Homework 2

Assigned October 14, 2019
Due October 2~~1~~3, 2019 on Canvas

Design and Analysis of Algorithms
(CSCE 423/823, Fall 2019)

CSCE 823 students have to do all problems for full credit. CSCE 423 students need to do only the Core Problems for full credit, and may do the Advanced Problem for bonus points.

For this homework assignment, you are to work in the team that you established in Homework 0. This will be your collaborative team for the rest of the term. You may freely discuss solutions to exercises within your team, and you are to submit a single pdf file from your team. **The internet is not an allowed resource on this homework!**

Clarity of presentation is important. You should give a clear description of all your algorithms, each with a proof of correctness and analysis of time complexity. You must submit your solutions in a single pdf file via Canvas, and are encouraged to prepare your solutions in LaTeX. **Only** pdf will be accepted, and you should submit only one pdf file for Questions 2–5. When you submit for Question 1, submit a second pdf file to the *Questions* assignment in Canvas.

Core Problems

1. **(bonus points, but mandatory submission)** Present one question that you have on the lectures and/or textbook on either Greedy Algorithms or BFS/DFS. This question should be thoughtful and nontrivial, and suggest depth of knowledge in the material. Also, present what you consider to be a reasonable (doesn't have to be completely correct) answer to this problem.

   Your question and answer should be submitted to the *Questions* assignment in Canvas in a pdf file separate from the rest of your homework submission.

2. **(20 Points)** Give an example of a directed graph $G = (V, E)$, a source vertex $s \in V$, and a set of tree edges $E_\pi \subseteq E$ such that for each vertex $v \in V$, the unique simple path in the graph (V, E_π) from s to v is a shortest path in G, yet the set of edges E_π cannot be produced by running BFS on G, no matter how the vertices are ordered in each adjacency list.

3. **(20 Points)** Describe an efficient algorithm that, given a set $X = \{x_1, x_2, \ldots, x_n\}$ of points on the real line, determine the smallest set of unit-length closed intervals that contain all of the given points. Prove that your algorithm is correct and argue its time complexity.

4. **(20 Points)** Give a linear time ($O(|V| + |E|)$) algorithm that takes as input a DAG (directed acyclic graph) in adjacency list format and two vertices s and t, and returns the number of directed paths from s to t. Prove that your algorithm is correct and argue its time complexity.

Advanced Problem

5. **(50 points)** You are a ballroom dancing instructor. To make the students as comfortable as possible, you strive to match the heights of each pair of dance partners. Your class has W women and M men. The height of the ith woman is w_i and the height of the jth man is m_j. In each of the problems below you are to assign men to women as dance partners in such a way as to minimize the sum of the absolute differences of the heights in each pair. In each case, prove your algorithm's correctness and time complexity.

(a) (25 pts) Give a greedy algorithm that runs in time $O(M \log M)$ that obtains an optimal solution to this problem when $W = M$. Be sure to give a clear high-level description of your algorithm, a proof that the solution output is optimal, and an analysis of the time complexity of your algorithm.

(b) (25 pts) Give a dynamic programming algorithm that runs in time $O(MW + M \log M)$ that obtains an optimal solution to this problem when you only know that $M \geq W$ (any men unpaired will sit out of this class). Again, give a clear high-level description of your algorithm, a proof that the solution output is optimal, and the time analysis.