CSCE 428/828 Midterm 1

Monday 2/20/2017
1. (25 pts) Design a **DFA** for the following language and briefly describe the strings corresponding to each state. You do not need to minimize your DFA

\[ L = \{ w \in \{0, 1\} \mid w \text{ is the binary representation of an integer that is divisible by 3} \} \]

Note that the right-most bit of \( w \) is the least-significant bit. For example, the following strings are members of \( L \): \( \varepsilon, 0, 11, 110, 1100, 1111, 10010, 10101, 00, 0011 \).

**Solution:**

![DFA Diagram](image-url)

**Figure 1:** The three states represent the integral value of the number read so far mod 3.
2. (25 points) Let $\Sigma = \{a, b\}$. Convert the following NFA to an equivalent DFA. You do not need to minimize your DFA.

Solution:
3. (25 points) Describe the following language and design a **regular expression** for it

\[ L = \{ 0^n w 0^n \mid n \geq 1, \text{ } w \text{ is a string over alphabet } \{0, 1\} \} \]

**Solution:**

\( L \) contains all strings starting and ending with symbol 0, and can be described by regular expression \( 0^*0 \).
4. (25 points) Let \( \Sigma = \{0, 1\} \). Use the **pumping lemma** to prove that language \( L \) is not regular.

\[
L = \{0^n1^n \Sigma^* \mid n > 0\}
\]

**Solution:**

Proof by contradiction

(a) Assume that \( L \) is regular.

(b) Let \( p \) be the pumping length.

(c) Consider string \( 0^p1^p \). It is in language \( L \) and its length is greater than \( p \). According to the pumping lemma, it can be divided into three parts \( xyz \) satisfying \( |y| > 0 \) and \( |xy| \leq p \) such that for any integer \( i \geq 0 \), \( xy^iz \in L \).

(d) Because \( |xy| \leq p \), \( y \) consists of only symbol 0s. Let \( y = 0^k \), and we have \( p \geq k > 0 \). If \( i = 2 \), we have the new string \( xy^2z = xyyz = 0^{p+k}1^p \), which \( \notin L \) because the number of 0s is more than the number of 1s. A contradiction!

Therefore, the assumption is wrong; that is, language \( L \) is not regular.

**NOTE 1:**

\( i = 0 \) does not work, because \( xy^iz = xz = 0^{p-k}1^p = 0^{p-k}1^{p-k}1^k \in L \)

**NOTE 2:**

string \( 01^p1^p \Sigma^* = 01^p \Sigma^* \) does not work, because if \( y = 1^k \), then this decomposition can be pumped.