Homework 4
Due on 25th Mar, 2005

Design and Analysis of Algorithms
(CSCE 423/823, Spring 2005)
URL: `http://www.cse.unl.edu/~vinod/823S05/index.html`
7th March, 2005

Name :

Student ID :

Course No : **423** **823**

Instructions

- Write your name and student ID on the space provided. Circle the course number in which you are registered. Return this sheet with your solutions.
- Clarity of presentation is utmost important. You should give a clear description of all your algorithms with an analysis of their time complexity.
- No collaboration with any one is permitted for CORE problems!. See the Instructor or the TA if you need additional guidance. You may discuss approaches to solving ADVANCED problems with your classmates. However, you must write up your own solutions in your own words and acknowledge any one with whom you collaborated.

Core Problems

1. (5 Points) Problem 25.2-8 from the text book (page number 635). Give an $O(VE)$-time algorithm for computing the transitive closure of a directed graph $G = (V, E)$.

2. (5 Points) Problem 26.1-5 from the text book (page number 650). For the flow network $G = (V, E)$ and flow f show nin Figure 26.1(b), find a pair of subsets $X, Y \subseteq V$ for which $f(X, Y) = -f(V - X, Y)$. Then, find a pair of subsets $X, Y \subseteq V$ for which $f(X, Y) \neq -f(V - X, Y)$.

3. (5 Points) Problem 26.2-2 from the text book (page number 663). Show the execution of Edmonds-Karp algorithm on the flow network of Figure 26.1(a).

4. (10 Points) Problem 26.2-9 from the text book (page number 664). The ***edge connectivity*** of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. **Show** how the edge connectivity of an undirected graph $G = (V, E)$ can be determined by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(V)$ vertices and $O(E)$ edges.

5. (5 Points) Problem 26.3-5 from the text book (page number 669). We say a bipartite graph $G = (V, E)$, where $V = L \bigcup R$, is ***d-regular*** if every vertex $v \in V$ has degree exactly d. Every d-regular bipartite graph has $|L| = |R|$. Prove that every d-regular graph has a matching of cardinality $|L|$ by arguing that a minimum cut of the corresponding flow network has capacity $|L|$.

6. (5 Points) Problem 15.2.1 from the text book (page number 338). Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$.

Advanced Problems

7. **Minimum path cover** (10 Points; Problem 26.2 of the text; page 692). A ***path cover*** of a directed graph $G = (V, E)$ is a set P of vertex-disjoint paths such that every vertex in V is included in exactly one path in P. Paths may start and end anywhere, and they may be of any length, including 0. A ***minimum path cover*** of G is a path cover containing the fewest possible paths.

   (a) Give an efficient algorithm to find a minimum path cover of a directed acyclic graph $G = (V, E)$.

   (b) Does your algorithm work for directed graphs that contain cycles? Explain.

8. (5 Points; Problem 15.4.2 from the text book; page number 356). Show how to reconstruct an LCS from the completed c table and the original sequences $X = \langle x_1, x_2, \ldots, x_m \rangle$ and $Y = \langle y_1, y_2, \ldots, y_n \rangle$ in $O(m + n)$ time, without using the b table.