CSCE 936: Cyber-Physical Systems: Design a Controller - Part II (HW #3)

Assigned: 2019-10-03
Due: 2019-10-24 upload to Canvas before class

Homework Sequence Overview

This is really part 2 of a multi-part homework sequence. There are multiple objectives of this homework sequence:

- Inform me about your strengths and weaknesses in designing a controller from beginning to end
- Discover for yourself your strengths and weaknesses in designing a controller from beginning to end

Note that I **do not** expect you to have all the tools and background to do each part of the sequence. I want you to work hard and **attempt** each part of the sequence. If you cannot complete a question entirely tell me what process you used to try and figure it out and show me what you were able to accomplish. Also, on this homework sequence you may collaborate with others if it will help. Also feel free to use Google, and any other resources at your disposal.

Problem

The CPS we will be designing in this homework is control of a Planar VTOL (vertical take-off and landing) or a multicopter. A sketch of the system can be seen in Figure 1.

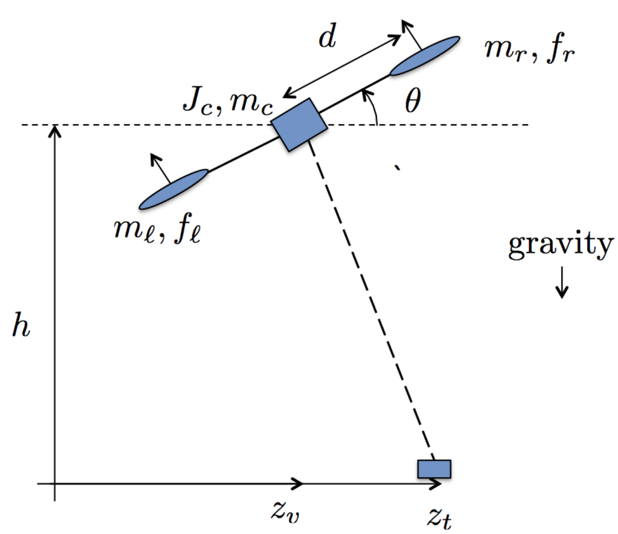

Figure 1: Planar VTOL

In this design study we will explore the control design for a simplified planar version of a quadrotor following a ground target. In particular, we will constrain the dynamics to be in a two dimension plane comprising vertical and one dimension of horizontal, as shown in Figure 1. The planar vertical take-off and landing (VTOL) system is comprised of a center pod of mass m_c and inertia J_c, a right motor/rotor that is modeled as a point mass m_r that exerts a force f_r at a distance d from the center of mass, and a left motor/rotor that is modeled as a point mass m_l that exerts a force f_l at a distance $-d$ from the center of mass. The position of the center of mass of the planar VTOL system is given by horizontal position z_v and altitude h. The airflow through the rotor creates a change in the direction of flow of air and causes what is called "momentum drag." Momentum drag can be modeled as a viscous drag force that is proportional to the horizontal velocity $\dot{z}_v$. In other words, the drag force is $F_{drag} = -\mu \dot{z}_v$. The target on the ground will be modeled as an object with position z_t and altitude $h = 0$. We will not explicitly model the dynamics of the target.

Use the following physical parameters: $m_c = 1\,\text{kg}$, $J_c = 0.0042\,\text{kg}\,\text{m}^2$, $m_r = 0.25\,\text{kg}$, $m_l = 0.25\,\text{kg}$, $d = 0.3\,\text{m}$, $\mu = 0.1\,\text{kg/s}$, $g = 9.81\,\text{m/s}^2$.

HW #4 Problems

Consider only the linear state-space equations we derived in HW #2. You can assume full state feedback. That is

$$\mathbf{y} = \mathbf{C}\mathbf{x}$$
$$\mathbf{C} = \mathbb{I}_{n\times n}$$

where n is the number of states in the state vector $\mathbf{x}$. Complete the following:

1. **(10 points)** Ensure the system is controllable. In your writeup describe how you drew your conclusion.
2. **(40 points)** Design a hover controller for the quadrotor by designing an LQR controller for both the longitudinal and lateral systems
   (a) Put the values of $\mathbf{Q}$ and $\mathbf{R}$ in your writeup, and the gain matrix $\mathbf{K}$ for each
   (b) Examine Bode plots and write a couple sentences about your analysis of the plots for the LQR controller. Does it have good margins of stability?
   (c) Provide a disturbance response (nonzero initial conditions) plot in your writeup. Show a plot of all the states for both lateral and longitudinal systems, as well as the control effort $\tilde{u}_{lat}$ and $\tilde{u}_{lon}$.
      i. Hint: don't forget that $\tilde{u}_{lon}$ has a F_e component in it that you must add to the controller design. That's what keep the quadrotor flying remember!
3. **(40 points)** Design a tracking controller for the lateral system (you will use the LQR controller to hold altitude)
   (a) You can use any control strategy you like (e.g. integral control, PID, feedforward, precompensation, etc.)
      i. Hint: feedforward with LQR is probably the easiest, though PID would be a good choice as well.
   (b) Put your design in your writeup
   (c) Provide plots for
      i. The quadrotor moving to a waypoint 3 meters away in the lateral direction. You need only show the position state
      ii. The quadrotor tracking an arbitrary signal (like a $\sin(t)$ wave). Can it track this?
      iii. Examine Bode plots and write a couple sentences about your analysis of the plots for the LQR controller. Does it have good margins of stability?
4. **(10 points)** Write a couple paragraphs on which parts were easy, which parts were hard and what you need to learn going forward to design this system well.

What to Submit

Please submit the following, on Canvas, by the specified time above:

1. **(100 points)** A PDF document with your answer to the questions above. I understand that many of you have not been exposed to this type of material, so this may be difficult. I expect you to make a reasonable effort for full credit. So convince me. Please observe the following:

    - Homework must be typed