Instructions  Follow instructions carefully, failure to do so may result in points being deducted. Clearly label each problem and submit the answers in order. Staple this cover page to the front of your assignment for easier grading. Be sure to show sufficient work to justify your answer(s). If you are asked to prove something, you must give as formal, rigorous, and complete proof as possible. The CSE academic dishonesty policy is in effect (see http://www.cse.unl.edu/undergrads/academic_integrity.php).

You are highly encouraged to typeset your homework using LATEX; if your answers are not legible, you may be required to use LATEX in future assignments. Print out a copy of the cover sheet and include it with your homework.

Partner Policy  You may work in groups of at most two students. This is optional and you may work alone if you wish. If you opt to work in pairs, you must follow these guidelines:

1. You must work on all problems together. You may not simply partition the work between you.
2. You should not discuss problems with other groups or individuals beyond general questions.
3. Hand in only one hard copy (and possibly soft copy) under the first author’s name/cse login.

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1. 7 points (Rosen 2.1.10) Determine whether these statements are true or false.
   1. $\emptyset \in \{\emptyset\}$
   2. $\emptyset \in \{\emptyset, \{\emptyset\}\}$
   3. $\{\emptyset\} \in \{\emptyset\}$
   4. $\{\emptyset\} \in \{\{\emptyset\}\}$
   5. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
   6. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
   7. $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

2. 5 points Prove or disprove:
   \[ A \cup B = \overline{A} \cap B \]

3. 5 points Prove or disprove:
   \[ (A \setminus B) \cap B = \emptyset \]

4. 4 points Let $A = \{1, 2, 6\}, B = \{0, 3\},$ and $C = \{1, 3, 9\}$. Find the following
   (a) $A \times C$
   (b) $A \times B \times C$
   (c) $B \times B \times B$
   (d) What would the cardinality of $A \times B \times C \times B \times A$ be?

5. 10 points Let $S = \{1, 2\}$ and $T = \{a, b, c\}$.
   (a) How many unique functions are there mapping $S \rightarrow T$?
   (b) How many unique functions are there mapping $T \rightarrow S$?
   (c) How many onto (surjective) functions are there mapping $S \rightarrow T$?
   (d) How many onto (surjective) functions are there mapping $T \rightarrow S$ (hint: think of how many non onto functions there are)?
   (e) How many one-to-one (injective) functions are there mapping $S \rightarrow T$?
   (f) How many one-to-one (injective) functions are there mapping $T \rightarrow S$?
   (g) Let $f : S \rightarrow T$, is it possible to define $f^{-1}$? Why or why not?

6. 8 points For each of the following functions: prove whether or not they are onto and/or one-to-one
   (a) $f : \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = 3x + 8$
   (b) $f : \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = 3x^2 + 2x + 7$
   (c) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x^2 + 2x + 7$
   (d) $f : \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = 3x^3 + 7$

7. 6 points Define the following functions (assume that the domains/codomains are defined such that each composition is valid): $f(x) = 2x, g(x) = \frac{x}{(1+x)}, h(x) = \sqrt{x}$. Find
   (a) $f \circ g \circ h$
   (b) $h \circ g \circ f$
   (c) $f \circ f$
8. **6 points** Find inverses of the following functions (assume that the domains/codomains are defined such that each function is a bijection).
   
   (a) \( f(x) = 5x - 3 \)
   
   (b) \( f(x) = 2x^3 \)
   
   (c) \( f(x) = (2x - 2)^2 \)
   
   (d) \( f(x) = \sqrt{x^6} \)

9. **4 points** Let \( f(x) \) and \( g(x) \) be two linear functions (a function is linear if there exist \( a, b \in \mathbb{R} \) such that \( f(x) = ax + b \)).
   
   a. Prove or disprove: \( f \circ g = g \circ f \).
   
   b. Prove or disprove: \( f \circ g \) and \( g \circ f \) are linear functions.

10. **8 points** Let \( R \) be a relation on a set \( A = \{a_1, \ldots, a_n\} \) of size \( n \). Let \( M_R \) be the 0-1 matrix representing \( R \) (i.e., the entry \( m_{ij} = 1 \) if \( (a_i, a_j) \in R \) and zero otherwise).

   (a) How many unique relations are there on \( A \) (in terms of \( n \))?

   (b) The complement relation is defined as

   \[ \overline{R} = \{(a, b) \mid (a, b) \not\in R\} \]

   Say that the number of nonzero entries in \( M_R \) (that is, the number of 1s) is \( k \). How many nonzero entries are there in \( M_{\overline{R}} \)? Briefly justify your answer.

   (c) How many reflexive relations are there on a set of size \( n \)? Briefly justify your answer.

   (d) How many symmetric relations are there on a set of size \( n \)? Briefly justify your answer.

11. **4 points** Prove or disprove: if \( R \) is antisymmetric, then \( R \) is asymmetric.

12. **8 points** Consider the relation \( R \) on \( A = \{a_0, a_1, a_2, a_3\} \) in Figure 1. Indicate whether or not this relation is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, and transitive. Give a brief justification for each property.

   ![Figure 1: A relation corresponding to the input file](image)

13. **25 points** **Programming Assignment** – In this programming assignment, you will implement several algorithms dealing with sets. Specifically, you will implement the methods in the SetUtils Java class provided. Note: this class utilizes a Pair utility class also provided for you. You may add any helper methods you wish, but to ensure proper grading you may not change the signatures of the methods already given.
14. **50 points** Programming Assignment – In this programming assignment, you will implement a Java class, `Relation` that will model a relation $R$ on a set $A$. You must correctly implement all of the methods as described in their javadoc comments.

How you choose to internally represent relations in your class is a design decision that is completely up to you. You may also add any other convenience methods that you may find useful, but we will only be testing the ones available in the skeleton class. Note, that as before this class is parameterized on a type (using Java generics). You will need to ensure that any helper methods that you do implement will follow that same convention.