CSCE 428/828 HW 3

due in class Monday 3/13/2017
1. (15 pts) Design CFG’s for the following languages (10 pts). Briefly describe the strings derived by each variable in the grammars (5 pts).

\[
\begin{align*}
L_1 & = \{a^i b^j \mid 3i < j, i \geq 0, j \geq 0\} \\
L_2 & = \{a^i b^j \mid i \leq j \leq 3i, i \geq 0, j \geq 0\} \\
L_3 & = \{a^i b^j \mid i < j < 3i, i \geq 0, j \geq 0\}
\end{align*}
\]
2. (10 points) Design a **CFG** for the following language (5 points). Briefly describe the strings derived by each variable of your grammar (5 points).

\[ L = \{ a^i b^j \mid i \neq j \text{ and } 3i \neq j, \ i \geq 0, \ j \geq 0 \} \]
3. (10 pts) Convert the following CFG to an equivalent PDA.

\[
\begin{align*}
S &\rightarrow aSb \\
S &\rightarrow bSa \\
S &\rightarrow SS \\
S &\rightarrow \epsilon
\end{align*}
\]
4. (10 points) Design a **PDA** recognizing the following language (5 points). Briefly describe how your PDA works (5 points). Directly design a PDA, and a PDA that is simply converted from a CFG is not allowed.

\[ L = \{ a^i b^j \mid i \leq j \leq 3i, \ i \geq 0, \ j \geq 0 \} \]
5. (10 points) Design a **PDA** recognizing the following language (5 points). Briefly describe how your PDA works (5 points). Directly design a PDA, and a PDA that is simply converted from a CFG is not allowed.

\[ L = \{a^i b^j \mid i < j < 3i, \ i \geq 0, \ j \geq 0\} \]
6. (10 points) Design a PDA for the following language. Briefly describe how your PDA works. Directly design a PDA, and a PDA that is simply converted from a CFG is not allowed.

\[ \{a^i b^j c^k d^l \mid (2i + l) \leq (k - 3j), \ i \geq 0, \ j \geq 0, \ k \geq 0, \ l \geq 0 \} \]
7. (10 points) Let $L$ be the language of all palindromes over $\{0, 1\}$ containing an equal number of 0s and 1s. Show that $L$ is not context free using the pumping lemma.
8. (10 points) Prove that the following language is not context free using **pumping lemma**.

\[
\{ x_1 \# x_2 \# \ldots \# x_n \mid n \geq 3, \text{ each } x_i \in (a \cup b)^*, \text{ and } x_j = x_k \text{ for some } j \neq k \}
\]

Examples: strings \textit{aab\#aab\#, ab\#aab\#a\#aab\#b, a\#\#b, and \#a\#} belong to the language.
9. **(EXTRA CREDIT: 15 points)** Prove or disprove that $L$ is a CFL. If prove, then use either a CFG or PDA (and as before, provide some brief descriptions). If disprove, then use the Pumping Lemma.

$$L = \text{the complement of } \{a^n b^n \mid n \geq 0\} \text{ over alphabet } \{a, b\}$$