Design and Analysis of Algorithms
(CSCE 423/823, Spring 2005)
URL: http://www.cse.unl.edu/∼vinod/823S05/index.html
26th Jan, 2005

Homework 1
Due on 4th Feb, 2005

Name :

Student ID :

Course No : **423** **823**

Instructions

- Write your name and student ID on the space provided. Circle the course number in which you are registered. Return this sheet with your solutions.
- Clarity of presentation is utmost important. You should give a clear description of all your algorithms with an analysis of their time complexity.
- No collaboration with any one is permitted for CORE problems!. See the Instructor or the TA if you need additional guidance. You may discuss approaches to solving ADVANCED problems with your classmates. However, you must write up your own solutions in your own words and acknowledge any one with whom you collaborated.

Core Problems

1. (10 Points) Solve Problems 4-1-a, b, c, g, and h, from the text book (page 85).
2. (5 Points) In the algorithm Select discussed in the class, the input elements are divided into groups of 5. Will the algorithm work if they are divided into groups of 7? Give an upper bound on the time complexity in this case. Give an upper bound on the time complexity if the inputs are divided into groups of 3.
3. (5 Points) Analyze select to show that if $n \geq 140$, then at least $\lceil n/4 \rceil$ elements are greater than median of medians x and at least $\lceil n/4 \rceil$ elements are less than x.
4. (5 Points) Given a "black-box" worst-case linear time median finding subroutine, give a simple linear-time algorithm that solves the selection problem for an arbitrary order statistic.
5. (10 Points) Let $X[1 \dots n]$ and $Y[1 \dots n]$ be two sorted arrays each containing n numbers. Give an $O(\log n)$-time algorithm to find the median of all the $2n$ elements in X and Y.

Advanced Problems

6. **Weighted Median.** For n distinct elements $x_1, x_2, \dots, x_n$ with positive weights $w_1, w_2, \dots, w_n$ such that $\sum_{i=1}^{n} w_i = 1$, the *weighted median* is the element x_k satisfying

$$\sum_{x<x_k} w_i < \frac{1}{2} \quad \text{and} \quad \sum_{x>x_k} w_i \leq \frac{1}{2}$$

   (a) (5 points) Show that the median of $x_1, x_2, \dots, x_n$ is same as the weighted median of $x_1, x_2, \dots, x_n$ with weights $w_i = \frac{1}{n}$ for $i = 1, 2, \dots, n$.
   (b) (10 Points) Show how to compute the weighted median in $O(n)$ worst-case time using a linear-time median-finding algorithm.