Clause Learning and Intelligent Backtracking in MiniSAT

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CSCE 235H Introduction to Discrete Structures (Honors)
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All questions: Piazza

Clause Learning

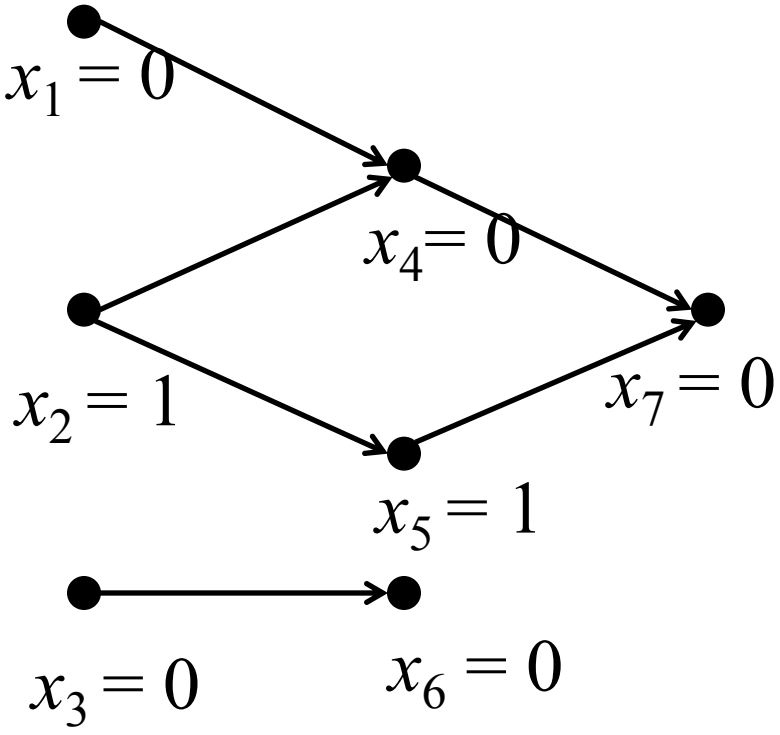
- At every conflict, determine the cause of the conflict
- Create a new clause to prevent the conflict from being reached in the future
- Tools
  - Implication graph to determine cause of conflicts
  - Added clause is a “learnt” no-good

Implication Graph (1)

- Nodes correspond to assignments
- Nodes with no incoming edges are decision variables (assignments)
- Nodes with incoming edges were assigned through propagation

$$x_1 \leftarrow 0$$
$$x_2 \leftarrow 1$$
$$x_3 \leftarrow 0$$

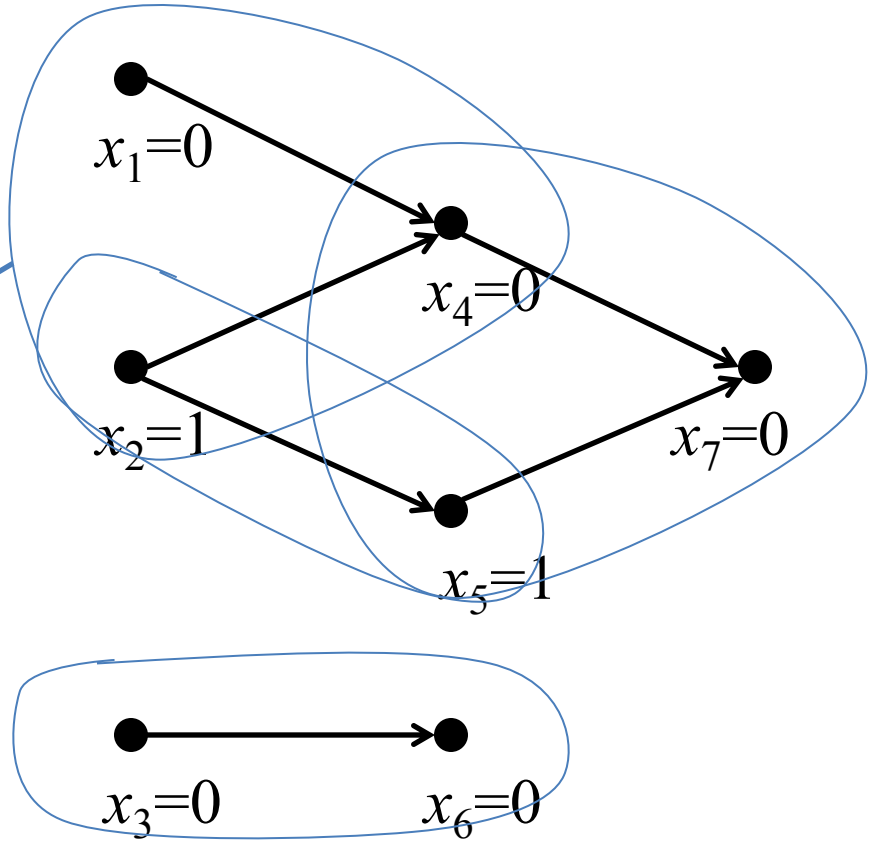
$$(x_1 \vee \neg x_2 \vee \neg x_4) \wedge (x_4 \vee \neg x_5 \vee \neg x_7) \wedge (x_3 \vee \neg x_6) \wedge (\neg x_2 \vee x_5)$$



Implication Graph (2)

- A node and its immediate predecessors correspond to a clause

$$(x_1 \vee \neg x_2 \vee \neg x_4)$$



Trail

- Series of assignments made up to current point in search
- Broken up by ‘decision levels’
- Each decision level includes propagations

Decision level	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_4 = 0$
	$x_5 = 1$
	$x_7 = 0$
3	$x_3 = 0$
	$x_6 = 0$

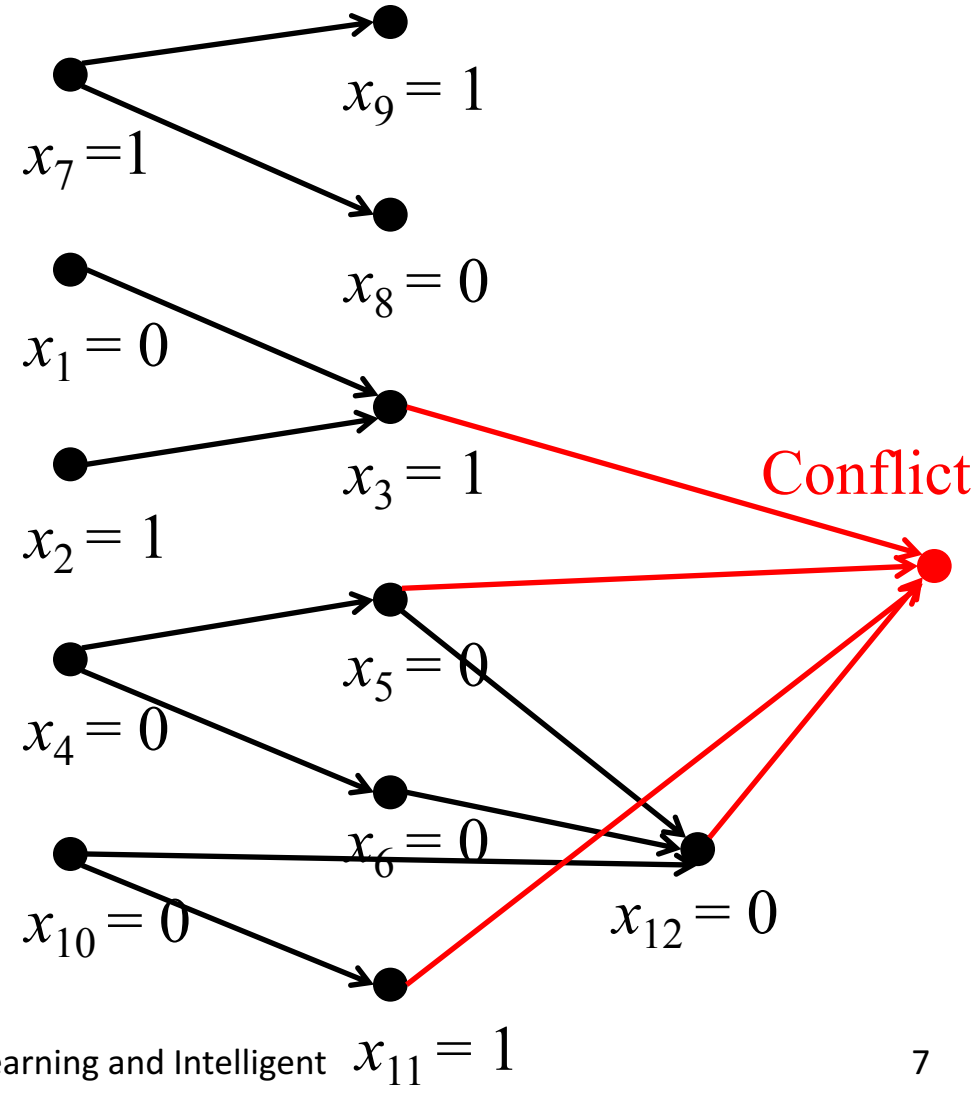
Clause Learning Example (1)

DL	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$x_{10} = 0$
	$x_{11} = 1$
	$x_{12} = 0$

$$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$$
$$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$$
$$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_3) \wedge$$
$$(\neg x_7 \vee x_9) \wedge$$
$$(\neg x_7 \vee \neg x_8) \wedge$$
$$(x_4 \vee \neg x_5) \wedge$$
$$(x_4 \vee \neg x_6) \wedge$$
$$(x_{10} \vee x_{11}) \wedge$$
$$(\neg x_{10} \vee x_{11})$$

Clause Learning Example (2)

DL	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$x_{10} = 0$
	$x_{11} = 1$
	$x_{12} = 0$



Clause Learning Example (3)

Decision variables

Conflict caused by assignment

$$(\neg x_1 \wedge x_2 \wedge \neg x_4 \wedge \neg x_{10})$$

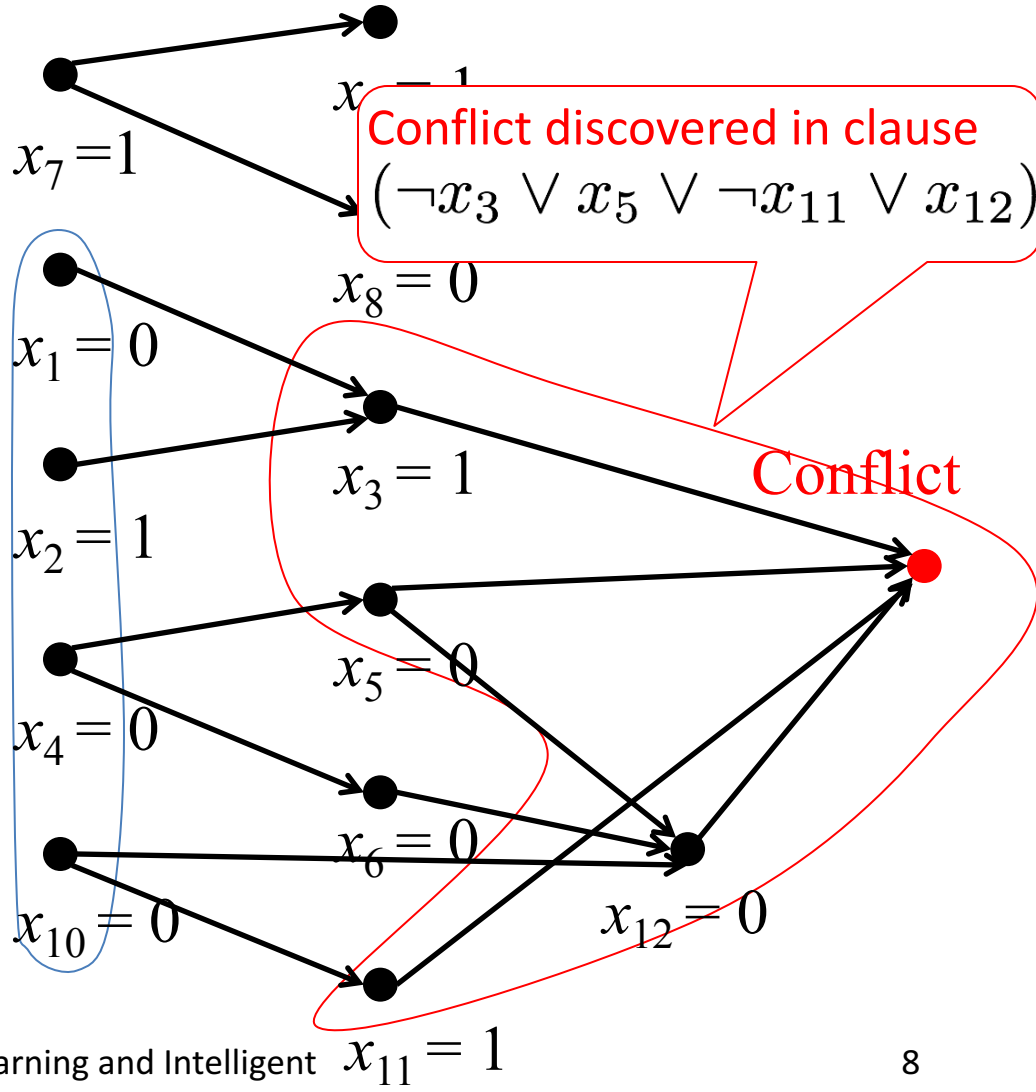
So, we negate it

$$\neg(\neg x_1 \wedge x_2 \wedge \neg x_4 \wedge \neg x_{10})$$

.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$$

to the formula to prevent this conflict in the future



Intelligent Backtracking

- When reaching a conflict, we
  - Consider conflicted clauses
  - Draw the implication graph
  - Identify the decision variables
  - Generate the learnt no-good
  - Add learned clause to the formula
- Undo assignments until the learned clause becomes a unit clause

Intelligent Backtracking Example (1)

DL	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$\boldsymbol{x_{10} = 0}$
	$x_{11} = 1$
	$x_{12} = 0$

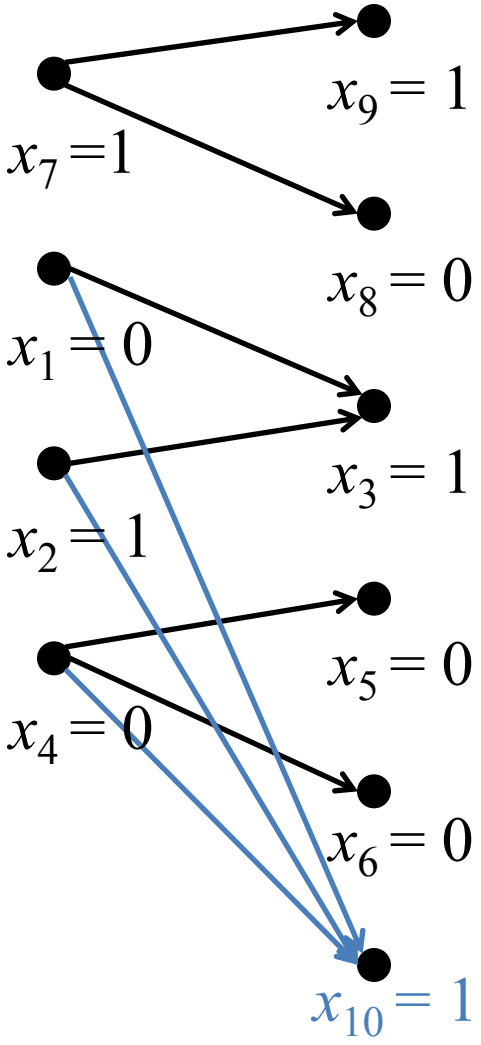
$$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$$
$$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$$
$$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_3) \wedge$$
$$(\neg x_7 \vee x_9) \wedge$$
$$(\neg x_7 \vee \neg x_8) \wedge$$
$$(x_4 \vee \neg x_5) \wedge$$
$$(x_4 \vee \neg x_6) \wedge$$
$$(x_{10} \vee x_{11}) \wedge$$
$$(\neg x_{10} \vee x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$$

x_{10} deepest decision variable

Intelligent Backtracking Example (2)

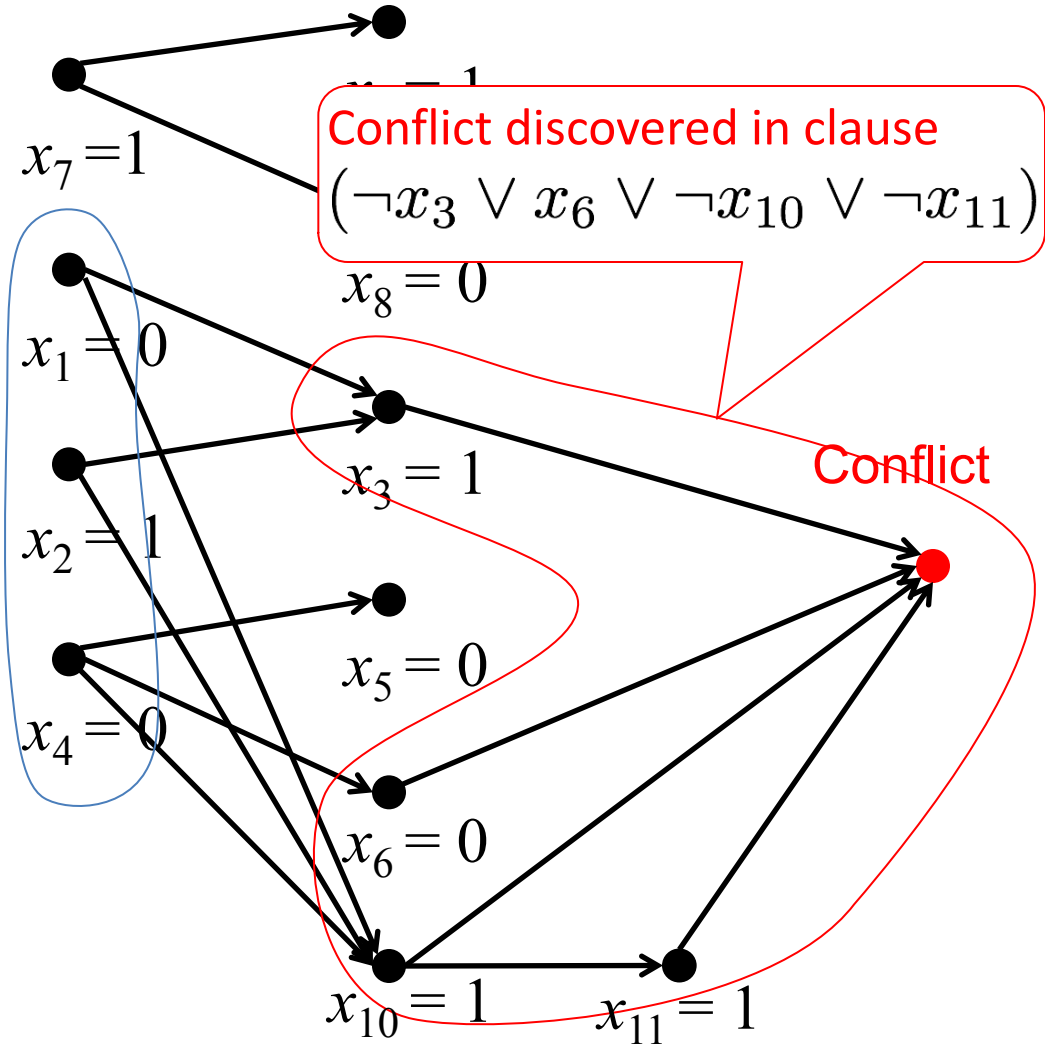
DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$x_{10} = 0$
	$x_{11} = 1$
	$x_{12} = 0$

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
	$\boldsymbol{x_{10} = 1}$



Intelligent Backtracking Example (3)

Conflict caused by these decision variables



.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4)$$

to the formula to prevent this conflict in the future

Intelligent Backtracking Example (4)

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
	$x_{10} = 1$

$$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$$
$$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$$
$$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_3) \wedge$$
$$(\neg x_7 \vee x_9) \wedge$$
$$(\neg x_7 \vee \neg x_8) \wedge$$
$$(x_4 \vee \neg x_5) \wedge$$
$$(x_4 \vee \neg x_6) \wedge$$
$$(x_{10} \vee x_{11}) \wedge$$
$$(\neg x_{10} \vee x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4)$$

Intelligent Backtracking Example (5)

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
	$x_{10} = 1$

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 1$

$$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$$
$$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$$
$$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_3) \wedge$$
$$(\neg x_7 \vee x_9) \wedge$$
$$(\neg x_7 \vee \neg x_8) \wedge$$
$$(x_4 \vee \neg x_5) \wedge$$
$$(x_4 \vee \neg x_6) \wedge$$
$$(x_{10} \vee x_{11}) \wedge$$
$$(\neg x_{10} \vee x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4)$$

Summary

- Search
  - Assign variable, Propagate
  - Detect conflict? Intelligent backtracking
- Intelligent backtracking
  - Identify decision variables source of conflict
  - Add no-good clause so conflict cannot arise in the future
  - Backtrack the deepest variables in the learnt clause
  - Flip assignment of deepest variable in learnt clause
  - Proceed
- Do you see any problem in this strategy?