Name(s) __________________________  CSE Login __________________________

Instructions Follow instructions carefully, failure to do so may result in points being deducted. Clearly label each problem and submit the answers in order. Print out a copy of this cover sheet and staple it to the front of your assignment for grading. Be sure to show sufficient work to justify your answer(s). When asked to prove something, you must give a formal, rigorous, and complete proof. The CSE academic dishonesty policy is in effect (see https://cse.unl.edu/academic_integrity). You are highly encouraged to typeset your assignment using \LaTeX; if your answers are not legible, you may be required to use \LaTeX in future assignments.

Partner/Group Policy The same policy as your semester project is in effect.

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1. **8 points** Let \( R \) be a relation on a set \( A = \{a_1, \ldots, a_n\} \) of size \( n \). Let \( M_R \) be the 0-1 matrix representing \( R \) (i.e., the entry \( m_{ij} = 1 \) if \((a_i, a_j) \in R\) and zero otherwise).

(a) How many unique relations are there on \( A \) (in terms of \( n \))?  
(b) The *complement* relation is defined as  
\[
\overline{R} = \{(a, b) \mid (a, b) \not\in R\}
\]

Say that the number of nonzero entries in \( M_R \) (that is, the number of 1s) is \( k \). How many nonzero entries are there in \( M_{\overline{R}} \)? Briefly justify your answer.

(c) How many reflexive relations are there on a set of size \( n \)? Briefly justify your answer.

(d) How many symmetric relations are there on a set of size \( n \)? Briefly justify your answer.

2. **5 points** Prove the following. A relation \( R \) is asymmetric if and only if \( R \) is irreflexive and antisymmetric. Note: a relation \( R \) on the set \( A \) is *irreflexive* if for every \( a \in A \), \((a, a) \not\in R\). That is, \( R \) is irreflexive if no element in \( A \) is related to itself.

Hint: write the definition of what it means to be asymmetric, then “add” the contradiction:  
\[
((a = b) \land (a \neq b))
\]

3. **2 points** Prove or disprove: if \( R \) is antisymmetric, then \( R \) is asymmetric

4. **65 points** **Programming Assignment** – In this programming assignment, you will implement a Java class, `Relation` that will model relations on a set. The skeleton class is available for download on Blackboard. You must correctly implement all of the methods as described in their javadoc comments.

How you choose to internally represent relations in your class is a design decision that is completely up to you. You may also add any other convenience methods that you may find useful, but we will only be testing the ones available in the skeleton class. Note, that as before this class is parameterized on a type (using Java generics). You will need to ensure that any helper methods that you do implement will follow that same convention.