Instructions Follow instructions carefully, failure to do so may result in points being deducted. Clearly label each problem and submit the answers in order. Print out a copy of this cover sheet and staple it to the front of your assignment for grading. Be sure to show sufficient work to justify your answer(s). When asked to prove something, you must give a formal, rigorous, and complete proof. The CSE academic dishonesty policy is in effect (see https://cse.unl.edu/academic_integrity). You are highly encouraged to typeset your assignment using \LaTeX; if your answers are not legible, you may be required to use \LaTeX in future assignments.

Partner/Group Policy The same policy as your semester project is in effect.

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1. **5 points** In Java, every object has an **equals** and a **hashCode** method that is used, among other things, by the standard Collections library. By default both are based on an object’s memory location, but best practice dictates that they should be overridden to be dependant on the object’s state. The rule that should be followed when doing this is: if two objects are equal, then they should have the same hash code value. Which of the following situations would be permissible and which would not under this rule? Provide a brief justification for each.

(a) Two objects have the same hash code and are equal
(b) Two objects are equal but have different hash codes
(c) Two objects have the same hash code but are not equal
(d) Two objects have different hash codes and are not equal

2. **8 points** (Rosen 1.1.32) Construct a truth table for each of these compound propositions.

1. \( p \to \neg p \)
2. \( p \leftrightarrow \neg p \)
3. \( (p \land q) \to (p \lor q) \)
4. \( (q \to \neg p) \leftrightarrow (p \leftrightarrow q) \)

3. **5 points** (BoP 2.5.10) Suppose the statement
\[
((p \land q) \lor r) \to (r \lor s)
\]

is false. Without using a truth table, determine the truth values for \( p, q, r, s \).

4. **5 points** Prove or disprove: \( (p \land q) \to (q \to p) \) is a tautology.

5. **5 points** Prove that the contrapositive holds (without using a truth table), that is that the following holds:
\[
p \to q \equiv \neg q \to \neg p
\]

6. **5 points** (Rosen 1.3.18) Show that \( \neg (p \oplus q) \) and \( p \leftrightarrow q \) are logically equivalent without using a truth table.

7. **5 points** (Rosen 1.3.30) Show that \( (p \lor q) \land (\neg p \lor r) \to (q \lor r) \) is a tautology without using a truth table.

8. **7 points** (Rosen 1.4.12) Let \( Q(x) \) be the statement “\( x + 1 > 2x \)”. If the domain consists of all integers, what are these truth values? Briefly justify your answers.

1. \( Q(0) \)
2. \( Q(-1) \)
3. \( Q(1) \)
4. \( \exists x Q(x) \)
5. \( \forall x Q(x) \)
6. \( \exists x \neg Q(x) \)
7. \( \forall x \neg Q(x) \)

9. **5 points** Show that
\[
\forall x P(x) \lor \forall x Q(x) \neq \forall x [P(x) \lor Q(x)]
\]
By defining predicts \( P, Q \) over the same universe of discourse but for which each side of the expression above have different truth values.
10. 10 points Can a premise imply contradictory statements? Can two contradictory premises imply the same conclusion? Determine the answers to these questions by proving or disproving the following.
   (a) Prove or disprove: the following is a contradiction
   \[(p \rightarrow q) \land (p \rightarrow \neg q)\]
   (b) Prove or disprove: the following is a contradiction
   \[(p \rightarrow q) \land (\neg p \rightarrow q)\]

11. 5 points Use a proof by contrapositive to prove that if \(x + y \geq 2\) where \(x, y \in \mathbb{R}\), then \(x \geq 1\) or \(y \geq 1\).

12. 5 points (Rosen 1.7.26) Prove that if \(n\) is a positive integer, then \(n\) is even if and only if \(7n + 4\) is even.

13. 10 points (Rosen 1.7.32) Show that these statements about the real number \(x\) are equivalent: (i) \(x\) is rational, (ii) \(x/2\) is rational, (iii) \(3x - 1\) is rational. (Note: to show that three statements are equivalent it is enough to show that (i) \(\Rightarrow\) (ii) \(\Rightarrow\) (iii) \(\Rightarrow\) (i); why? Because of hypothetical syllogism!)