Robust Affine-Invariant Similarity Measures for Patterned Triangles*

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Abstract

A robust image similarity measure is presented for patterned triangles. A novel feature of the new similarity measure is the reduction of the patterns to a simple set of affine-invariant bar-codes.

1 Introduction

Different images of the same object are *affine-invariant* transformations of each other under the so-called *weak perspective assumption* [12]. Hence recognizing that a new image and a stored image show the same object requires an *affine-invariant similarity measure* between pairs of images.

Previously proposed affine-invariant similarity measures consider the similarity between pairs of points and contour lines, for example the *minimum Hausdorff distance measure* [3], the *geometric hashing* [14] technique, and least squares distance-based similarity measures [9], and in general ignore the complex colored patterns that are present in the pictures.

In line with these methods, in [8], we proposed an affine triangulation method for spatial data that is composed of a set of triangles. Our method is based on the computation of barycentric points in convex polygons obtained by the so-called *sketch* of the image. A spatial triangulation method is called *affine-invariant*, if whenever it is applied to two spatial figures A and B that can be mapped to each other by an affinity α of the plane, also the resulting triangulations can be mapped to each other by that same affinity α.

Unfortunately, our computer experiments on a set of over 400 bird images led to poor results. It appears that the colorful and richly patterned bird images need to be considered in their entirety. The images can be abstracted or reduced to a set of triangles that are affine-invariant, but apparently too much valuable information is lost in the process. In fact, consider two objects whose overall shapes are single triangles. If one ignores the patterns within the triangles and only concentrates on their contours, then one cannot say anything definite because two black triangles are always affine-invariant to each other.

The present paper is motivated by enriching the method in [8] by considering the colorful patterns within the individual triangles. Our goal is to achieve a good recognition even if all objects have a triangular shape. For complex objects that are composed of several triangles, the the new method can be easily combined with the earlier method in [8] that considers the spatial relationships of the set of affine-invariant triangles into which the pictures can be decomposed.

The outline of this article is as follows. Section 2 discusses barcodes in patterned triangles. In this section, Theorem 2.1 shows that the barcodes are affine-invariant. Section 3 describes the barcode-based similarity measure for patterned triangles and presents some examples on various different types of patterns. Section 4 presents ways of adding color information to the barcode-based similarity measure.

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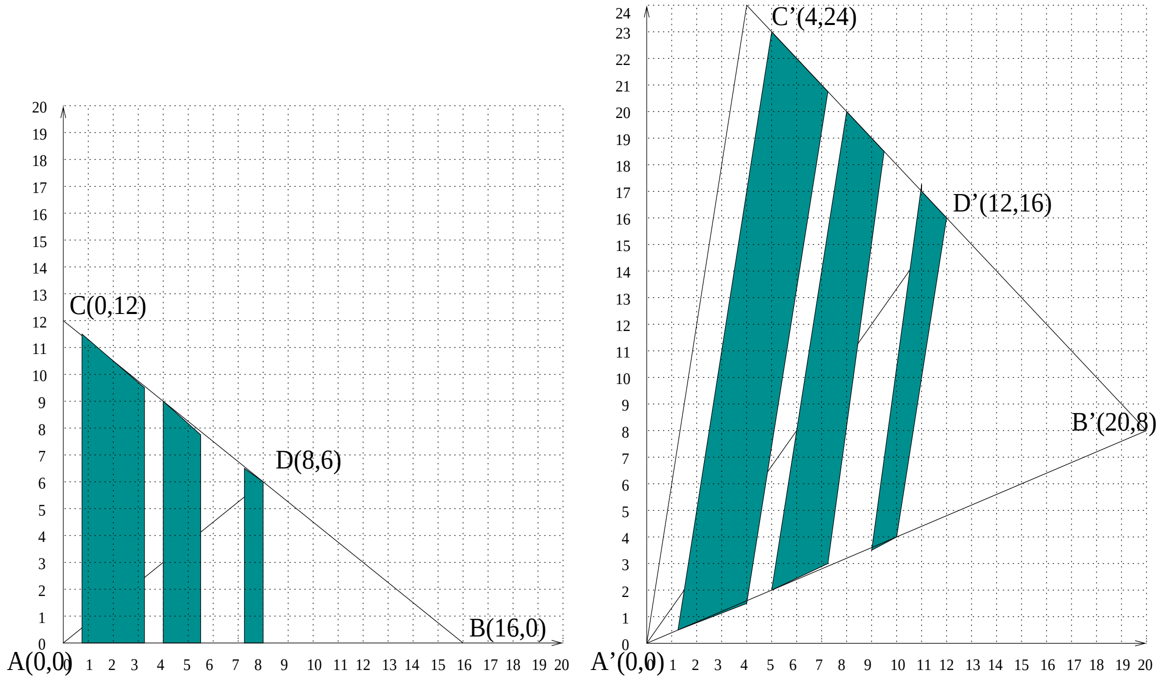

Figure 1. A striped triangle ABC (left) and its affine-transformation $A'B'C'$ (right).

2 Barcode-based similarity measures

2.1 Barcodes in patterned triangles

Barcodes provide an inspiration for robust affine-invariant similarity measures. Barcodes encode information in a robust machine-readable way using sequences of dark and light bars with different widths. The optical scanners which read barcodes are quite robust and already allow the presentation of barcodes from slightly different angles.

Although natural objects do not have barcodes, they have rich patterned and textured surfaces with a variety of colors. For example, the feathers of many birds have interesting patterns, such as the feathers of the parrot shown in Figure **??**.

The surface of objects can be broken into a set of triangles that each contain some interesting pattern. Let us concentrate on just one triangular area ABC with some unique pattern as shown on the left side of Figure 1. Triangle ABC is transformed by the affine motion:

$$f(x,y) = \begin{bmatrix} \frac{5}{4} & \frac{1}{3} \\ \frac{1}{2} & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

into another triangle $A'B'C'$ shown on the right side of Figure 1. The shape of triangle ABC becomes highly distorted. However, in triangle ABC one can identify the point D as the midpoint on the edge CD. Similarly, one can identify also the corresponding midpoint D' on the edge $C'D'$.

Now let us consider scanning the line segment AD from point A to point D. During the scan one sees a series of lighter and darker areas. In particular, one sees in sequence 1 unit light area, 3 units dark area, 1 unit light area, 2 units dark area, 2 units light area, and 1 unit dark area. Hence the barcode of the AD segment can be represented as $(1, \mathbf{3}, 1, \mathbf{2}, 2, \mathbf{1})$, with the boldface numbers representing the dark areas.

Interestingly, when one scans the line segment $A'D'$, one finds also a similar sequence of light and dark areas. In particular, one sees 2 units light area, 6 units dark area, 2 unit light area, 4 units dark area, 4 units light area, and 2 units dark area. Hence the barcode of $A'D'$ can be represented as $(2, \mathbf{6}, 2, \mathbf{4}, 4, \mathbf{2})$.

The two barcodes are similar because they have the same number of light and dark areas and those have the same length ratios, which are equal to the ratio of the lengths of the two line segments.

Let the length of a line segment l be denoted as $length(l)$. The similarity of barcodes is a general feature of affine transformations as expressed in Theorem 2.1.

Theorem 2.1 Let $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_m)$ be the barcodes of two corresponding line segments l_1 and l_2 in two affine transformations of a patterned triangle. Then $n = m$ and the following hold for each $1 \leq i \leq n$:

$$\frac{a_i}{b_i} = \frac{length(l_1)}{length(l_2)}$$

Example 2.1 Consider again the barcodes of AD and $A'D'$ of the two affine transformations shows in Figure 1. In this case the barcode of AD is:

$$(a_1, a_2, a_3, a_4, a_5, a_6) = (1, \mathbf{3}, 1, \mathbf{2}, 2, \mathbf{1})$$

and the barcode of $A'D'$ is:

$$(b_1, b_2, b_3, b_4, b_5, b_6) = (2, \mathbf{6}, 2, \mathbf{4}, 4, \mathbf{2})$$

We have that $length(AD) = 10$ and $length(A'D') = 20$. Further, as expected by Theorem 2.1,

$$\frac{a_i}{b_i} = \frac{10}{20} = 0.5 \quad \forall\ 1 \leq i \leq 6$$

3 Similarity measures for barcodes

Next we present a similarity measure for two barcodes. Let $\overline{a} = (a_1, \ldots, a_n)$ and $\overline{b} = (b_1, \ldots, b_m)$ be the barcodes of two corresponding line segments l_1 and l_2 in two affine transformations of a patterned triangle. If $n \neq m$, then we simply say that the two barcodes are not similar. Otherwise, let

$$d = \frac{length(l_1)}{length(l_2)}$$

Equalize the total length of the two line segments by scaling l_2 by the factor d. After scaling we obtain a barcode $\overline{c} = (c_1, \ldots, c_n)$ where each $c_i = b_i \times d$ for $1 \leq i \leq n$.

Next compare $\overline{a}$ and $\overline{c}$. If one is an affine transformation of the other, then by Theorem 2.1 and the choice of the scaling factor d, the following holds:

$$\frac{a_i}{c_i} = \frac{a_i}{b_i \times d} = \frac{length(l_1)}{length(l_2) \times d} = 1 \quad \forall\ 1 \leq i \leq n$$

In general, one cannot expect two barcodes to be perfect affine transformations of each other, that is to have $a_i = c_i$ for each $1 \leq i \leq n$. Hence one needs to consider how much a_i and c_i deviate from each other. One can use a root mean square error measurement as follows:

$$E(\overline{a}, \overline{c}) = \sqrt{\frac{\sum_{k=1}^{N} (a_i - c_i)^2}{n}} \tag{1}$$

Example 3.1 Let $\overline{a} = (1, \mathbf{2.5}, 1, \mathbf{2}, 2.5, \mathbf{1})$ and $\overline{b} = (2, \mathbf{5.5}, 2, \mathbf{4.5}, 4, \mathbf{2})$. Then $length(\overline{a}) = 10$ and $length(\overline{b}) = 20$ hence $d = 0.5$ and $\overline{c} = (1, \mathbf{2.75}, 1, \mathbf{2.25}, 2, \mathbf{1})$. Further,

$$\begin{aligned} E(\overline{a}, \overline{c}) &= \sqrt{\tfrac{0^2 + (-.25)^2 + 0^2 + (-.25)^2 + .5^2 + 0^2}{6}} \\ &= 0.375 \end{aligned}$$

Since the root mean square error is small, the two barcodes are quite similar.

3.1 Different patterns

The barcode-based similarity measure can accommodate other patterns beside striped patterns. For example, Figure 2 shows that if the triangles are spotted with ovals instead of having stripes, then the barcodes are still similar after the same affine transformation.

Note that the barcodes in Figures 1 and 2 are the same. That means that a single barcode for a triangle is incapable of distinguishing between the striped and the spotted patterns within the triangle.

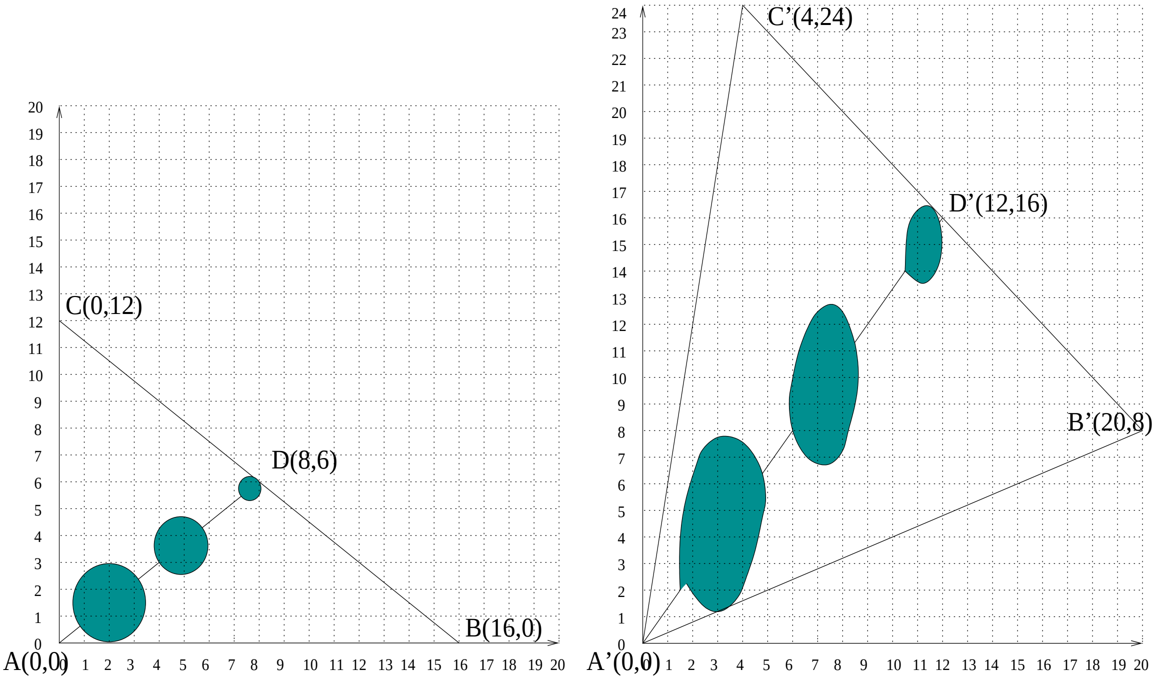

Figure 2. A spotted triangle ABC (left) and its affine-transformation $A'B'C'$ (right).

However, one can improve the situation by considering not one but two or three barcodes for a single triangle. Each barcode is scanned along the line segment whose endpoints are a corner vertex of the triangle and the midpoint vertex on the opposite side of the triangle. Figure **??** shows the three line segments $A'D'$, $B'E'$ and $C'F'$ in the striped and the spotted triangles.

4 Addition of color

In [8], we described two affine-invariant color measures: the *primary color ratio measure* and the *rainbow color ratio measure*. The primary color ratio measure finds the ratios $\frac{R}{G}$ and $\frac{B}{G}$ for the total amount of R, G, and B in the triangles.

The rainbow color ratio measure uses 9 different colors: *red, green, blue, yellow, turquoise, purple, white, gray and black,* which are defined precisely in [8]. The areas colored with these are denoted R, G, B, Y, T, P, W, Gr and Bl, respectively. For each image, we take out the background. Let I be the total area of the picture without its background. The rainbow color ratio vector of each image consists of the ratios $\frac{R}{I}$, $\frac{G}{I}$, $\frac{B}{I}$, $\frac{Y}{I}$, $\frac{T}{I}$, $\frac{P}{I}$, $\frac{Gr}{I}$ and $\frac{Bl}{I}$.

The experiments in [8] showed that the primary color ratio measure performs well with respect to the rainbow color ratio measure under the same lightning conditions.

There are other ways to add color to the barcode-based similarity measure. For example, the barcode can be enhanced with a sensor of the color (or average color) in each “bar” instead of sensing only white and black. Assume that in Figure 1 non-white stripes (shown as black) are from left to right red, blue, and red. Then a barcode representation of the striped triangle would be $(1-white, 3-red, 1-white, 2-blue, 2-white, 1-red)$. Such a colored barcode representation would be distinguishable form another colored barcode representation such as $(1-white, 3-blue, 1-white, 2-red, 2-white, 1-blue)$ based purely on the color differences.

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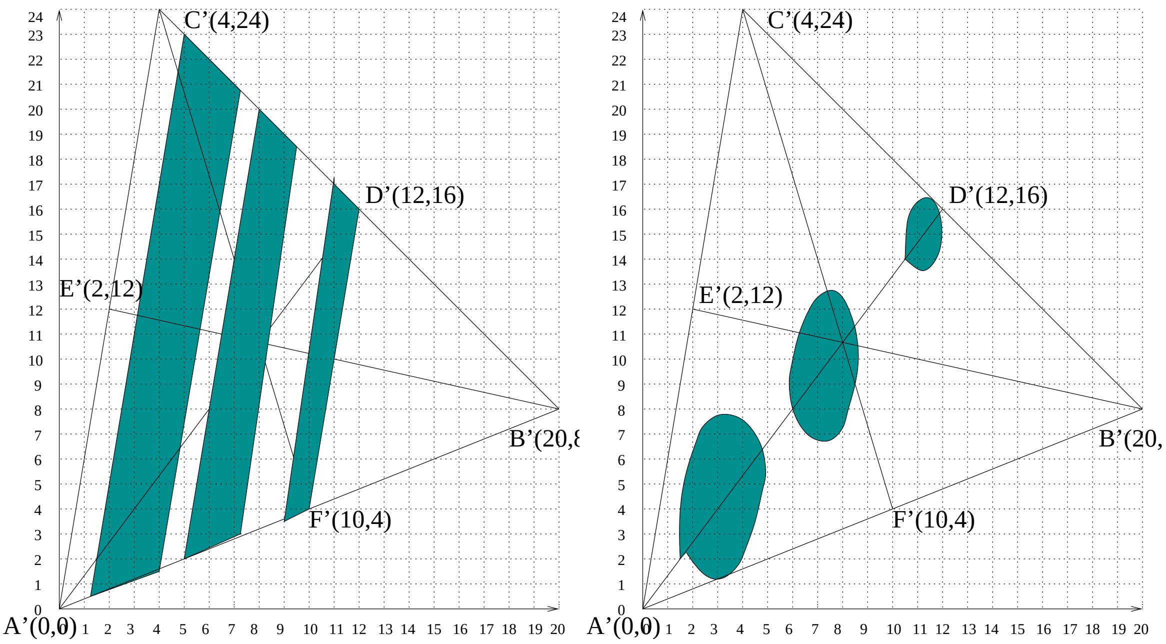

Figure 3. Midpoints of a striped triangle ABC (left) and a spotted triangle $A'B'C'$ (right).

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