

Statistical Thinking

Based on C. J. Wild and M. Pfannkuch (1999). Statistical thinking in Empirical Enquiry, *International Statistical Review*, **67**(3):223-265.

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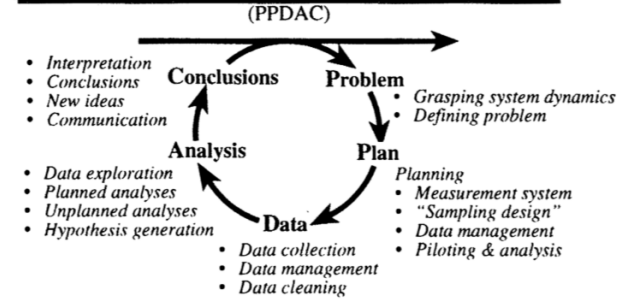
Professor Matt Waite's notes

Basic Ideas

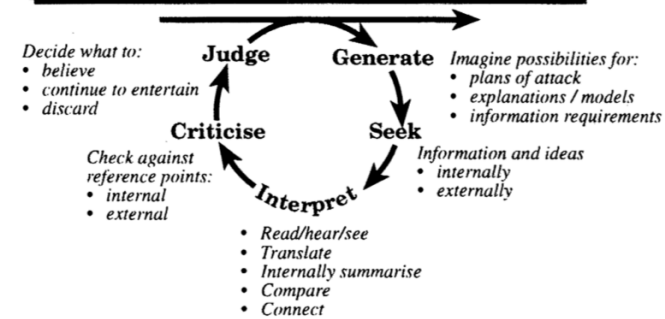
- Thought processes involved in statistical problem solving
 - From problem formulation to conclusions
- A four-dimensional framework for statistical thinking in empirical enquiry
 - Investigative cycle
 - Interrogative cycle
 - Types of thinking
 - Dispositions
- Central element: “**variation**”

Four-Dimensional Framework

(a) DIMENSION 1 : THE INVESTIGATIVE CYCLE



(c) DIMENSION 3 : THE INTERROGATIVE CYCLE



(b) DIMENSION 2 : TYPES OF THINKING

GENERAL TYPES

- Strategic**
 - planning, anticipating problems
 - awareness of practical constraints
- Seeking Explanations**
- Modelling**
 - construction followed by use
- Applying Techniques**
 - following precedents
 - recognition and use of archetypes
 - use of problem solving tools

TYPES FUNDAMENTAL TO STATISTICAL THINKING (Foundations)

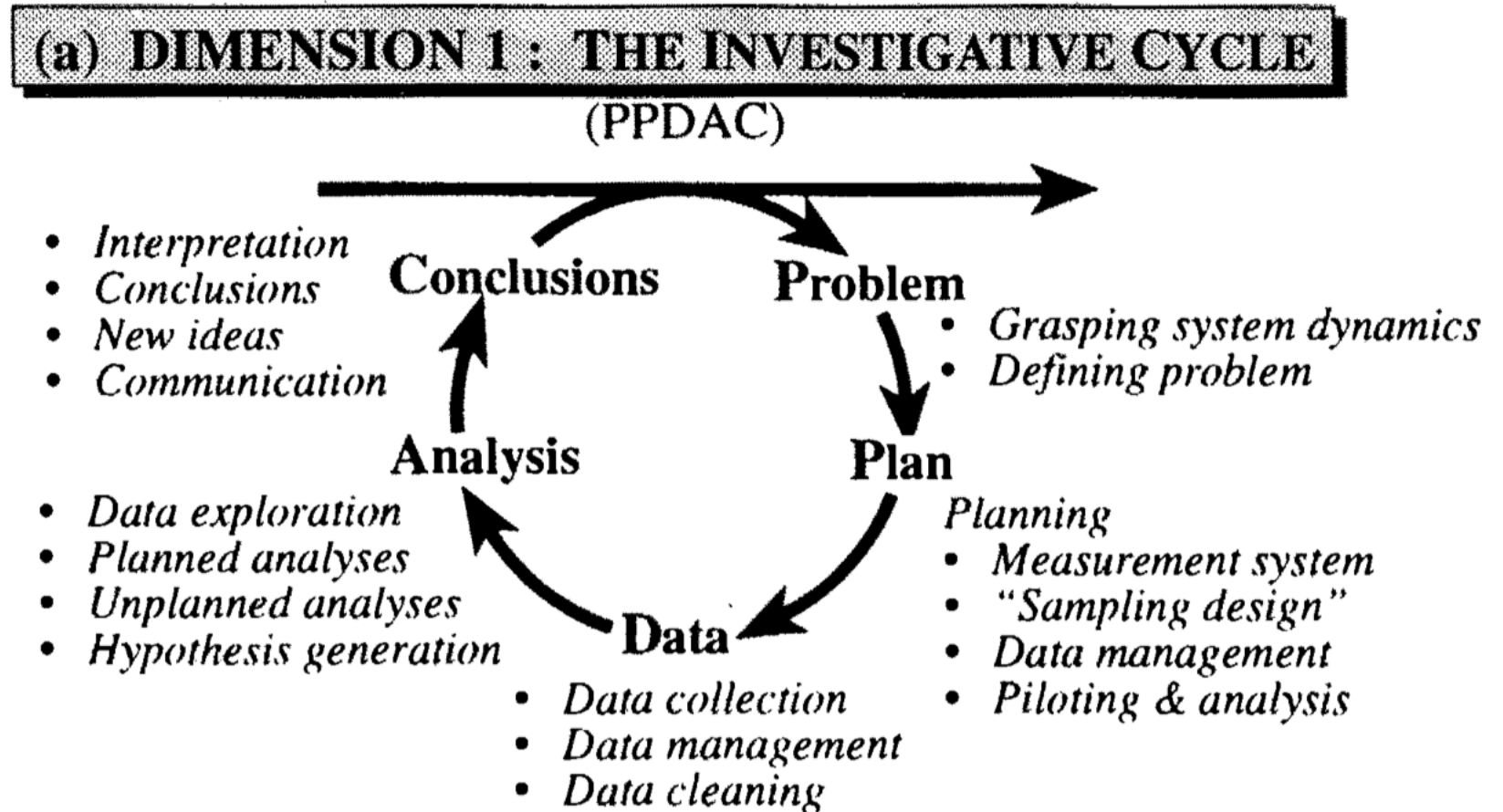
- Recognition of need for data**
- Transnumeration**
 - (Changing representations to engender understanding)
 - capturing "measures" from real system
 - changing data representations
 - communicating messages in data
- Consideration of variation**
 - noticing and acknowledging
 - measuring and modelling for the purposes of prediction, explanation, or control
 - explaining and dealing with
 - investigative strategies
- Reasoning with statistical models**
- Integrating the statistical and contextual**
 - information, knowledge, conceptions

(d) DIMENSION 4 : DISPOSITIONS

- Scepticism**
- Imagination**
- Curiosity and awareness**
 - observant, noticing
- Openness**
 - to ideas that challenge preconceptions
- A propensity to seek deeper meaning**
- Being Logical**
- Engagement**
- Perseverance**

Dimension 1: The Investigative Cycle

- Concerned with abstracting and solving a statistical problem grounded in a larger "real" problem
- Based on the PPDAC model (Problem, Plan, Data, Analysis, Conclusions)



Dimension 2: Types of Thinking

- **Variation**
 - Thinking which is statistical is concerned with learning and decision making under uncertainty
 - for the purposes of explanation, prediction, or control

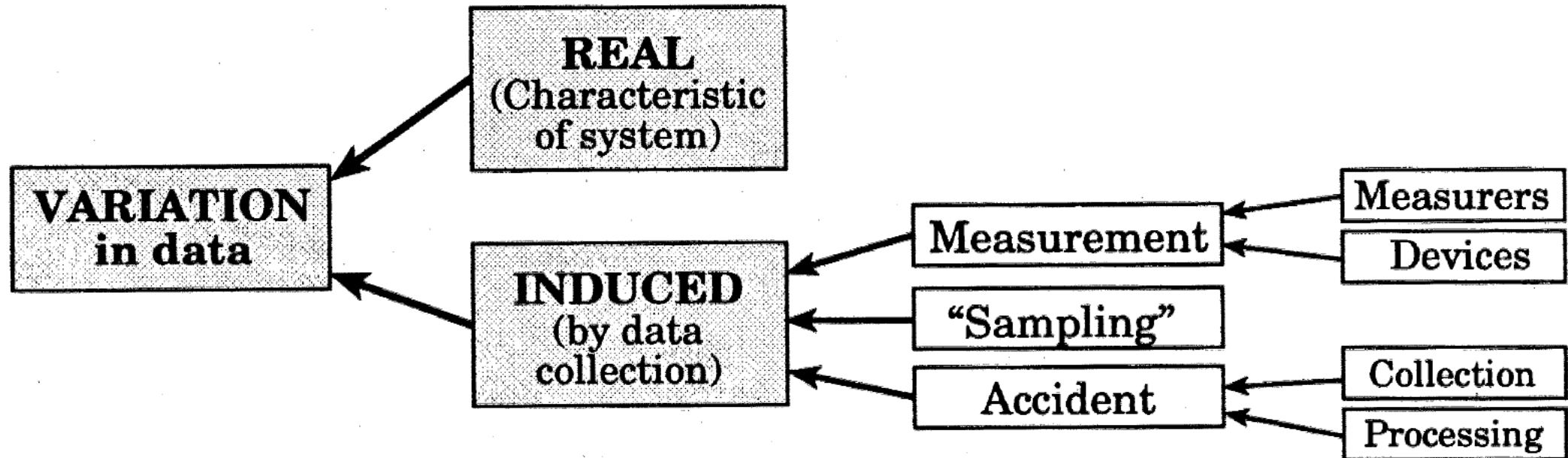
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TYPES FUNDAMENTAL TO *STATISTICAL* THINKING (Foundations)

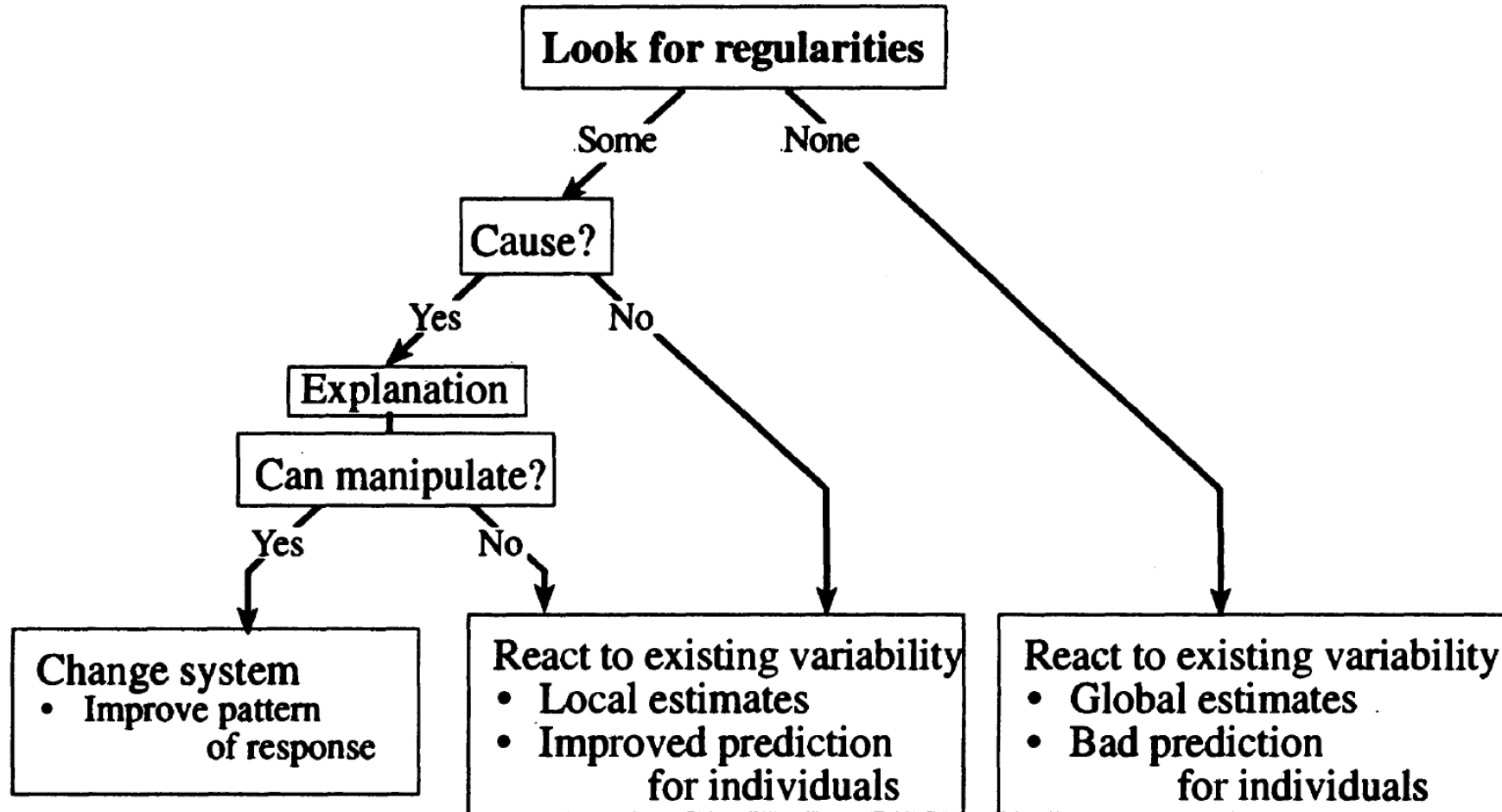
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Dimension 2: More on Variation | Sources



Dimension 2: More on Variation |

Prediction, Explain, Control

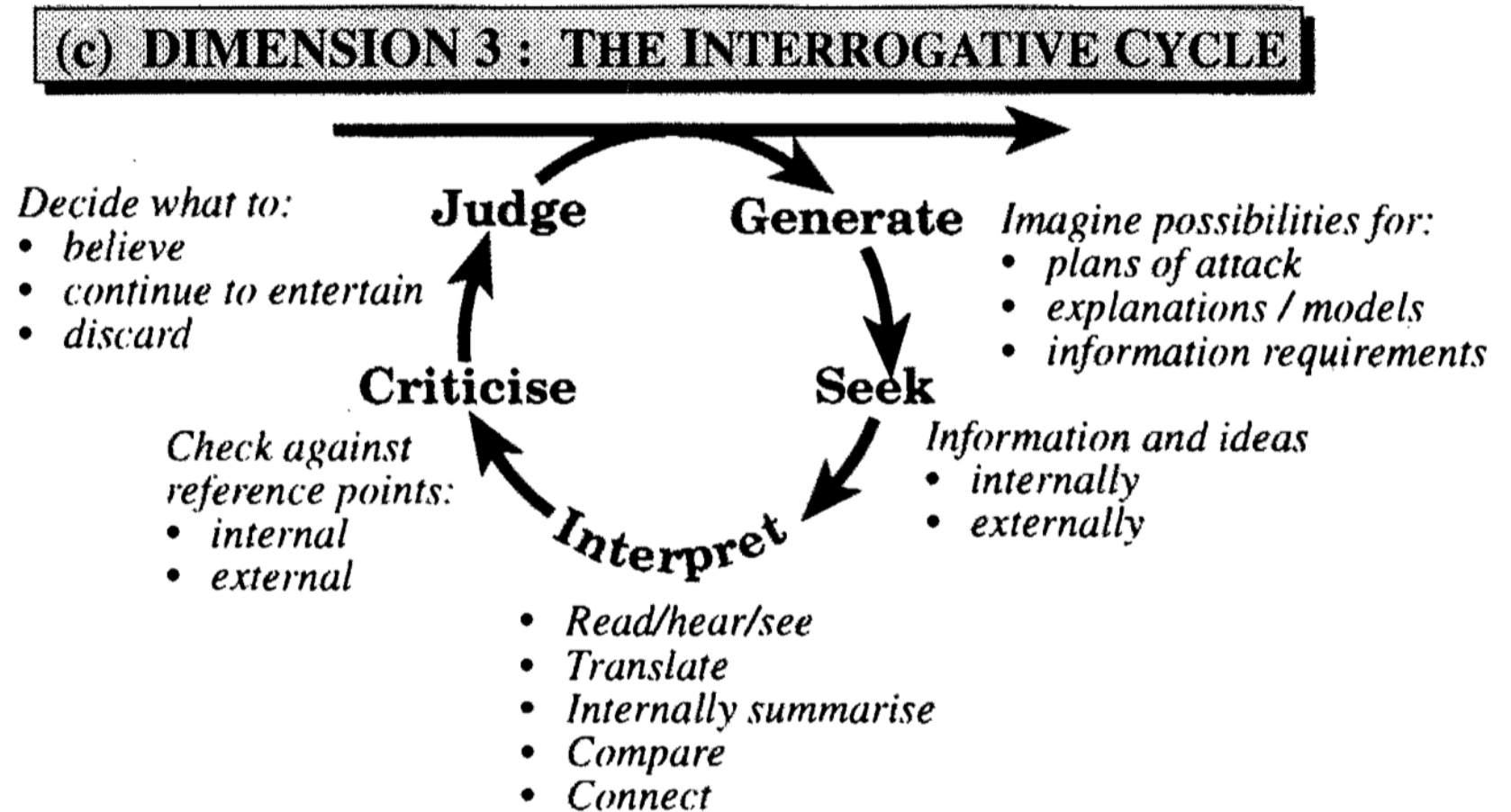


Dimension 2: Summary on Variation

- **Special-cause** vs. **common cause** variation
 - Useful when looking for causes
- **Explained** vs. **unexplained** variation
 - Useful when exploring data & building a model for them
- Suppositions
 - Variation is an **observable reality**
 - Some variation can be explained; other variation **cannot** be explained on current knowledge
 - **Random** variation is the way in which statisticians model unexplained variation
 - This unexplained variation may in part or in whole be produced by the process of observation through **random sampling**
 - Randomness is a **convenient** human construct which is used to deal with variation in which patterns cannot be detected

Dimension 3: The Interrogative Cycle

- Applies at macro levels
- Applies also at very detailed levels of thinking
 - **Recursive**
 - Subcycles are initiated within major cycles



Dimension 4: Dispositions

- When authors become intensely interested in a problem or are, *a heightened sensitivity and awareness* develops towards information on the peripheries of our experience that might be related to the problem
 - *People are most observant in areas they find most interesting*
- **Engagement** intensifies each dispositional element

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Types of Analytics

- **Descriptive**

- Describing characteristics or properties in the data

- **Predictive**

- Predicting the types of outcomes given new sets of data, usually based on a classifier trained using labelled, existing datasets

- **Prescriptive**

- Deciding on the best route or option or decision to make given data

Types of Data

- **Categorical** (cf. wikipedia)

- Variable that can take on one of a limited, and usually fixed number of possible values, assigning each individual or other unit of observation to a particular group or nominal category on the basis of some qualitative property
- The blood type of a person: A, B, AB or O
- The state that a person lives in
- The political party that a voter might vote for
- The type of a rock: igneous, sedimentary or metamorphic
- **Ordinal** data?

- **Numerical**

- Can be subdivided into discrete data (things that can be counted) and continuous data (all possible numbers).
- # of children, age, scores, temperatures, etc.

Descriptive Statistics

- There are three main groups of descriptive statistics
- The **distribution**
 - Works well with categorical data. How many of each thing is there?
- The **central tendency**
 - Only works with numerical data. What is the mean, median and mode?
- The **dispersion**
 - Only works with numerical data. How spread out is the data?

Descriptive Statistics: Distribution

- Grouping and counting by categorical data – group and count by town, or zip code or something like that
 - Often called a **frequency distribution**
 - **Histogram**
- With numerical data, **minimum** and **maximum** values are useful

Descriptive Statistics: Central Tendency

- **Mean**

- ***Average or norm:*** add up all values to find a total, and then divide the total by the number of values

- **Median**

- ***Middle value:*** Sort all values into order, and the median is the middle value; if there are 2 values in the middle, find the mean of these two

- **Mode**

- ***Most frequent value:*** Count how many each value appears, the mode is the value that appears the most
- Can have more than one mode

Descriptive Statistics: Dispersion

- **Range**

- Difference between the lowest and highest values
- Subject to extremes (e.g., ***outliers***)

- **Standard deviation**

- It is the relation that a set of scores has to the mean
- Subject to ***skewness*** in distribution

- For a Gaussian/normal distribution

- 68% of all values will be within 1 standard deviation
- 95% will be within 3 standard deviation

Dirty Data

- **Missing** data
 - Blanks in the database or spreadsheet.
 - Data missing from a period of time.
 - Missing states, counties, zip codes.
- **Wrong** data
 - Wrong type – numbers where they should be text and vice versa
 - Sharp curves – trends that continue normally that suddenly jump in one year
 - Conflicting data within a dataset or across datasets (race, percentages, etc)
- **Unusable** data
 - Non-standardized data
 - Inconsistent data
 - Abbreviations
 - Unit consistency

Correlation

- Pearson correlation coefficients (or Pearson product-moment correlation coefficient)
- It is a measure of how LINEARLY related two entities are.
- How often is a change in A related to a change in B? And is that positive or negative?

Correlation: For a population

$$\rho_{X,Y} = \frac{\mathbb{E}[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

where:

- cov and σ_X are defined as above
- μ_X is the **mean** of X
- μ_Y is the **mean** of Y
- $\mathbb{E}$ is the **expectation**.

Standard deviation of X ; standard deviation of Y

The formula for ρ can be expressed in terms of uncentered moments. Since

- $\mu_X = \mathbb{E}[X]$
- $\mu_Y = \mathbb{E}[Y]$
- $\sigma_X^2 = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - [\mathbb{E}[X]]^2$
- $\sigma_Y^2 = \mathbb{E}[(Y - \mathbb{E}[Y])^2] = \mathbb{E}[Y^2] - [\mathbb{E}[Y]]^2$
- $\mathbb{E}[(X - \mu_X)(Y - \mu_Y)] = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X] \mathbb{E}[Y],$

Correlation: For a sample

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

where:

- n is the sample size
- x_i, y_i are the individual sample points indexed with i
- $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ (the sample **mean**); and analogously for $\bar{y}$

Correlation: What it means?

- It is based on a range from -1 to 1.
- **1 = perfect positive correlation**
 - A goes up 1, B goes up 1
 - In the real world, almost never happens outside of a mistake
- **0 = no correlation at all**
 - 0 rarely ever happens
 - NEAR zero happens all the time
- **-1 = perfect negative correlation**
 - A goes up 1, B goes down 1
 - It is just like 1: rare, probably a mistake

Correlation is NOT causation

Significance: *t*-test

- The ***t*-test** is any statistical hypothesis test in which the test statistic follows a Student's *t*-distribution under the ***null*** hypothesis.
- A *t*-test is most commonly applied when the test statistic would follow a ***normal*** distribution if the value of a scaling term in the test statistic were known
 - When the scaling term is unknown and is replaced by an estimate based on the data, the test statistics (under certain conditions) follow a Student's *t* distribution
 - ***The t-test can be used, for example, to determine if two sets of data are significantly different from each other***

Significance: p -value & null hypothesis

- In the context of null hypothesis testing: to quantify the idea of statistical significance of evidence
 - In essence, *a claim is assumed valid if its counter-claim is improbable*
- The only hypothesis that needs to be specified in this test and which embodies the counter-claim is referred to as the ***null hypothesis***
 - i.e., the hypothesis to be nullified
- A result is said to be ***statistically significant*** if it allows us to ***reject*** the null hypothesis
 - *The statistically significant result should be highly improbable if the null hypothesis is assumed to be true*
 - *The rejection of the null hypothesis implies that the correct hypothesis lies in the logical complement of the null hypothesis*
- ***Caveat***: Unless there is a single alternative to the null hypothesis, the rejection of null hypothesis does *not* tell us which of the alternatives might be the correct one