

# CSCE 936: Cyber-Physical Systems: HW #5

Assigned: 2018-11-08  
Due: 2018-11-29 upload to Canvas before class

## Problems

### Timed Automata

For each of the following systems construct a timed automaton that accepts only valid real-time executions. For each automaton, you can assume an alphabet  $\Sigma = \{a, b, c, a', b', c'\}$  where  $a$ ,  $b$ , and  $c$  represent the arrival of real-time computing tasks  $a$ ,  $b$ , and  $c$  respectively, while  $a'$ ,  $b'$ , and  $c'$  represent the completion of tasks  $a$ ,  $b$ , and  $c$ . You may assume that upon initialization no tasks are awaiting completion. Also, you can only “complete” a task after it has “arrived.” Specify each solution as a timed automaton graph (no formal machine need otherwise be defined), but be careful to define the meaning of each timer/counter in your text as well as defining all counter resets and constraints in your graphs.

1. **(25 points)** Define a timed automaton that imposes an execution deadline of 30 seconds for each of the three independent tasks. Note that “deadline” is defined as the maximum delay between task arrival and completion.
2. **(25 points)** Now define a timed automaton that imposes no delay constraints on task  $c$  but that requires task  $a$  to be completed at least 5 seconds before task  $b$  is completed when both are simultaneously active (i.e., both have arrived but neither has executed). Impose the same 30 second deadline for tasks  $a$  and  $b$  as was used in #1 above.

### Hybrid Automata

1. **(50 points)** A high-Isp electric propulsion system enables a spacecraft to survive long-term with minimal fuel. This propulsion system is “bang-bang” (on/off) and can be turned on long-term as needed, providing a low-magnitude force  $F$  to the spacecraft when on and 0 when off. The force,  $F$ , is in the direction of forward velocity (i.e. tangential velocity). Model drag by a simple squared relationship with total velocity (i.e. tangential and radial), and presume constant drag coefficient  $c_d$  throughout the designated altitude interval. Remember that drag should be a force in the direction opposite that of the velocity. Use Lagrangian mechanics to develop the equations of motion. Define, formally and graphically (that is, in both set notation and as a graph), a hybrid automaton to maintain a spacecraft’s circular orbit between altitudes  $h_{\min}$  and  $h_{\max}$ .

### Hints:

- Define  $\mathbf{q} = [r \ \dot{\theta}]$  where  $r = h + R$  is the radius of the orbit (i.e.  $h$  is the altitude of the spacecraft above the surface of the Earth, and  $R$  is the radius of the Earth).
- Let  $\mathbf{v} = [\dot{r} \ r\dot{\theta}]$  be the velocity of the spacecraft. Then the kinetic energy is  $K(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}m\mathbf{v}^\top \mathbf{v}$
- The gravitational potential energy (not near Earth) is  $P(\mathbf{q}) = -\frac{GMm}{r}$  where  $G$  and  $M$  are the gravitational constant and mass of the Earth respectively, and  $m$  is the mass of the spacecraft.
- For the right hand side of the Lagrangian you need to provide generalized forces, torques, frictions, etc. In this case the forces are gravity (be sure to use the gravitational force, not  $g$ ), the force from the spacecraft,  $F$ , and drag proportional to the squared velocity in each direction.

## What to Submit

Please submit the following, on Canvas, by the specified time above:

1. **(100 points)** A PDF document with your plots, graphs, and answers to the problems above. Please observe the following:

- Homework must be typed
- Protips:
  - Typesetting automata in L<sup>A</sup>T<sub>E</sub>X becomes fairly easy with the “automata” part of the tikz library of the tikz package.
  - You could also use PowerPoint or some other drawing tool and paste images in your document.