Local Polynomial Regression Models for Vehicle Speed Estimation and Forecasting in Linear Constraint Databases

Hang Yue, Elizabeth Jones*
Mid-America Transportation Center
Civil Engineering Department
University of Nebraska-Lincoln
Email: yuehang366@gmail.com
*ejones1@unl.edu

Peter Revesz
Computer Science & Engineering Department
University of Nebraska-Lincoln
Email: revesz@unl.cse.edu

Abstract

Constraint databases have the specific advantage of being able to represent infinite temporal relations by linear equations, linear inequalities, polynomial equations, and so on. This advantage can store a continuous time-line that naturally connects with other traffic attributes, such as vehicle speed. In most cases, vehicle speed varies over time, that is, the speed is often non-linear. However, the infinite representations allowed in current constraint database systems are only linear. This article presents a new approach to estimate and forecast continuous non-linear vehicle speed using linear constraint database systems. Our new approach to represent and query non-linearly moving vehicles is based on a combination of local polynomial regression models and piecewise-linear approximation algorithms. Experiments using the MLPQ constraint database system and queries show that our method has a high accuracy in predicting the speed of the vehicle. The actual accuracy is controllable by a parameter. We compare the local linear regression model with the local cubic model by using a field experiment. It is found that the local cubic fit can implement a better estimation in the peak and valley of the data patterns.

1. Introduction

Today relational databases are popularly applied into many traffic systems [1] [2], such as traffic management systems, public transit systems and Advanced Traveler Information Systems (ATISs). However, when the relational model is used to handle spatial data, the points, lines and polygons in space are discretely saved in tables and often lose spatial relationships in databases [3]. However, having only a finite set of tuples in the relational tables may make it difficult to see the intuitive relationship among the core traffic attributes [4].

Time discontinuity is another significant deficiency in relational databases. Everyone expects that, regardless of the magnitude of change, the complete snapshot produced at each time slice could duplicate all the unchanged data in the database. However, relational databases with time discontinuity [3] cannot store the complete information of moving objects, such as moving vehicles and pedestrians.

Constraint databases are viewed as a special kind of post-relational databases, although they share with relational database some important features, such as, formal, model-theoretic semantics, various high-level query languages like SQL and Datalog [5]. On the other hand, constraint databases have some specific features such as the ability to represent infinite relations by various types of constraints, to describe continuous temporal and arbitrarily high-dimensional and continuous spatial or spatiotemporal data [6].

Constraint databases may allow many different types of constraints, such as, linear equations and linear inequalities over rational numbers or polynomial equations over real numbers. Constraint databases have the potential to serve as a useful tool for traffic data archiving and operation, although they were not designed for this particular purpose and need to be adapted for such a task.

The aim of this paper is to develop the local polynomial regression models to estimate and predict non-linear vehicle speed with the continuous time-line in linear constraint databases. The development of these new models means that constraint databases have the capability to model and store continuous non-linear data. In addition, constraint databases have far-reaching potentials to evaluate and analyze the information of traffic moving objects (vehicles and pedestrians) on the basis of statistical nonparametric methods.

This paper is structured as follows. Section 2 discusses the local polynomial regression models. Section 3 describes piecewise-linear approximations. Section 4 presents the experiments that test the accuracy of using constraint databases to predict the speed of vehicles. Section 5 discusses the literature review. Finally, Section 6 gives a brief discussion and some concluding remarks.

2. Local polynomial regression

2.1. Definition

As an important data analytic approach, nonparametric density estimation can effectively describe the important structure in a set of data. There is a basic difference between the parametric and nonparametric approaches. The former assumes that some parameters can represent the density estimator; the latter does not assume a pre-specified functional form for the density estimator. Suppose that in a sample of random pairs $(x_1, y_1), \ldots, (x_n, y_n)$, the response variable is assumed to satisfy [22]:

$$Y_i = m(x_i) + \upsilon^{1/2}(x_i)\varepsilon_i \qquad (1)$$

where $m(\cdot)$ is the function to be estimated; $\upsilon(\cdot)$ is the variance function; ε_i is an independent random variable with zero mean and unit variance; x_i is a random variable having common density f; $i = 1, \ldots, n$.

Local polynomial kernel estimators $\hat{m}(x; p, h)$ [23] [24] [25] can be developed via "locally" fitting a p^{th} degree polynomial $\sum_{j=0}^{p} \beta_j (x_i - x)^j$ to the (x_i, Y_i) using weighted least squares. The bandwidth h is a nonnegative number controlling the size of the local neighborhood; and h is assumed to approach zero, but at a rate slower than n –1, that is:

$\lim_{n\to\infty} h = 0 \quad \lim_{n\to\infty} nh = \infty$, $\hat{\beta} = (\hat{\beta}_0, \cdots, \hat{\beta}_p)^T$ is able to minimize the locally weighted polynomial regression $\sum_{i=1}^{n} \{Y_i - \sum_{j=0}^{p} \beta_j (x_i - x)^j\}^2 K_h(x_i - x)$, and $K_h(\cdot) = K(\cdot / h)/h$ is a kernel function scaled by h. The weights are chosen according to the height of the kernel function centered about the particular point x, and the kernel weight $K_h(x_i - x)$ is the weight assigned to Y_i. The data closer to x carry more influence in the value of m(x), not assuming a specific form of the regression function m. The functions of the estimators are listed below:

$$\hat{m}(x; p, h) = e_1^T (X_x^T W_x X_x)^{-1} X_x^T W_x Y = e_1^T \hat{\beta} = \hat{\beta}_0 \qquad (2)$$

where e_1 is the $(p+1) \times 1$ vector having 1 in the first entry and zero elsewhere;

$Y = (Y_1, \cdots, Y_n)^T$ is the vector of responses;

$$W_x = diag\{K_h(x_1 - x), \cdots, K_h(x_n - x)\}$$

is an $n \times n$ diagonal matrix of weights;

$$X_x = \begin{bmatrix} 1 & x_1 - x & \cdots & (x_1 - x)^p \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n - x & \cdots & (x_n - x)^p \end{bmatrix}$$

is an $n \times (p+1)$ design matrix, n is the number of observations.

The shape choice about the kernel function is not as important for the data estimation and analysis as the bandwidth selection (see Section 2.4). There are some choices [26], such as Epanechnikov, Biweight, Triweight, Normal, Uniform, Triangular, and so on. "Normal"– the Gaussian density function – is used in this research.

2.2. Order choice

In terms of the order of polynomial fit for the asymptotic performance of $\hat{m}(\cdot; p, h)$, [22] shows that fitting the polynomials of higher order leads to a possible bias reduction and a variance increase, and the odd order fits are preferable to the even order fits in the problem of the variability augment. Furthermore, the even order fits achieve lower efficiency in a bias reduction, especially in the boundary regions and highly clustered design regions. According to the practical performance in many cases, the order of polynomial fits, which are beyond cubic fit, need a very large sample to actualize a significant improvement. Therefore, this study proposes to use p=1 and p=3. The local cubic fit (when p=3) can implement a better estimation in the peak and valley of m, although the cubic fit has a higher requirement concerning its calculation and sample variability than the local linear model [26].

2.3. Bandwidth selection

The choice of value for the bandwidth is particularly important to highlight the significant structure in a set of data. [27] executes a survey of several bandwidth selections for the density estimation, and these selectors are Biased Cross-validation (BCV) [28], Least Squares Cross-validation (LSCV) [29], Rule-of-Thumb (ROT), Solve-the-equation (STE) [23] [30] [31] [32] [33], and Smoothed Bootstrap [34], and summarizes that ROT has a small variance, yet an unacceptable large mean; LSCV has a good mean, yet too large a variance; BCV suffers from unstable performance; both STE and smoother bootstrap have a correctly centered distribution in mean and an acceptable variance. [35] compares three plug-in bandwidth selection strategies [22], such as ROT, STE, and Direct Plug-in (DPI) via the data simulation and analysis, and the result is that DPI has the same appealing performance as STE. Moreover, it does not need the extra complication of requiring a root-finding procedure and minimization.

There are several assumptions for the calculation of DPI bandwidth [35]: the random and independent pairs (X_1, Y_1), …, (X_n, Y_n) have the common density f with support confined to a compact set $S = R$, $S= [a, b]$; the errors are homoscedastic, and $v(x) = \sigma^2$ for all x; p is an integer greater than r and s (where r, $s \geq 0$; $r + s$ is even), and both $p-r$ and $p-s$ are odd; K is a second-order symmetric kernel with $R(K) = \int K(x)^2 dx$. The cubic fit (p=3) has more degrees of freedom for estimating a high curve region in a set of data than the linear fit (p=1), and [35] clarifies the DPI rule by the particular case (r=s=2 and p=3) and the calculation steps about the direct plug-in bandwidth selector $\hat{h}_{DPI}$.

3. Piecewise-linear approximation

Piecewise-linear approximation is the approximation of a nonlinear function by a set of line segments. Piecewise-linear approximation considered below creates the line segments based on the known discrete data points. In addition, the compressed data can speed up answering database queries. In a time series (t_i, y_i) for i =1, 2,…, n, the maximum error threshold Ψ controls the maximum difference between the original data and piecewise-linear approximation. That means that we assure that the original data is always within a narrow band with width Ψ around piecewise-linear approximation as shown in Figure 1:

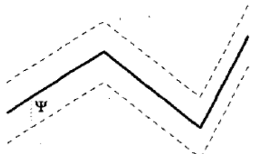

Figure 1. Piecewise-linear approximation [41]

The relation between a piecewise-linear function $f(t_i)$ and y_i satisfies:

$$| f(t_i) - y_i | \leq \Psi \text{ for each } (t_i, y_i) \quad (3)$$

Given enough known conditions, a piecewise-linear function can be automatically produced using the following algorithm [41]:

Input: time series S and maximum error threshold Ψ.
Output: a piecewise-linear approximation function.
Local variables: Begin is start, S_L min and S_U max slope of a piece.

```
Begin: = (t1, y1)
SL: = – ∞
SU: = + ∞
For  i = 1 to (n –1) do
    S'L: = max [SL, slope (Begin, (ti+1, yi+1 –Ψ)]
    S'U: = min [SU, slope (Begin, (ti+1, yi+1+Ψ))]
   if S'L ≤ S'U then
    SL: = S'L
    SU: = S'U
   else
    add line f(t) = 0.5 (SL +SU)*(t – Begin.t) +Begin.y
    Begin: = (ti, f(ti))
    SL: = slope (Begin, (ti+1, yi+1 –Ψ))
    SU: = slope (Begin, (ti+1, yi+1+Ψ))
   end-if
end-for
     add line f(t) = 0.5 (SL+SU)*(t– Begin.t) +Begin.y
```

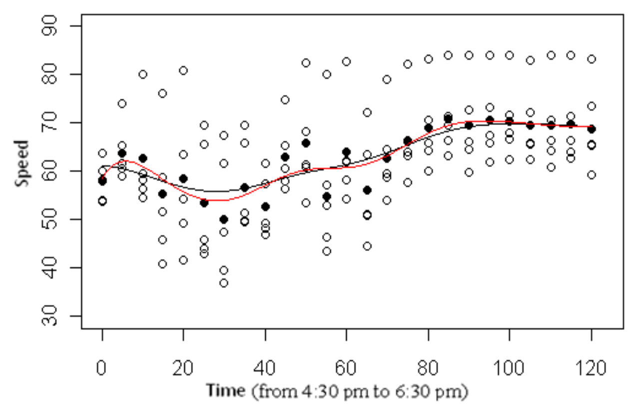

Figure 2. Local linear and cubic fits for vehicle speed data

4. Experimental results

4.1. Data collection

The values of traffic speeds were collected by Cambridge Systematics, Inc. at the detector station (717490) of U.S. Highway 101 in Los Angeles, California on June 8, 2005 [36]. The data points (the circles in Fig. 2) represent the sequential five-minute average speed values (speed = g × flow / occupancy, unit: mph). The values of flow and occupancy were reported by the loop detectors in five different lanes.

[43] provides the algorithm to track the g-factor, which depends on the actual vehicle length and the loop's electrical circuit.

4.2.Model implementation

The local polynomial regression models figure out the estimation of the average speed for all lanes. Meanwhile, the models alter the discrete vehicle speed points into the continuous speed curves: local linear regression model with p=1 (black curve) and local cubic regression model with p= 3 (red curve). The solid circles represent the average speed reported at the detector station.

Figure 3 and 4 respectively display the XY pairs about time and speed in the local linear and cubic models with those average speed values from the detector station (the number of the XY pairs is 361). Mean Square Error (MSE), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE) are applied for the accuracy estimation, their definitions are shown in the following equations, where n is the number of the average speed reported by the detector station from 4:30 pm to 6:30 pm (n=25), Y_i is the average speed of the detector station, and $\hat{Y}_i$ is the speed of XY pairs in the local linear and cubic models:

$$MSE = \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n} \qquad RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n}} \qquad MAE = \frac{\sum_{i=1}^{n}|Y_i - \hat{Y}_i|}{n}$$

Table 1. Model estimation

Model	MSE	RMSE	MAE
Linear	15.014	3.875	0.046
Cubic	7.561	2.75	0.0331

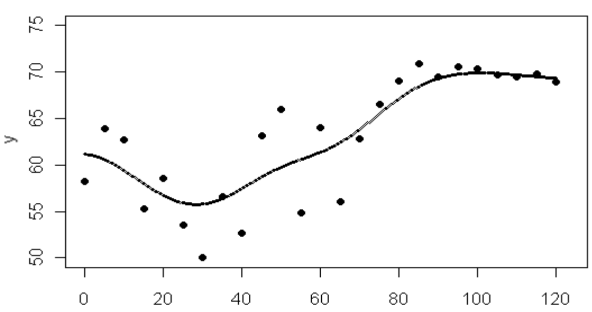

Figure 3. XY pairs in local linear model and average speed from detector station

The accuracy estimations about the linear and cubic models are given in Table 1, and the results show that the cubic model is closer to the speed values recorded by the detector station compared with the linear model. The data patterns in Figure 2, 3, and 4 display that the local cubic fit (when p=3) can implement a better estimation in the peak and valley, which is consistent with the description in [26]. Due to the traffic congestion at the rush hours, the speed is the lowest at almost 5:00 pm, i.e. x=30, and then the speed begins to rise and has a tendency to level off after 6:00 pm, i.e., x=90. From 4:30 pm (x=0) to 4:40 pm (x=10) and from 5:20 pm (x=50) to 5:40 pm (x=70), the local linear and cubic models show significant different information (see Fig. 3 and 4), and the local cubic model is more sensitive to follow the raw data.

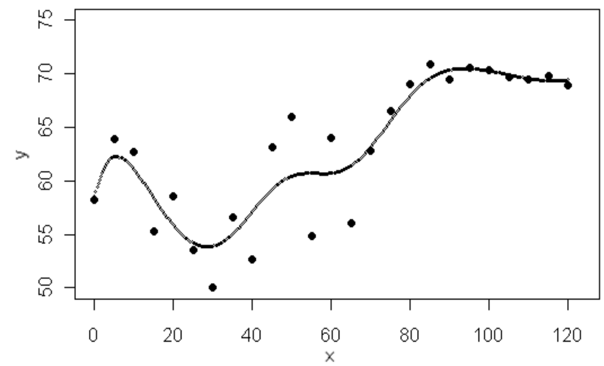

Figure 4. XY pairs in local cubic model and average speed from detector station

4.3. Database implementation and queries

The combination of XY pairs in the local polynomial models and piecewise-linear approximation can change the continuous speed curves into the corresponding linear arithmetic constraints with high accuracy for data storage and query. More importantly, the accuracy can be adjusted and controlled via the error threshold Ψ, and the accuracy is higher with a smaller error threshold.

The query design and results are displayed in Management of Linear Programming Queries (MLPQ) [6]: MLPQ allows Datalog queries, minimum and maximum aggregation operators over linear objective functions, and some other operators. MSE, RMSE, and MAE are also applied for the accuracy estimation, where n is the number of XY pairs in every piecewise-linear function, Y_i is the speed of XY pair, and $\hat{Y}_i$ is the speed calculated by the piecewise-linear function.

Based on piecewise-linear algorithm with the error threshold Ψ=0.05, the XY pairs' data points in Figure 3 and 4 are compressed into some linear functions, which are respectively shown in Table 2 and 3. The MSE, RMSE, and MAE columns summarize the accuracy analysis of every piecewise-linear function calculated by piecewise-linear approximation algorithm.

Now, the piecewise-linear segments listed in Table 2 and 3 are actualized to exert the data analysis in constraint databases. In Figure 5, the software (MLPQ) shows the similar curves as Figure 2. Table 4 lists the model speed values evaluated by the local linear and cubic regressions and the query results from constraint databases. They are very close to each other, and even some query results are the same as the velocity values in the two models, i.e. error is zero. It also displays that the cubic regression has a better result than the linear regression.

Table 2. Local linear model

Function	Piecewise-linear function		MSE	RMSE	MAE
1	$Y=-0.11278*X+61.20767$	$X\in[0.0, 4.2]$	0.001	0.0317	0.00045
2	$Y=-0.24021*X+61.74287$	$X\in[4.2, 11.7]$	0.00106	0.0325	0.00048
3	$Y=-0.27471*X+62.14656$	$X\in[11.7, 18.6]$	0.0015	0.0387	0.00060
4	$Y=-0.16853*X+60.17166$	$X\in[18.6, 24.6]$	0.00114	0.0337	0.00053
5	$Y=-0.02836*X+56.72327$	$X\in[24.6, 29,7]$	0.00099	0.0314	0.0005
6	$Y=0.10796*X+52.67477$	$X\in[29.7, 35.1]$	0.001	0.0316	0.0005
7	$Y=0.22043*X+48.72692$	$X\in[35.1, 51.0]$	0.00093	0.0306	0.00047
8	$Y=0.16662*X+51.47116$	$X\in[51.0,62.4]$	0.0014	0.0374	0.00059
9	$Y=0.26155*X+45.54756$	$X\in[62.4, 69.9]$	0.00106	0.0325	0.00046
10	$Y=0.32257*X+41.28253$	$X\in[69.9, 81.3]$	0.00142	0.0377	0.00055
11	$Y=0.21982*X+49.63603$	$X\in[81.3, 88.2]$	0.00116	0.0341	0.00045
12	$Y=0.10476*X+59.78426$	$X\in[88.2, 95.4]$	0.00111	0.0333	0.00043
13	$Y=0.00948*X+68.87408$	$X\in[95.4, 105.3]$	0.0011	0.0331	0.00042
14	$Y=-0.03706*X+73.77512$	$X\in[105.3, 120.0]$	0.00031	0.0177	0.00021

Table 3. Local cubic model

Function	Piecewise-linear function		MSE	RMSE	MAE
1	$Y=1.2905*X+58.61258$	$X\in[0.0, 1.2]$	0.00095	0.03083	0.00043
2	$Y=0.82242*X+59.17428$	$X\in[1.2, 2.7]$	0.00109	0.03296	0.0005
3	$Y=0.40767*X+60.29409$	$X\in[2.7, 4.2]$	0.00123	0.03505	0.00052
4	$Y=0.06462*X+61.73487$	$X\in[4.2, 6.3]$	0.00117	0.03426	0.00046
5	$Y=-0.23598*X+63.62872$	$X\in[6.3, 8.7]$	0.00084	0.02895	0.0004
6	$Y=-0.4859*X+65.803$	$X\in[8.7, 13.8]$	0.00107	0.0327	0.00048
7	$Y=-0.62621*X+67.73928$	$X\in[13.8, 15.3]$	0.00122	0.03499	0.00051
8	$Y=-0.48699*X+65.60911$	$X\in[15.3, 20.7]$	0.00107	0.03275	0.00051
9	$Y=-0.31532*X+62.05559$	$X\in[20.7, 24.6]$	0.00109	0.03308	0.00054
10	$Y=-0.11443*X+57.11376$	$X\in[24.6, 28.2]$	0.00108	0.03288	0.00054
11	$Y=0.0943*X+51.22758$	$X\in[28.2, 31.8]$	0.00104	0.03225	0.00053
12	$Y=0.28838*X+45.05575$	$X\in[31.8, 36.0]$	0.00109	0.033	0.00054
13	$Y=0.41237*X+40.59212$	$X\in[36.0, 44.7]$	0.00092	0.03026	0.00048
14	$Y=0.26747*X+47.06927$	$X\in[44.7, 48.9]$	0.00092	0.0304	0.00045
15	$Y=0.11142*X+54.69993$	$X\in[48.9, 53.4]$	0.00086	0.02937	0.00043
16	$Y=0.02482*X+59.32448$	$X\in[53.4, 60.9]$	0.00126	0.03553	0.00055
17	$Y=0.17216*X+50.35145$	$X\in[60.9, 65.1]$	0.00099	0.03151	0.00045
18	$Y=0.33456*X+39.77924$	$X\in[65.1, 69.3]$	0.00096	0.03101	0.00044
19	$Y=0.46527*X+30.72121$	$X\in[69.3, 80.1]$	0.00075	0.02731	0.00036
20	$Y=0.32499*X+41.95741$	$X\in[80.1, 84.9]$	0.0011	0.0331	0.00043
21	$Y=0.16552*X+55.49663$	$X\in[84.9, 89.7]$	0.00103	0.03216	0.00041
22	$Y=0.02861*X+67.777$	$X\in[89.7, 95.7]$	0.00108	0.0329	0.00042
23	$Y=-0.06537*X+76.77111$	$X\in[95.7, 113.7]$	0.00081	0.02847	0.00035
24	$Y=0.01361*X+67.79179$	$X\in[113.7, 120.0]$	0.00037	0.01912	0.00023

Table 4. Result comparison

Time	Local linear model			Local cubic model		
	Model speed	Query result	Error	Model speed	Query result	Error
0	61.14	61.21	−0.07	58.61	58.61	0.00
5	60.56	60.54	0.02	62.10	62.06	0.04
10	59.38	59.34	0.04	60.97	60.94	0.03
15	57.97	58.03	−0.06	58.37	58.35	0.02
20	56.71	56.8	−0.09	55.86	55.87	−0.01
25	55.91	56.01	−0.1	54.26	54.25	0.01
30	55.8	55.91	−0.11	54.01	54.06	−0.05
35	56.39	56.45	−0.06	55.13	55.15	−0.02
40	57.49	57.54	−0.05	57.11	57.09	0.02
45	58.72	58.65	0.07	59.08	59.11	−0.03
50	59.78	59.75	0.03	60.29	60.27	0.02
55	60.6	60.64	−0.04	60.66	60.69	−0.03
60	61.39	61.47	−0.08	60.81	60.81	0.00
65	62.43	62.55	−0.12	61.58	61.54	0.04
70	63.86	63.86	0.0	63.30	63.29	0.01
75	65.53	65.48	0.05	65.64	65.62	0.02
80	67.16	67.09	0.07	67.91	67.96	−0.05
85	68.47	68.32	0.15	69.53	69.58	−0.05
90	69.34	69.21	0.13	70.33	70.35	−0.02
95	69.79	69.74	0.05	70.48	70.5	−0.02
100	69.94	69.82	0.12	70.26	70.23	0.03
105	69.87	69.87	0.0	69.89	69.91	−0.02
110	69.69	69.7	−0.01	69.54	69.58	−0.04
115	69.48	69.51	−0.03	69.36	69.36	0.00
120	69.3	69.33	−0.03	69.43	69.42	0.01

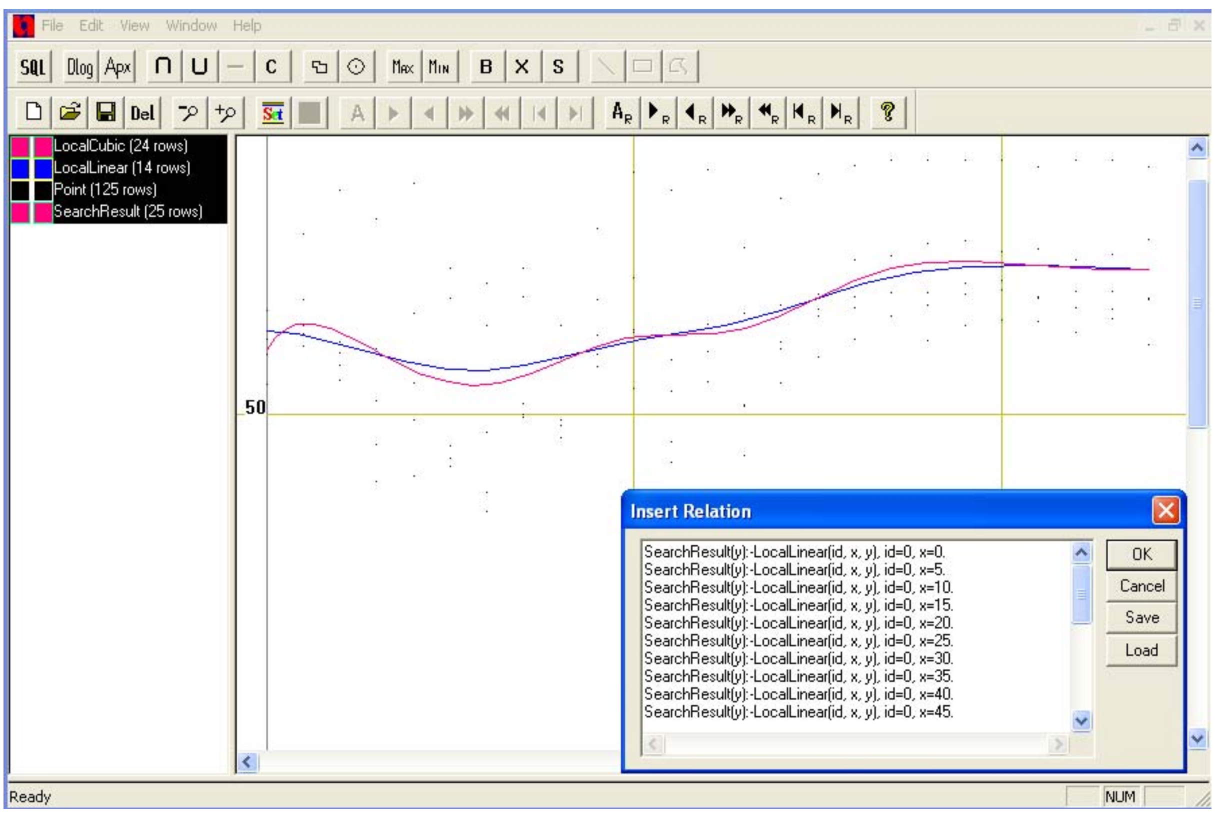

Figure 5. Piecewise models speed in MLPQ

5. Literature review

As a nonparametric method, the local regression follows the curved tendency of the data over the entire estimating region, not implement the model selection depending on the response. The local model is a real-time model without data pre-classification and learns functions from the raw data [7]. Many approaches have been formed to offer the fast computation for one or more independent variables. However, the global model, as opposed to the local model, deviates from the data pattern and requires the offline training [7], such as neural networks and time series models. A complicated global function can be easily approximated into the local model via the design of band widths and weights. [8] depicts that the local modeling is not the approximate function with more accuracy from it, and this feature avoids negative interference exhibited by the global models. This is the primary fascination in the local modeling.

There exist a few distinguishing advantages in the local polynomial regression smoothing. This approach can avoid the drawbacks of the traditional kernel regression methodologies, such as the Nadaraya-Watson estimator [9] [10] and the Gasser-Müller estimator [11]. The Nadaraya-Watson estimator produces an undesirable bias, and the Gasser-Müller estimator must pay a price in variance to manipulate a random design model. Also, the local polynomial fitting is competent for different models, such as random design, fixed design, highly clustered design, and highly uniform design, without boundary effects. Boundary modifications [12] [13] in multi-dimension are a tough task for other approaches, but the local polynomial fitting with high curvature adapts well to the bias problems at boundaries, so no boundary modification in this approach has remarkable merit. In addition, [14] proves that local polynomial smoothers of general orders achieve the mini-max efficiency over some well interpreted class of functions. The polynomial order selection is in a straightforward manner.

At present there are many methods and models concerning the short-term prediction in the domain of transportation. [7] summarizes these approaches, as follows: short-term forecasting algorithms [15], time series models [37], Kalman filtering models [38] [16], simulation models [39] [17] [18], dynamic traffic assignment models [19] [20], neural network models [40], and nonparametric methods [41] [21]. In terms of local constant regression in transportation, [21] and [42] respectively implement the k-nearest neighbor method and kernel estimator. [22] shows that the local linear method is better than the local constant methods in data distribution, and it is consistent with the conclusion in [7]: the local linear method is preferable to the k-nearest neighbor and the kernel smoothing method in the analysis of vehicle speed data.

6. Conclusions

This article details the definition of local polynomial regression models, their bandwidth selection, and their order choice. The combination of the two models (the local linear or cubic fit) and piecewise-linear approximation algorithm is proposed as a new approach for estimating and predicting vehicle speed in constraint databases. The experiment results prove that this approach has a high accuracy in the storage of continuous non-linear data in linear constraint databases for transportation application.

The local cubic fit can implement a better estimation in the peak and valley of traffic data sources than the local linear fit, and yet the local cubic fit has a higher requirement concerning its calculation. Fortunately, the development of the software package can execute the complex calculations concerning the local cubic model and overcome this demerit to update the current transportation systems. Meanwhile, it can satisfy traffic data operation in a large data size.

Future research would concentrate on the non-linear simulation of traffic moving objects and perform spatiotemporal data analysis in dynamic transportation environment.

References

[1]. Bob McQueen, Kan Chen and Rick Schuman. Advanced Traveler Information Systems. Artech House Publishers, 2002.

[2]. Schofer, J.L., F.S. Koppelman, and W.A. Charlton. Perspectives on Driver Preferences for Dynamic Route Guidance Systems. Transportation Research Record 1588, pp. 26-31, 1997.

[3]. Linxin Li, Xingyou Zhang, Reinhard Piltner. A Spatiotemporal Database for Ozone in the Conterminous U.S. Proceeding of the Thirteenth International Symposium on Temporal Representation and Reasoning (TIME'06). Budapest, Hungary, June 15-17, 2006.

[4]. Fleming CC, Von Halle B. Handbook of relational database design. Addison-Wesley, Reading, pp.605, 1989.

[5]. Peter Revesz. Constraint Databases: A Survey. In: Semantics in Databases, L. Libkin and B. Thalheim, eds., Springer LNCS 1358, pp. 209-246, 1998.

[6]. Peter Revesz. Introduction to Database: From Biological to Spatiotemporal, Springer, New York, 2009.

[7]. Hongyu Sun, Henry X. Liu, Heng Xiao, Rachel R. He, and Bin Ran. Use of local linear regression model for short-Term traffic forecasting. Transportation Research Board, No. 1836, Vol. 18, pp.59-71, 2003.

[8]. Atkeson, C., Moore, A. & Schaal, S. Locally weighted learning. Artificial Intelligence Review, 11, pp.11-73,

1997.
[9]. Nadaraya, E.A. On estimating regression. Theory Probab. Appl. 10, 186-90, 1964.
[10]. Wathson, G. W. and Leadbetter, M.R. Hazard analysis I. Biometrika 51, 175-84, 1964.
[11]. Gasser, T. and Müller, H.-G. Kernel estimation of regression functions. In Smoothing Techniques for Curve Estimation (eds. T. Gasser and M. Rosenblatt). Springer-Verlag, Heidelberg, pp. 23-68, 1979.
[12]. Silverman, B.W. Density estimation for statistics and data analysis. Chapman and Hall, London, 1986.
[13]. Fan, J. and Marron, J.S. Fast implementations of nonparametric curve estimators. J. Comput. Graphical Statist. 3, 35-56, 1994.
[14]. Kani Chen. Linear minimax efficiency of local polynomial regression smoothers. Journal of Nonparametric Statistics, 1029-0311, Volume 15, Issue 3, Pages 343 – 353, 2003.
[15]. Van Arem, B, Kirby, H., Van Der Vlist, M&Whittaker, J. Recent advances and applications in the field of short-term traffic forecasting." International Journal of Forecasting 13, 1997, 1-12.
[16]. Chen, M. & Chien, S. Dynamic freeway travel time prediction using probe vehicle data: link-based vs. path-based. TRB Paper No. 01-2887, 2001.
[17]. Van Vliet, D. Saturn: a simulation-assignment model for the evaluation of traffic management schemes. Traffic Engineering and Control, Vol.21 (4), 1980.
[18]. Duncan, G. & Littlejohn, J. High Performance Microscopic Simulation for Traffic Forecasting. Computing and Control Division Colloquium on Strategic Control of Inter-Urban Road Networks, London, England: Institution of Electrical Engineers, 1997.
[19]. Ben-Akiva, M., Cascetta, E. & Gunn, H. An on-line dynamic traffic prediction model for an inter-urban motorway network. Urban traffic networks: dynamic flow modeling and control. Berlin : Springer-Verlag, 1995.
[20]. Ran, B. Using traffic prediction models for providing predictive traveler information. Int. J. Technology Management, Vol. 20. Nos. 3/4, 2000.
[21]. Smith, B., Williams, B. & Oswald, K. Parametric and nonparametric traffic volume forecasting. TRB Preprints CDROM, 1999.
[22]. Fan J. and Gijbels I. Local polynomial modeling and its applications. Chapman & Hall, London, New York, 1996.
[23]. Scott, D.W., Tapia, R.A. and Thompson, J.R. Kernel density estimation revisited. Nonlinear Anal. Theory Meth. Applic. 1, 339-72, 1977.
[24]. Cleveland, W. Robust locally weighted regression and smoothing scatterplots. J. Amer. Statist. Assoc. 74, 829-36, 1979.
[25]. Fan, J. Design-adaptive nonparametric regression. J. Amer. Statist. Assoc. 87, 998-1004, 1992.
[26]. Wand, M. P. and Jones, M. C. Kernel Smoothing. Chapman and Hall, London, 1995.
[27]. Jones, M.C. Marron J.S. and Sheather S. J. A brief survey of bandwidth selection for density estimation. Amer. Statist. Assoc. 91, No. 433, 1996.
[28]. Scott, D.W. and Terrell, G.R.. Biased and unbiased cross-validation in density estimation. J. Amer. Statist. Assoc. 82, 1131-46, 1987.
[29]. Bowman, A. W. An alternative method of cross-validation for the smoothing of density estimates. Biometrika 71, 353-60, 1984.
[30]. Sheather, S.J. An improved data-based algorithm for choosing the window width when estimating the density at a point. Comp. Statist. Data Anal. 4, 61-5, 1986.
[31]. Park, B. U. and Marron, J.S. Comparison of data-driven bandwidth selectors. J. Amer. Statist. Assoc. 85, 66-72, 1990.
[32]. Sheather, S.J. and Jones, M.C. A reliable data-based bandwidth selection method for kernel density estimation. J. Roy. Statist. Soc. Ser. B 53, 683-90, 1991.
[33]. Engel, J., Herrmann, E. and Gasser, T. An iterative bandwidth selector for kernel estimation of densities and their derivative. J. Nonparametric Statist. to appear, 1995.
[34]. Faraway, J.J., and Jhun, M. Bootstrap choice of bandwidth for density estimation. Journal of the American Statistical Association. 85, 1119-1122, 1990.
[35]. Ruppert, D., Sheather, S. J. and Wand, M. P. An effective bandwidth selector for local least squares regression. Journal of the American Statistical Association, 90, 1257–1270, 1995.
[36]. Cambridge Systematics, Inc. Prepared for Federal Highway Administration. NGSIM U.S. 101 Data Analysis (4:30 p.m. to 6:30 p.m.), June 8, 2005.
[37]. Ahmed, M. & Cook, A. Analysis of freeway traffic time series data by using Box-Jenkins techniques. Transportation Research Board, No. 722, pp.1-9, 1979.
[38]. Okutani, I. & Stephanedes, J. Dynamic prediction of traffic volume through Kalman filter theory. Transportation Research, Part B, Vol. 18B, 1-11, Feb 1984.
[39]. Rathi, A. & Santiago, A. The New NETSIM: TRAF-NETSIM 2.00 Simulation Program. Paper Presented at the Transportation Research Board 68th Annual Meeting, Washington, D.C, 1989.
[40]. Park, D. & Rilett, L. Forecasting multiple-period freeway link travel times using modular neural networks. TRB 1998.
[41]. Davis, G. & Nihan, N. Nonparametric regression and short-term freeway traffic forecasting. Journal of Transportation Engineering, p178-188, 1991.
[42]. Faouzi, E. Nonparametric Traffic Flow Prediction Using Kernel Estimator. Transportation and Traffic Theory: Proceedings of the 13th International Symposium on Transportation and Traffic Theory, Lyon, France, 24-26, July, 19, 1996.
[43]. Jia, Z., Chen, C., Coifman, B., Varaiya, P. (2001). Pems algorithms for accurate, real-time estimates of g-factors and speeds from single-loop detectors. In Proceeding of IEEE ITS Annual Meeting, 536-541.