Instructions Follow instructions carefully, failure to do so may result in points being deducted. Clearly label each problem and submit the answers in order. Print out a copy of this cover sheet and staple it to the front of your assignment for grading. Be sure to show sufficient work to justify your answer(s). When asked to prove something, you must give a formal, rigorous, and complete proof. The CSE academic dishonesty policy is in effect (see https://cse.unl.edu/academic_integrity). You are highly encouraged to typeset your assignment using \LaTeX; if your answers are not legible, you may be required to use \LaTeX in future assignments.

Partner/Group Policy The same policy as your semester project is in effect.

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1. 5 points For each of the following sets, 1) determine whether there is a one-to-one function \( f : S \to T \); 2) determine whether there is an onto function \( f : S \to T \); and 3) determine if there is a bijective function \( f : S \to T \). For each (1–3) if such a function exists, explicitly give it. If no function exists give a short explanation of why.

(a) \( S = \{1, 2, 3\}, T = \{a, b, c\} \)
(b) \( S = \{a, b\}, T = \{1, 2, 3, 4\} \)
(c) \( S = \{1, 2, 3, 4\}, T = \{a, b\} \)

2. 14 points Let \( S = \{1, 2\} \) and \( T = \{a, b, c\} \).

(a) How many unique functions are there mapping \( S \to T \)?
(b) How many unique functions are there mapping \( T \to S \)?
(c) How many onto (surjective) functions are there mapping \( S \to T \)?
(d) How many onto (surjective) functions are there mapping \( T \to S \) (hint: think of how many non onto functions there are)?
(e) How many one-to-one (injective) functions are there mapping \( S \to T \)?
(f) How many one-to-one (injective) functions are there mapping \( T \to S \)?
(g) Let \( f : S \to T \), is it possible to define \( f^{-1} \)? Why or why not?

3. 8 points Determine whether each of the functions below is onto, and/or one-to-one for \( f : \mathbb{Z} \to \mathbb{Z} \), prove your answers.

(a) \( f(x) = 5x - 3 \)
(b) \( f(x) = 2x^3 \)
(c) \( f(x) = (2x - 2)^2 \)
(d) \( f(x) = \sqrt[3]{x} \)

4. 10 points Determine whether each of the functions below is onto, and/or one-to-one for \( f : \mathbb{R} \to \mathbb{R} \), prove your answers.

(a) \( f(x) = 5x - 3 \)
(b) \( f(x) = 2x^3 \)
(c) \( f(x) = (2x - 2)^2 \)
(d) \( f(x) = \sqrt[3]{x} \)

5. 10 points Define the following functions (assume that the domains/codomains are defined such that each composition is valid): \( f(x) = 2x, g(x) = \frac{x}{(1 + x), h(x) = \sqrt{x}} \). Find

(a) \( f \circ g \circ h \)
(b) \( h \circ g \circ f \)
(c) \( f \circ f \)
(d) \( g \circ g \)

6. 8 points Find inverses of the following functions (assume that the domains/codomains are defined such that each function is a bijection).

(a) \( f(x) = 5x - 3 \)
(b) \( f(x) = 2x^3 \)
(c) \( f(x) = (2x - 2)^2 \)
(d) \( f(x) = \frac{\sqrt{x}}{2} \)

7. **5 points** Let \( f(x) \) and \( g(x) \) be two linear functions (a function is linear if \( f(x) = ax + b \) for \( a, b \in \mathbb{R} \)).

   a. Prove or disprove: \( f \circ g = g \circ f \).

   b. Prove or disprove: \( f \circ g \) and \( g \circ f \) are linear functions.