Homework 2

Due on 16th Feb, 2005

Name :

Student ID :

Course No : **423** **823**

Instructions

- Write your name and student ID on the space provided. Circle the course number in which you are registered. Return this sheet with your solutions.
- Clarity of presentation is utmost important. You should give a clear description of all your algorithms with an analysis of their time complexity.
- No collaboration with any one is permitted for CORE problems!. See the Instructor or the TA if you need additional guidance. You may discuss approaches to solving ADVANCED problems with your classmates. However, you must write up your own solutions in your own words and acknowledge any one with whom you collaborated.

Core Problems

1. (5 Points, Problem 22.1-3 from the text, page number 530). The transpose of a directed graph $G = (V, E)$ is the graph $G^T = (V, E^T)$, where $E^T = \{(v, u) \in V \times V : (u, v) \in E\}$. Thus G^T is G with all its edges reversed. Describe efficient algorithms for computing G^T from G, for both the adjacency list and adjacency matrix representations of G. Analyze the running times of your algorithms.

2. (10 Points) Draw a 10 node graph with 20 directed edges of your choice. Make sure that your graph is as *random looking* as possible.

   (a) Simulate DFS on this directed graph recording discovery and finish times for all vertices. Also redraw the graph by showing DFS forest and all other edges with their classification.

   (b) Consider the undirected version of the same graph and simulate BFS from a source of your choice and find out the shortest distance from the source to all other vertices.

3. (5 Points, Problem 22.3-8 from the text book, page number 548). Give a counterexample to the conjecture that if there is a path from u to v in a directed graph G, then any depth-first search must result in $d[v] \leq f[u]$.

4. (5 Points, Problem 22.4-1 from the text book, page number 551). Show the ordering of vertices produced by TOPOLOGICAL-SORT when it is run on dag of Figure 22.8 (page 551). Assume that the vertices are considered in the alphabetical order and that each adjacency list is ordered alphabetically.

5. (10 Points, Problem 22.4-2 from the text book, page number 552). Give a linear-time algorithm that takes as input a directed acyclic graph $G = (V, E)$ and two vertices s and t, and returns the number of paths from s to t in G. For example, in the directed acyclic graph of Figure 22.8, there are exactly four paths from vertex p to vertex v: $pov, poryv, posryv, psruyv$. (Your algorithm only needs to count the paths, not list them.)

Advanced Problems

6. **Diameter of a tree** (5 Points, Problem 22.2-7 from the text book, page 539). For a tree $T = (V, E)$, the diameter of T is given by $\max_{u,v \in V} \delta(u, v)$. That is, the diameter is the largest of all shortest-path distances in the tree. Give a $O(|V|)$ algorithm to compute the diameter of a tree and argue that your algorithm is correct.

   Hint: You may try using two BFSs on the tree.

7. **Reachability** (10 Points, Problem 22-4 from the text book, page 559). Let $G = (V, E)$ be a graph with each vertex $u \in V$ labeled with a unique integer $L(u)$ from the set $\{1, 2, \ldots, |V|\}$. For each vertex $u \in V$ define the set $R(u) = \{v \in V | u \leadsto v\}$, the set of vertices that are reachable from u. Define $\min(u)$ to be the vertex in $R(u)$ whose label is minimum. That is, $\min(u)$ is the vertex $v \in R(u)$ such that $L(v) = \min\{L(W)|w \in R(u)\}$. Give an $O(|V| + |E|)$ algorithm that computes $\min(u)$ for all vertices $u \in V$.