$

3. Take weighted average for final action value:

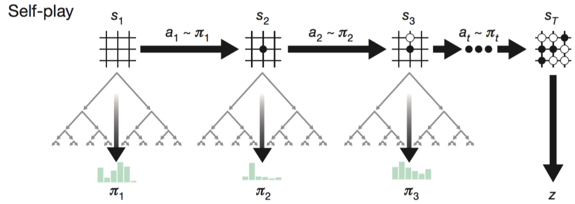
$$Q(s, a) = (1 - \lambda) \left(\frac{W_v(s, a)}{N_v(s, a)} \right) + \lambda \left(\frac{W_r(s, a)}{N_r(s, a)} \right)$$

AlphaGo Zero (Silver et al., 2017)

Overview

- The "Zero" refers to zero human knowledge
- No supervised training from KGS Go data
  - Trained only via RL in self-play
  - Trained a single network $(\boldsymbol{p}, v) = f_\theta$ for both policy and value
- Integrated MCTS into training as well as live play
  - Folded lookahead search into training loop
  - Did not rollout to end of game
- Input: $19 \times 19 \times 17$ image stack:
  - Eight of 17 binary planes indicate locations of player's stones the past 8 time steps
  - Eight of 17 binary planes indicate locations of opponent's stones the past 8 time steps
  - Final plane indicates color to play
- Discovered new Go knowledge during self-play, including previously unknown tactics

AlphaGo Zero (Silver et al., 2017)
Self-Play

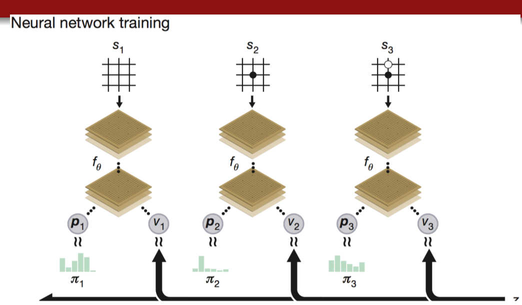


- Play games against self, choosing actions $a_t \sim \boldsymbol{\pi}_t$ via MCTS
- Outcome of game recorded as $z = \pm 1$

AlphaGo Zero (Silver et al., 2017)
Training

- Training is a form of **policy iteration:** Alternating between
  - **Policy evaluation:** Estimating value v of policy $\boldsymbol{p}$
  - **Policy improvement:** Improving policy wrt v
- Use MCTS to map NN policy $\boldsymbol{p}$ to search policy $\boldsymbol{\pi}$
- Self-play outcomes inform updates to v

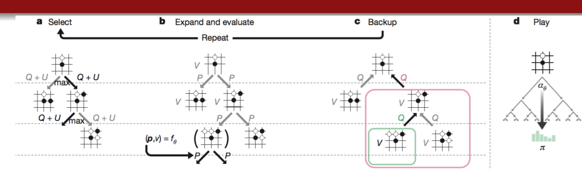
AlphaGo Zero (Silver et al., 2017)
Training



- State s_t's targets are distribution $\boldsymbol{\pi}_t$ and reward z_t
- Update network using loss function

$$\overbrace{(z_t - v(s_t))^2}^{\text{sq loss}} - \overbrace{\boldsymbol{\pi}_t^\top \log \boldsymbol{p}_t}^{\text{CE}} \overbrace{+ c\|\boldsymbol{\theta}\|^2}^{\text{regularizer}}$$

AlphaGo Zero (Silver et al., 2017)
MCTS



- MCTS similar to that of AlphaGo, but drop N_r and W_r since no rollout: $[N(s,a), W(s,a), Q(s,a), P(s,a)]$

(a) Select: same as before, but $u(s,a)$ uses N instead of N_r

(b) Expand + evaluate: $f_{\boldsymbol{\theta}}$ compute value $v(s)$ (modulo symmetry) for backup instead of rollout to game end

(c) Backup: same as before, but no N_r or W_r

(d) Play policy: $\pi(a \mid s_0) = N(s_0,a)^{1/\tau} / \sum_b N(s_0,b)^{1/\tau}$ (τ controls exploration)

AlphaZero (Silver et al., 2017b)

- AlphaGo Zero's approach applied to chess and shogi
- Same use of $(\boldsymbol{p}, v) = f_{\boldsymbol{\theta}(s)}$ and MCTS
- Go-specific parts removed + other generalizations
- No game-specific hyperparameter tuning
  - Similar framework as Atari

Game	White	Black	Win	Draw	Loss
Chess	*AlphaZero*	*Stockfish*	25	25	0
	Stockfish	*AlphaZero*	3	47	0
Shogi	*AlphaZero*	*Elmo*	43	2	5
	Elmo	*AlphaZero*	47	0	3
Go	*AlphaZero*	*AG0 3-day*	31	–	19
	AG0 3-day	*AlphaZero*	29	–	21

Table 1: Tournament evaluation of *AlphaZero* in chess, shogi, and Go, as games won, drawn or lost from *AlphaZero*'s perspective, in 100 game matches against *Stockfish*, *Elmo*, and the previously published *AlphaGo Zero* after 3 days of training. Each program was given 1 minute of thinking time per move.