CSCE 478/878 Lecture 8: Combining Classifiers: Weighted Majority, Boosting, and Bagging

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(Adapted from Tom Mitchell's slides and Rob Schapire)

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Outline

- Combining classifiers to improve performance
- Combining arbitrary classifiers: Weighted Majority algorithm
- Combining while learning:
  - Boosting
  - Bagging

Combining Classifiers

- Sometimes a single classifier (e.g. neural network, decision tree) won't perform well, but a weighted combination of them will

- Each classifier (or expert) in the pool has its own weight

- When asked to predict the label for a new example, each expert makes its own prediction, and then the master algorithm combines them using the weights for its own prediction (i.e. the "official" one)

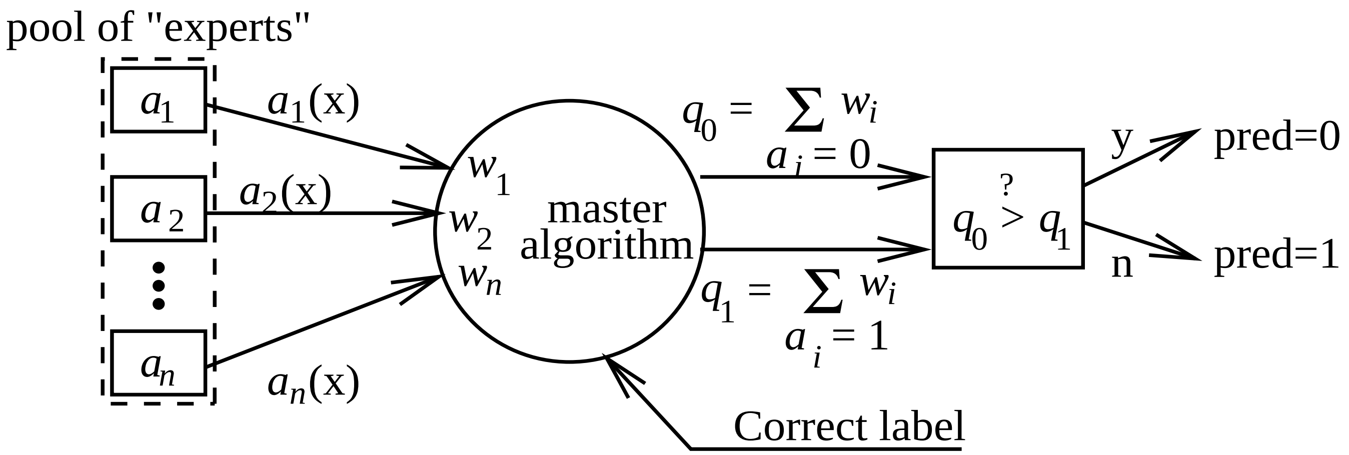
- If the classifiers themselves cannot learn (e.g. heuristics) then the best we can do is to learn a good set of weights

- If we are using a learning algorithm (e.g. ANN, dec. tree), then we can rerun the algorithm on different subsamples of the training set and set the classifiers' weights during training

Weighted Majority Algorithm (WM)

[Sec. 7.5.4]

A = pool of fixed "experts"



Weighted Majority Algorithm (WM)
(cont'd)

a_i is ith prediction algorithm in pool A of algorithms; each algorithm is arbitrary function from X to $\{0, 1\}$

w_i is weight the master alg associates with a_i

$\beta \in [0, 1)$ is parameter

- $\forall\, i$ set $w_i \leftarrow 1$
- For each training example (or trial) $\langle x, c(x) \rangle$
  - Set $q_0 \leftarrow q_1 \leftarrow 0$
  - For each algorithm a_i
    * If $a_i(x) = 0$ then $q_0 \leftarrow q_0 + w_i$
      else $q_1 \leftarrow q_1 + w_i$
  - If $q_1 > q_0$ then predict 1 for $c(x)$, else predict 0 (case for $q_1 = q_0$ is arbitrary)
  - For each $a_i \in A$
    * If $a_i(x) \neq c(x)$ then $w_i \leftarrow \beta\, w_i$

Setting $\beta = 0$ yields Halving Algorithm over A

Weighted Majority
Mistake Bound (On-Line Model)

- Let $a_{opt} \in A$ be expert that makes fewest mistakes on arbitrary sequence S of exs; let $k =$ number of mistakes a_{opt} makes
- Let $\beta = 1/2$ and $W_t = \sum_{i=1}^{n} w_{i,t} =$ sum of wts before trial t ($W_1 = n$)
- On trial t such that WM makes a mistake, the total weight reduced is

  $$W_t^{mis} = \sum_{a_i(x_t) \neq c(x_t)} w_i \geq W_t/2$$

  so

  $$W_{t+1} = \left(W_t - W_t^{mis}\right) + W_t^{mis}/2 = W_t - W_t^{mis}/2 \leq 3W_t/4$$

- After seeing all of S, $w_{opt,|S|+1} = (1/2)^k$ and $W_{|S|+1} \leq n(3/4)^M$ where $M =$ total number of mistakes, yielding

  $$\left(\frac{1}{2}\right)^k \leq n\left(\frac{3}{4}\right)^M ,$$

  so

  $$\boxed{M \leq \frac{k + \log_2 n}{-\log_2(3/4)} \leq 2.4\,(k + \log_2 n)}$$

Weighted Majority

Mistake Bound (cont'd)

- Thus for any arbitrary sequence of examples, WM guaranteed to not perform much worse than best expert in pool plus $\log$ of number of experts

- Other results:
  - Bounds hold for general values of $\beta \in [0, 1)$
  - Better bounds hold for more sophisticated algorithms, but only better by a constant factor (worst-case lower bound: $\Omega(k + \log n)$)
  - Get bounds for real-valued labels and predictions
  - Can track shifting concept, i.e. where best expert can suddenly change in S; key: don't let any weight get too low relative to other weights, i.e. don't overcommit

Bagging Classifiers

[Breiman, ML Journal, ’96]

Bagging = Bootstrap aggregating

Bootstrap sampling: given a set D containing m training examples:

- Create D_i by drawing m examples uniformly at random with replacement from D
- Expect D_i to omit $\approx 37\%$ of examples from D

Bagging:

- Create k bootstrap samples $D_1, \ldots, D_k$
- Train a classifier on each D_i
- Classify new instance $x \in X$ by majority vote of learned classifiers (equal weights)

Result: An ensemble of classifiers

Bagging Experiment

[Breiman, ML Journal, ’96]

Given sample S of labeled data, Breiman did the following 100 times and reported avg:

1. Divide S randomly into test set T (10%) and training set D (90%)
2. Learn decision tree from D and let e_S be its error rate on T
3. Do 50 times: Create bootstrap set D_i and learn decision tree (so ensemble size = 50). Then let e_B be the error of a majority vote of the trees on T

Results

Data Set	$\bar{e}_S$	$\bar{e}_B$	Decrease
waveform	29.0	19.4	33%
heart	10.0	5.3	47%
breast cancer	6.0	4.2	30%
ionosphere	11.2	8.6	23%
diabetes	23.4	18.8	20%
glass	32.0	24.9	27%
soybean	14.5	10.6	27%

Bagging Experiment
(cont'd)

Same experiment, but using a nearest neighbor classifier (Chapt. 8), where prediction of new example x's label is that of x's nearest neighbor in training set, where distance is e.g. Euclidean distance

Results

Data Set	$\bar{e}_S$	$\bar{e}_B$	Decrease
waveform	26.1	26.1	0%
heart	6.3	6.3	0%
breast cancer	4.9	4.9	0%
ionosphere	35.7	35.7	0%
diabetes	16.4	16.4	0%
glass	16.4	16.4	0%

What happened?

When Does Bagging Help?

When learner is unstable, i.e. if small change in training set causes large change in hypothesis produced

- Decision trees, neural networks

- Not nearest neighbor

Experimentally, bagging can help substantially for unstable learners; can somewhat degrade results for stable learners

Boosting Classifiers

[Freund & Schapire, ICML ’96; many more]

Similar to bagging, but don’t always sample uniformly; instead adjust resampling distribution over D to focus attention on previously misclassified examples

Final classifier weights learned classifiers, but not uniform; instead weight of classifier h_t depends on its performance on data it was trained on

Repeat for $t = 1, \ldots, T$:

1. Run learning algorithm on examples randomly drawn from training set D according to distribution $\mathcal{D}_t$ ($\mathcal{D}_1 =$ uniform)
2. Output of learner is hypothesis $h_t : X \rightarrow \{-1, +1\}$
3. Compute $error_{\mathcal{D}_t}(h_t)$ = error of h_t on examples drawn according to $\mathcal{D}_t$ (can compute exactly)
4. Create $\mathcal{D}_{t+1}$ from $\mathcal{D}_t$ by increasing weight of examples that h_t mispredicts

Final classifier is weighted combination of $h_1, \ldots, h_T$, where h_t’s weight depends on its error w.r.t. $\mathcal{D}_t$

Boosting
(cont'd)

- Preliminaries: $D = \{(\vec{x}_1, y_1), \ldots, (\vec{x}_m, y_m)\}$, $y_i \in \{-1, +1\}$, $\mathcal{D}_t(i)$ = weight of $(\vec{x}_i, y_i)$ under $\mathcal{D}_t$

- Initialization: $\mathcal{D}_1(i) = 1/m$
  (in general, require $\forall t$, $\sum_{i=1}^m \mathcal{D}^t(i) = 1$)

- Error Computation: $\epsilon_t = \Pr_{\mathcal{D}_t}[h_t(\vec{x}_i) \neq y_i]$
  (easy to do since we know $\mathcal{D}_t$)

- If $\epsilon_t > 1/2$ then halt; else:

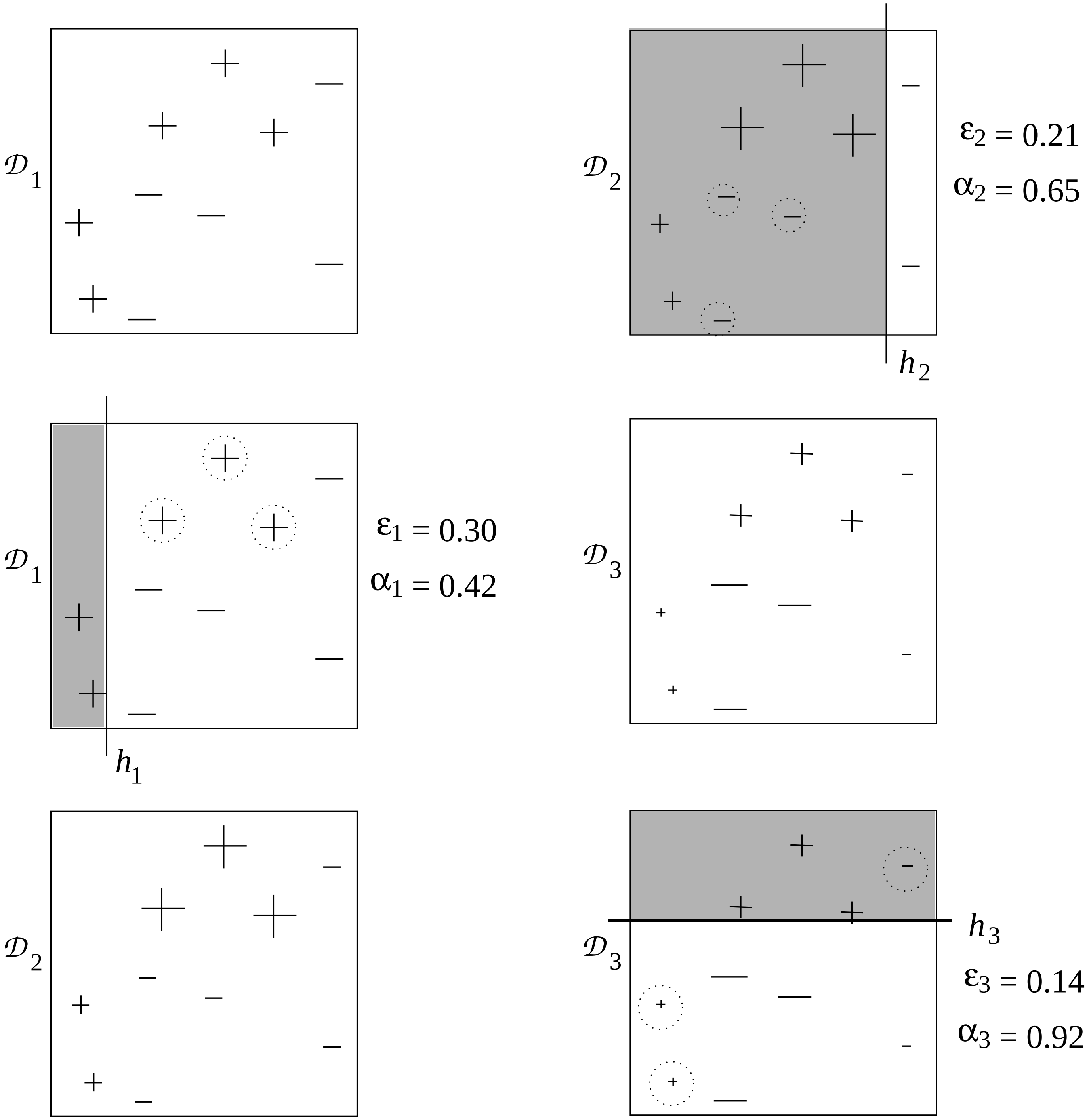
- Weighting Factor: $\alpha_t = \frac{1}{2} \ln \left(\frac{1 - \epsilon_t}{\epsilon_t} \right)$
  (grows as ϵ_t decreases)

- Update: $\mathcal{D}_{t+1}(i) = \frac{\mathcal{D}_t(i) \exp(-\alpha_t \, y_i \, h_t(\vec{x}_i))}{\underbrace{Z_t}_{\text{normalization factor}}}$
  (increase weight of mispredicted examples, decrease wt of correctly predicted exs)

- Final Hypothesis: $H(\vec{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t \, h_t(\vec{x}) \right)$

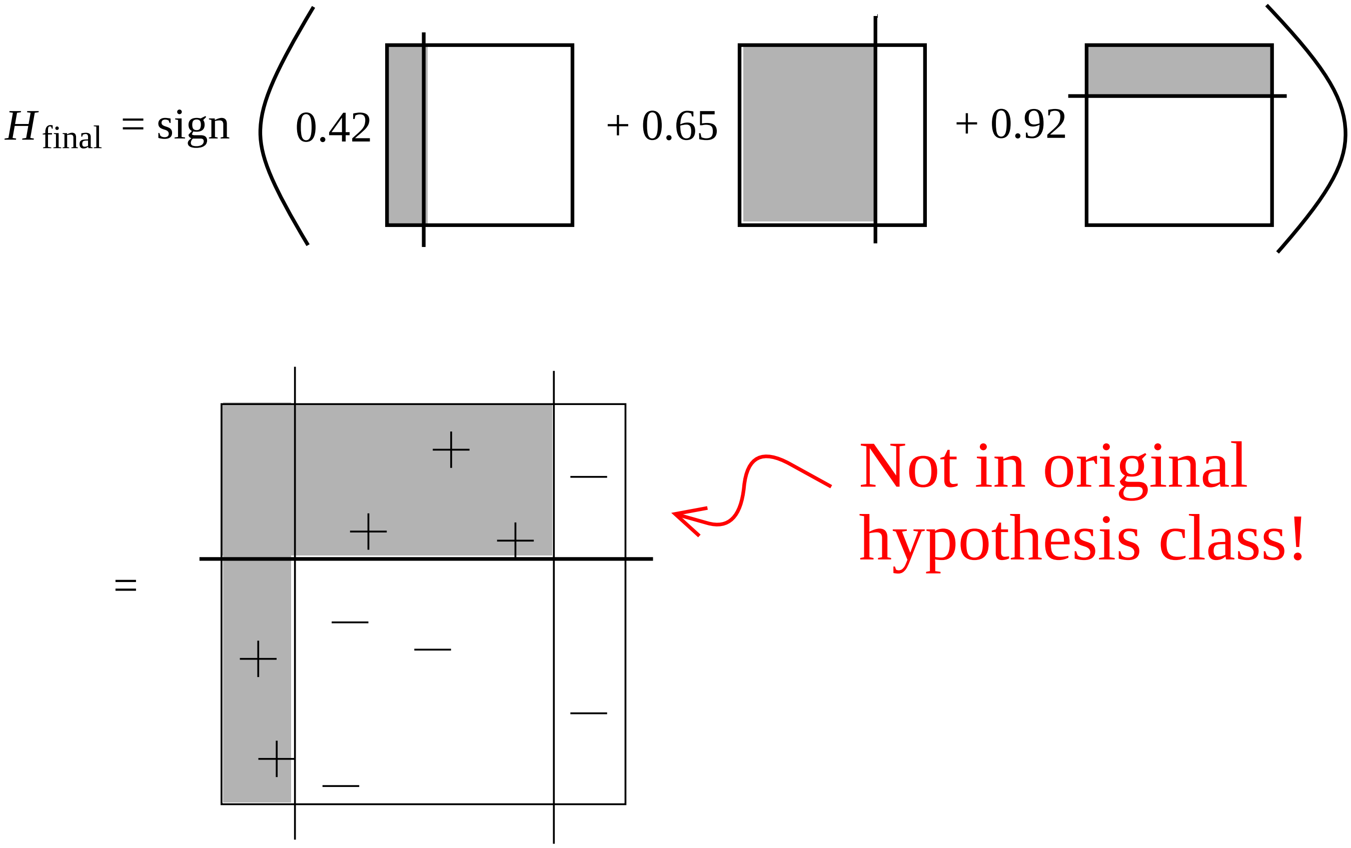
Boosting

Example



Boosting

Example (cont'd)



Not in original hypothesis class!

Other advantages to ensembles (boost/bag):

- Helps with problem of choosing one of several consistent hypotheses
- Compensates for imperfect search algorithms (e.g. it is hard to find smallest decision tree or a consistent ANN)

Boosting

Miscellany

- If each $\epsilon_t < 1/2 - \gamma_t$, error of $H(\cdot)$ on D drops exponentially in $\sum_{t=1}^{T} \gamma_t$

- Can also bound generalization error of $H(\cdot)$ independent of T

- Also successful empirically on neural network and decision tree learners
  - Empirically, generalization sometimes improves if training continues after $H(\cdot)$'s error on D drops to 0 [cf. generalization error's independence of T]
  - Contrary to intuition: would expect overfitting
  - Related to increasing the combined classifier's margin (confidence in prediction)

- Can apply to labels that are multi-valued using e.g. error-correcting output codes