Instructions Follow all instructions carefully, failure to do so may result in points being deducted. Clearly label each problem and submit the answers in order. Print out a copy of this cover sheet and staple it to the front of your assignment for grading. Be sure to show sufficient work to justify your answers. When asked to prove something, you must give a formal, rigorous, and complete proof. The CSE academic dishonesty policy is in effect (see https://cse.unl.edu/academic-integrity). You are highly encouraged to typeset your assignment using \LaTeX; if your answers are not legible, you may be required to use \LaTeX in future assignments.

Partner Policy You have the option of working in pairs for this assignment. You always have the option of working alone if you chose. If you choose to work as a pair, you must adhere to these guidelines:

1. You must work on all problems together. You may not simply partition the work between you.

2. You should not discuss problems with other groups or individuals beyond general questions.

3. Hand in only one hard copy with both of your names and logins. The first author should hand in any soft copies of programs.

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1. **4 points** In Java, every object has an `equals` and a `hashCode` method that is used, among other things, by the standard Collections library. By default both are based on an object’s memory location, but best practice dictates that they should be overridden to be dependent on the object’s state. The rule that should be followed when doing this is: if two objects are equal, then they should have the same hash code value. Which of the following situations would be permissible and which would not under this rule? Provide a brief justification for each.

(a) Two objects have the same hash code and are equal
(b) Two objects are equal but have different hash codes
(c) Two objects have the same hash code but are not equal
(d) Two objects have different hash codes and are not equal

2. **6 points** Suppose that \( \neg p \rightarrow \neg q \) is known to be false. Give the truth values for

(a) \( p \land q \)
(b) \( p \lor q \)
(c) \( q \rightarrow p \)

3. **4 points** Indicate the order in which each of the logical operators in the following expression are evaluated. Rewrite the expression and add parentheses to make it clear what the order of operations is.

\[ p \land q \rightarrow r \leftrightarrow p \land q \rightarrow r \]

4. **3 points** How many rows appear in a truth table for each of these compound propositions?

(a) \( (q \rightarrow p) \land (p \rightarrow \neg q) \)
(b) \( (\neg p \lor \neg r) \lor (p \lor \neg s) \)
(c) \( (p \rightarrow r) \lor (\neg s \lor r) \lor (\neg u \rightarrow v) \)

5. **8 points** (Rosen 1.1.32) Construct a truth table for each of these compound propositions.

(a) \( p \rightarrow \neg p \)
(b) \( p \equiv \neg p \)
(c) \( (p \land q) \rightarrow (p \lor q) \)
(d) \( (q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q) \)

6. **8 points** All propositional statements can be written using only the logical connectives \( \neg \) and \( \lor \). Justify this fact by showing the following.

(a) Give a propositional statement that is equivalent to \( p \land q \) using only \( \neg \) and \( \lor \) and prove that they are equivalent.
(b) Give a propositional statement that is equivalent to \( p \leftrightarrow q \) using only \( \neg \) and \( \lor \) and prove that they are equivalent.

7. **9 points** The Sheffer Stroke is a logical connective denoted as \(|\) and is defined by the truth table in Table 1. It is equivalent to a logical NAND gate (negated and gate) and is interesting because all other logical connectives can be written in terms of only the Sheffer Stroke.

(a) Prove that \( \neg p \equiv p|p \)
(b) Prove that \( p \lor q \equiv (p|p)|(q|q) \)
(c) Find a proposition equivalent to \( p \rightarrow q \) using only the Sheffer Stroke
Table 1: Sheffer Stroke Truth Table

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8. 4 points (BoP 2.5.10) Suppose the statement

\[ ((p \land q) \lor r) \rightarrow (r \lor s) \]

is false. Without using a truth table, determine the truth values for \( p, q, r, s \).

9. 4 points Prove or disprove: \( (p \land q) \rightarrow (q \rightarrow p) \) is a tautology.

10. 3 points Prove that the contrapositive holds (without using a truth table), that is that the following holds:

\[ p \rightarrow q \equiv \neg q \rightarrow \neg p \]

11. 5 points Prove the following holds (that is, the following is a tautology) without using a truth table (recall that \( \Rightarrow \) is the same thing as \( \rightarrow \)).

\[ (p \rightarrow q) \Rightarrow [(q \rightarrow r) \rightarrow (p \rightarrow r)] \]

12. 5 points (Rosen 1.3.18) Show that \( \neg(p \oplus q) \) and \( p \leftrightarrow q \) are logically equivalent without using a truth table.

13. 5 points (Rosen 1.3.26) Show that \( \neg p \rightarrow (q \rightarrow r) \) and \( q \rightarrow (p \lor r) \) are logically equivalent without using a truth table.

14. 5 points (Rosen 1.3.30) Show that \( (p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r) \) is a tautology without using a truth table.

15. 7 points (Rosen 1.4.12) Let \( Q(x) \) be the statement “\( x + 1 > 2x \)”. If the domain consists of all integers, what are these truth values?
   - (a) \( Q(0) \)
   - (b) \( Q(-1) \)
   - (c) \( Q(1) \)
   - (d) \( \exists x Q(x) \)
   - (e) \( \forall x Q(x) \)
   - (f) \( \exists x \neg Q(x) \)
   - (g) \( \forall x \neg Q(x) \)

16. 5 points Goldbach’s conjecture states that “every even integer greater than 2 is the sum of two primes.” Identify a universe of discourse and quantified propositions to “translate” this statement into a logical proposition.

17. 5 points Show that

\[ \forall x P(x) \lor \forall x Q(x) \neq \forall x [P(x) \lor Q(x)] \]

By defining a universe of discourse and predicts \( P, Q \) over it such that one side of the expression evaluates to true and the other side evaluates to false.
18. **5 points** Many mathematic statements have the form of “A if and only if B”. To prove such statements, you generally need to show that if A is true then B is true and if B is true then A is true. However, it is not necessary to show that if A is false then B is false and if B is false, then A is false. This is because the second two conditions follow by exclusion. That is,

\[ p \iff q \equiv \neg p \iff \neg q \]

Prove the above equivalence.

19. **10 points** Can a premise imply contradictory statements? Can two contradictory premises imply the same conclusion? Determine the answers to these questions by proving or disproving the following.

(a) Prove or disprove: the following is a contradiction

\[ (p \to q) \land (p \to \neg q) \]

(b) Prove or disprove: the following is a contradiction

\[ (p \to q) \land (\neg p \to q) \]

20. **20 points** The logical connectives \( \land, \lor, \oplus, \to \) are binary operators: they have two operands, say \( p, q \). Each one had a unique truth table column.

(a) How many possible binary logical operators are there?

(b) Identify all possible logical binary operators by giving the truth table for each one. Identify those that have an established “name” or symbol (such as \( \land \) and \( \lor \)) or their negation.

21. **10 points (bonus)** Suppose you have a predicate \( P \) on \( n \) boolean variables,

\[ P(x_1, x_2, \ldots, x_n) \]

We say that a truth assignment of the variables \( x_1, \ldots, x_n \) satisfies \( P \) if, when evaluated, evaluates to true. For example, the predicate

\[ P(x_1, x_2, x_3) = (\neg x_1 \lor x_2 \lor \neg x_3) \land (\neg x_1 \lor \neg x_2) \land (\neg x_2 \lor x_3) \]

has 3 variables. The truth assignment \((0, 1, 1)\) satisfies \( P \) but the truth assignment \((1, 0, 1)\) does not. Give a high-level description of an algorithm that, given a predicate \( P \) on \( n \) variables, determines how many truth assignments satisfy \( P \). How many truth assignments must your algorithm evaluate before it reaches its answer?