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Classical and Weighted Knowledgebase Transformations

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Abstract—In this paper, there is a review of some knowledgebase change operators, namely the revision, update, (symmetrical) model-fitting well known in the propositional case and some new problems concerning them. There is an extended set of axioms to avoid a certain problem in connection with revision. Based on the propositional case, we give some generalization of revision for first-order case. Furthermore we define an extension of the propositional knowledgebase to weighted knowledgebase. Finally we deal with the weighted knowledgebase transformations.

Keywords—Mathematical logic, Knowledgebases, Minimal model changes, Revision, Model-fitting.

1. INTRODUCTION

Generally, knowledgebases may be treated as some logical theory. For simplicity we suppose that *classical knowledgebases* are represented by a propositional (later first-order) well-formed formuli, and they are denoted by Greek letters. In the following, we refer to *classical knowledgebase* simply as knowledgebase. Later, when weighted knowledgebases occur, we will always precisely punctuate it by the word *weighted*.

The problem is the following: given knowledgebases φ (describing the originally stored information) and μ (the new knowledge) what should be the result of modification of φ by μ?

There are several theory change operators (see a review in [1,2]) which give different answers for the question. In this paper we deal with three types of them: the update, the revision and the model-fitting operators characterized in an axiomatic way by Katzuno and Mendelzon in [1,2], and Revesz in [3].

It turns out that these axioms imply a special minimality property: each operator picks up exactly those interpretations, which are minimal with respect to a previously defined preorder among the interpretations.

Section 2 is an overview of the propositional knowledgebase change operators and the problems occuring with them. Section 3 gives first order extensions of the update, revision and model-

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Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX fitting operators. After a brief preliminary (in 3.1) in 3.2 we review the first-order update of Grahne, Mendelzon and Revesz [4]. In 3.3 we give a new concrete operator for first-order revision. Section 4 deals with the weighted knowledgebases. In 4.1 we modify the original idea of weighted knowledgebase in [3]. The revision transformation is defined for weighted knowledgebases and a minimality theorem is proved in 4.2. A special solution is given for the model-fitting for weighted knowledgebases in 4.3. Finally Section 5 concludes with some open problems.

2. PROPOSITIONAL KNOWLEDGEBASE CHANGE OPERATORS

2.1. Motivation

This section is a brief survey on the background of the propositional knowledgebase change operators, namely the update, revision, and (symmetrical) model-fitting, as they were originally introduced.

The propositional formulas φ and μ represent two knowledgebases. Let φ be the original knowledgebase which will be modified by μ. μ represents the new information about the world initially described by φ. This modification is carried out by a theory change operator denoted by $\blacklozenge$. The resulting knowledgebase $\varphi\blacklozenge\mu$ can be defined in several ways depending on our expectations fixed in advance.

In [1–3] the authors gave the axioms (U1)–(U8) for the update, the axioms (R1)–(R6) for revision, and the axioms (M1)–(M8) for model-fitting. These axioms express the following ideas about the particular operators.

The update operator will be applied for φ, if the world—described correctly by φ—changes and we have some partial information about the new state of the world.

For the situation in which the world given by φ is static, but there is some new information about this static world represented by μ, the revision operator should be applied.

In these cases, the knowledgebase μ is supposed to be "truer" than the original knowledgebase φ, in the sense that after performing the update or revision operation, the resulting formula $\varphi\blacklozenge\mu$ implies μ.

Similarly, this property is still valid in the case of model-fitting, but the symmetrical model-fitting differs from the two above at this point. The symmetrical model-fitting operator is an application of model-fitting. It handles the knowledgebases φ and μ in an equivalent way. Neither of them is more important than the other; they play the same role from the point of view of modification. The aim of the symmetrical model-fitting is to find the best fit models for both knowledge bases.

2.2. Basic Notions and Notations

Let L_0 be a propositional language. The finite set of propositional terms is T. The subset of T is an interpretation. The set of all interpretations is $\Im$. The well-formed formulas can be constructed in the usual way. The models of a formula φ are denoted by $\mathrm{Mod}(\varphi)$. If φ is a propositional term t, then $\mathrm{Mod}(t) := \{I \mid I \in \Im,\ t \in I\}$. For the composed formula φ, $\mathrm{Mod}(\varphi)$ is the following:

$$\begin{aligned}\mathrm{Mod}(\neg\varphi) &= \Im \setminus \mathrm{Mod}(\varphi),\\ \mathrm{Mod}(\varphi \vee \mu) &= \mathrm{Mod}(\varphi) \cup \mathrm{Mod}(\mu),\\ \mathrm{Mod}(\varphi \wedge \mu) &= \mathrm{Mod}(\varphi) \cap \mathrm{Mod}(\mu).\end{aligned}$$

If $I_1, I_2, \dots, I_k$ are interpretations, *form* $(I_1, I_2, \dots, I_k)$ means those formulas whose models are exactly $I_1, I_2, \dots, I_k$. The set of all propositional formulas is denoted by F.

We say that φ implies μ if and only if $\text{Mod}(\varphi) \subseteq \text{Mod}(\mu)$.

In the following we will need the notion of a preorder among the interpretations. A preorder $\leq$ over $\Im$ is a reflexive and transitive relation on $\Im$. It is total, if for every pair $I, J \in \Im$ either $I \leq J$ or $J \leq I$ holds. $I < J$ if and only if $I \leq J$ but $J \not\leq I$ does not hold. The set of preorders over $\Im$ is denoted by PO.

The set of minimal interpretations in a subset $S \subseteq \Im$ with respect to the preorder $\leq$ is denoted by $\text{Min}\{S, \leq\}$ and defined as follows: $\text{Min}\{S, \leq\} := \{I \mid I \in S$, and there does not exist $J \in S$ for which $J < I\}$.

2.3. Propositional Update Operators

Based on the AGM-postulates in [5], Katzuno and Mendelzon gave a set of axioms for propositional revision operators [1,2], and to express the real practical needs, the set of axioms for the propositional update operators. First we deal with the update operators.

Let $\Diamond : F \times F \to F$ be a knowledgebase change operator. $\Diamond$ is called an update operator if and only if it satisfies the following axioms.

(U1) $\varphi\Diamond\mu$ implies μ.
(U2) If φ implies μ then $\varphi\Diamond\mu$ is equivalent to φ.
(U3) If both φ and μ are satisfiable then $\varphi\Diamond\mu$ is also satisfiable.
(U3) If $\varphi_1 \leftrightarrow \varphi_2$ and $\mu_1 \leftrightarrow \mu_2$ then $\varphi_1\Diamond\mu_1 \leftrightarrow \varphi_2\Diamond\mu_2$.
(U5) $(\varphi\Diamond\mu) \wedge \nu$ implies $\varphi\Diamond(\mu \wedge \nu)$.
(U6) If $\varphi\Diamond\mu_1$ implies μ_2 and $\varphi\Diamond\mu_2$ implies μ_1 then $\varphi\Diamond\mu_1 \leftrightarrow \varphi\Diamond\mu_2$.
(U7) If $|\text{Mod}(\varphi)| = 1$, then $(\varphi\Diamond\mu_1) \wedge (\varphi\Diamond\mu_2)$ implies $\varphi\Diamond(\mu_1 \wedge \mu_2)$.
(U8) $(\varphi_1 \wedge \varphi_2)\Diamond\mu \leftrightarrow (\varphi_1\Diamond\mu) \vee (\varphi_2\Diamond\mu)$.

The intuitive meaning behind these axioms are detailed in [1,2]. The main idea is that each possible world (the models) can be updated independently, and then the result should consist of some information from each of them. It is important that inconsistent knowledgebases cannot be corrected by an update operator.

In [1,2], Katzuno and Mendelzon proved the following minimality property.

THEOREM 2.3.1. *The knowledgebase change operator $\Diamond : F \times F \to F$ satisfies the axioms (U1)–(U8) if and only if there is a function f mapping each interpretation I to a partial preorder $\leq_I$ such that for any pair $I, J \in \Im$, if $I \neq J$ then $I <_I J$, and*

$$\text{Mod}(\varphi\Diamond\mu) = \bigcup_{I \in \text{Mod}(\varphi)} \text{Min}\{\text{Mod}(\mu), \leq_I\}.$$

2.4. Propositional Revision Operators

The other set of axioms introduced by Katzuno and Mendelzon is the restriction of the AGM-postulates to the propositional case. That is, the knowledgebase change operator $^\circ : F \times F \to F$ is called a revision operator, if it satisfies the following axioms.

(R1) $\varphi^\circ\mu$ implies μ.
(R2) If $\varphi \wedge \mu$ is satisfiable then $\varphi^\circ\mu \leftrightarrow \varphi \wedge \mu$.
(R3) If μ is satisfiable, then $\varphi^\circ\mu$ is also satisfiable.
(R4) If $\varphi_1 \leftrightarrow \varphi_2$ and $\mu_1 \leftrightarrow \mu_2$ then $\varphi_1 \circ \mu_1 \leftrightarrow \varphi_2 \circ \mu_2$.
(R5) $(\varphi^\circ\mu) \wedge \nu$ implies $\varphi^\circ(\mu \wedge \nu)$.
(R6) If $(\varphi^\circ\mu)^\wedge\nu$ is satifiable then $\varphi^\circ(\mu^\wedge\nu)$ implies $(\varphi^\circ\mu) \wedge \nu$.

In order to show a model-theoretic characterization of propositional revision operators we have to introduce first the concept of faithful functions, which are defined as follows.

DEFINITION 2.4.1. *The function $f : F \to \mathrm{PO}$ is said to be faithful if the following properties hold.*

(i) *If $M, M' \in \mathrm{Mod}(\varphi)$ then $M <_\varphi M'$ does not hold.*
(ii) *If $M \in \mathrm{Mod}(\varphi)$ and $I \notin \mathrm{Mod}(\varphi)$ then $M <_\varphi I$ holds.*
(iii) *If $\varphi \leftrightarrow \mu$ then $f(\varphi) = f(\mu)$.*

Similarly to 2.3.1 the following theorem holds [1,2].

THEOREM 2.4.2. *The knowledgebase change operator $\circ : F \times F \to F$ satisfies the axioms (R1)–(R6) if and only if there is a faithful function f mapping each knowledgebase φ to a total preorder $\leq_\varphi$ for which*

$$\mathrm{Mod}\,(\varphi^\circ \mu) = \mathrm{Min}\,\{\mathrm{Mod}(\mu), \leq_\varphi\}.$$

For example Dalal's [6,7] operator is a real revision operator, since it satisfies axioms (R1)–(R6). Dalal introduced the following distance function between two interpretations: $\mathrm{dist}\,(I, J) := |I \oplus J|$ where $\oplus$ is the symmetric set difference

$$I \oplus J := (I \setminus J) \cup (J \setminus I).$$

The distance between the knowledgebase φ and an interpretation I is the minimum distance between I and the models of φ :

$$\mathrm{dist}\,(\varphi, I) := \min_{J \in \mathrm{Mod}(\varphi)} \{\mathrm{dist}\,(I, J)\}.$$

Based on this distance, the following preorder can be defined: $I \leq_\varphi J$ if and only if $\mathrm{dist}\,(\varphi, I) \leq \mathrm{dist}\,(\varphi, J)$. Clearly the function f_D, which maps φ to $\leq_\varphi$, is faithful, so the operator defined by $\mathrm{Mod}(\varphi^\circ\mu) = \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_\varphi\}$ is a revision operator. This operator satisfies our expectations: the interpretations, which are picked up by this revision operator, are not only formally the closest models of μ to φ with respect to the preorder $\leq_\varphi$, but they are also intuitively acceptable. So we "feel" that the function f_D and the corresponding preorder are correct in this sense. Unfortunately, it is easy to construct formally correct, but intuitively unacceptable faithful functions, with the help of the minimality theorem for the revision. For example, suppose that the arrangement of the interpretations according to a faithful function f is the increasing sequence $I_1 \leq_\varphi I_2 \leq_\varphi \cdots \leq_\varphi \cdots I_k$. The models $I_1, I_2, \ldots, I_n$ of φ should lead the sequence. Let us fix the first n places for these first n interpretations in the arrangement for each formula φ. Then the arrangement among the remaining $k - n$ interpretations can be defined nearly arbitrarily; the only criterion is that equivalent formulae should have the same arrangement. Let us define the preorder $\leq^*_\varphi$ among the interpretations as follows:

$$I \leq^*_\varphi J \begin{cases} \text{if } I \leq_\varphi J, \text{ and } I, J \notin \mathrm{Mod}(\varphi), \text{ and } |\mathrm{Mod}(\varphi)| \text{ is odd,} \\ \text{if } I \leq_\varphi I, \text{ and } I, J \notin \mathrm{Mod}(\varphi), \text{ and } |\mathrm{Mod}(\varphi)| \text{ is even,} \\ \text{if } I \in \mathrm{Mod}(\varphi),\ J \notin \mathrm{Mod}(\varphi), \end{cases}$$
$$I =^*_\varphi J \quad \text{if } I, J \in \mathrm{Mod}(\varphi),$$

where the preorder $\leq_\varphi$ means Dalal's preorder as described above. Then the function f^*, which assigns to each knowledgebase φ the total preorder $\leq^*_\varphi$, is clearly faithful. Hence the operator $*$ defined by $\mathrm{Mod}(\varphi^*\mu) = \mathrm{Min}\{\mathrm{Mod}(\mu), \leq^*_\varphi\}$ is a revision operator. Now compare the results of the operators $*$ and Dalal's operator. For the knowledgebases which have an even number of models, applying the operator $*$, we get just the furthest models of μ to φ with respect to the Dalal's operator, if they have no common models. Although the function f^* is faithful, so the operator $*$ satisfies the axioms (R1)–(R6), this result should not be acceptable, because we feel that the function f^* is incorrect in the following sense: the operator corresponding to f^* picks up not the intuitively closest models of μ.

It turns out that we need further axioms to avoid the problems mentioned above. The class of revision operators can be restricted by adding new axiom(s) to the original ones. For example, the following axiom can be attached to (R1)–(R6):

(R7) $(\varphi_1 \bullet \mu) \wedge (\varphi_2 \bullet \mu)$ implies $(\varphi_1 \vee \varphi_2) \bullet \mu$.

Clearly (R1)–(R7) are consistent. So the class of revision operators can be refined in this way. Introducing the notion of loyality, a minimality theorem holds.

DEFINITION 2.4.3. *The function $f : F \to \mathrm{PO}$ is said to be loyal, if*

(i) *$I \leq_\varphi J$ and $I \leq_\mu J$ then $I \leq_{\varphi\vee\mu} J$;*
(ii) *$\varphi \leftrightarrow \mu$, then $f(\varphi) = f(\mu)$.*

REMARK. The property (ii) seems to be redundant since it appears in the definition of faithfulness (2.4.1) (as the property (iii)), but it is necessary later for the operation of model-fitting.

THEOREM 2.4.4. *The knowledgebase change operator $\bullet : F \times F \to F$ satisfies the axioms (R1)–(R7) if and only if there is a faithful and loyal function f mapping each knowledgebase φ to a total preorder $\leq_\varphi$ for which*

$$\mathrm{Mod}(\varphi \bullet \mu) = \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_\varphi\}.$$

PROOF. Only if: suppose that there exists the operator $\bullet$, which satisfies the axioms (R1)–(R7). Let the function f assign to each knowledgebase φ the preorder $\leq_\varphi$ for which $I \leq_\varphi J$, if and only if $I \in \mathrm{Mod}(\varphi \bullet \mathrm{form}(I,J))$. We shall prove the following properties:

(i) f is faithful;
(ii) $\mathrm{Mod}(\varphi \bullet \mu) = \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_\varphi\}$;
(iii) f is loyal.

The points (i) and (ii) can be proved similarly to the proof of the original theorem in [2, Theorem 3.1].

For (iii), suppose that $I \leq_{\varphi_1} J$ and $I \leq_{\varphi_2} J$. Then $I \in \mathrm{Mod}(\varphi_1 \bullet \mathrm{form}(I,J))$ and $I \in \mathrm{Mod}(\varphi_2 \bullet \mathrm{form}(I,J))$. Applying axiom (R7), $I \in \mathrm{Mod}((\varphi_1 \vee \varphi_2) \bullet \mathrm{form}(I,J))$; that is, $I \leq_{\varphi_1\vee\varphi_2} J$, and hence f is loyal.

If: suppose that there is a faithful and loyal function f, which assigns to each knowledgebase φ the preorder $\leq_\varphi$. Then the following operator $\bullet$ satisfies the axioms (R1)–(R7): $\mathrm{Mod}(\varphi \bullet \mu) = \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_\varphi\}$.

The axioms (R1)–(R6) follow from the faithful property and the minimal model. The proof can be carried out similarly to the original theorem in [2, Theorem 3.1].

(R7) follows from the loyality: if $I \in \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_{\varphi_1}\}$, and $I \in \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_{\varphi_2}\}$, then $I \leq_{\varphi_1} J$ and $I \leq_{\varphi_2} J$ for any other interpretation $J \in \mathrm{Mod}(\mu)$. Because of loyality, $I \leq_{\varphi_1\vee\varphi_2} J$ holds, and hence $I \in \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_{\varphi_1\vee\varphi_2}\}$; that is, the axiom (R7) also holds.

Clearly, the Dalal's revision operator satisfies the extended set of axioms (R1)–(R7) as well, since the function f_D is faithful and loyal.

With the loyalty requirement some of the faithful but unintuitive functions have been eliminated, e.g., the function f^*. To prove this, suppose that $I \leq_\varphi J$ and $I \leq_\mu J$, where $\leq_\varphi$ and $\leq_\mu$ are the functional values of f_D at φ and μ, respectively. Then by loyality, $I \leq_{\varphi\vee\mu} J$ holds. Suppose furthermore that both $|\mathrm{Mod}(\varphi)|$ and $|\mathrm{Mod}(\mu)|$ are odd, and $\mathrm{Mod}(\varphi) \wedge \mathrm{Mod}(\mu) = \emptyset$. Then $|\mathrm{Mod}(\varphi \vee \mu)|$ is even. The function f^* assigns to the knowledgebases φ, μ and $\varphi \vee \mu$ the the preorders $\leq^*_\varphi, \leq^*_\mu$ and $\leq^*_{\varphi\vee\mu}$, respectively. By the definition of f^*, $I \leq^*_\varphi J$ and $I \leq^*_\mu J$, since $|\mathrm{Mod}(\varphi)|$ and $|\mathrm{Mod}(\mu)|$ are odd. But $I \leq_{\varphi\vee\mu} J$, and the fact that $|\mathrm{Mod}(\varphi \vee \mu)|$ is even implies that $J \leq^*_{\varphi\vee\mu} I$ does not hold. So f^* cannot be loyal. Furthermore this example shows that the axiom (R7) is independent of the axioms (R1)–(R6).

The axiom (R7) was originally introduced in [3] for one of the axioms of the operator called model-fitting. This operator is discussed in Section 2.5, and in 4.3 for weighted knowledgebases.

2.5. Propositional Model-Fitting Operators

As we have already mentioned, the axiom (R7) was introduced originally in [3], as an axiom—the (M7) below—for model-fitting. Here we give a restricted set of axioms for model-fitting. The knowledgebase change operator $\nabla : F \times F \to F$ is a model-fitting operator if it satisfies the following axioms.

(M1) $\varphi\nabla\mu$ implies μ.
(M2) If φ is unsatisfiable then $\varphi\nabla\mu$ is unsatisfiable.
(M3) If both φ and μ are satisfiable then $\varphi\nabla\mu$ is also satisfiable.
(M4) If $\varphi_1 \leftrightarrow \varphi_2$ and $\mu_1 \leftrightarrow \mu_2$ then $\varphi_1\nabla\mu_1 \leftrightarrow \varphi_2\nabla\mu_2$.
(M5) $(\varphi\nabla\mu) \wedge \nu$ implies $\varphi\nabla(\mu \wedge \nu)$.
(M6) If $(\varphi\nabla\mu) \wedge \nu$ is satisfiable then $\varphi\nabla(\mu \wedge \nu)$ implies $(\varphi\nabla\mu) \wedge \nu$.
(M7) $(\varphi_1\nabla\mu) \wedge (\varphi_2\nabla\mu)$ implies $(\varphi_1 \vee \varphi_2)\nabla\mu$.

The minimality theorem also holds in this case.

Theorem 2.5.1. *The knowledgebase change operator $\nabla : F \times F \to F$ satisfies the axioms (M1)–(M7), if and only if there is a loyal function which maps each knowledgebase φ to a total preorder $\leq_\varphi$ such that*

$$\mathrm{Mod}(\varphi\nabla\mu) = \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_\varphi\}.$$

The proof can be found in [3].

The class of model-fitting operators and the revision operators are not disjoint, since the function f_D is loyal as well.

An example for model-fitting is the following: let the distance $\mathrm{dist}\,(I, J)$ of two interpretations I, J be equal to $|I \oplus J|$. Then the distance between the knowledgebase φ and an interpretation I can be defined as

$$o\_\mathrm{dist}\,(\varphi, I) := \operatorname*{Max}_{J \in \mathrm{Mod}(\varphi)} \{\mathrm{dist}\,(I, J)\}.$$

Then $I \leq'_\varphi J$ if and only if $o\_\mathrm{dist}\,(\varphi, I) \leq o\_\mathrm{dist}\,(\varphi, J)$. Clearly the function which maps φ to $\leq'_\varphi$ is loyal. o_dist can be interpreted as an overall distance between the knowledgebase φ and the interpretation I.

For the completeness we should touch upon the symmetrical model-fitting operation. This operation is also referred to as arbitration in [3]. It is an application of model-fitting.

Definition 2.5.2. *The symmetrical model-fitting operator $\Delta : F \times F \to F$ is defined by*

$$\mathrm{Mod}(\varphi\Delta\mu) := \mathrm{Mod}((\varphi \vee \mu)\nabla(\mathrm{form}\,(\Im))).$$

Clearly in case of symmetrical model-fitting the roles of the knowledgebases are symmetrical.

3. FIRST-ORDER KNOWLEDGEBASE CHANGE OPERATORS

In this section we define and interpret a restricted first-order language. We follow the presentation in [4].

3.1. Preliminaries

The first order function-free language L_1 contains symbols of the following kind.

Variables:	$X := \{x_i \mid i \in N\}$
Constants:	$C := \{c_i \mid i \in N\}$
Predicates:	$R := \{R_i \mid i \in N\}$
Punctuation signs:	(,)
Logical connectives:	$\wedge; \vee; \neg$
Quantifier:	$\exists$
Equality:	$=$

The notation $\mathrm{ar}(i)$ means the *arity* of R_i. Variables and constants are *terms*. If $\mathrm{ar}(i) = n$ and $t_1, t_2, t_3, \ldots, t_n$ are terms then $R(t_1, t_2, t_3, \ldots, t_n)$ and $t_k = t_l$ are *atoms*. If $t_1, t_2, t_3, \ldots, t_n$ are all constants, then $R(t_1, t_2, t_3, \ldots, t_n)$ and $t_k = t_l$ are *ground atoms*.

The *well-formed* formulas are defined in the usual way. The set of sentences is S. A *database d* is a finite set of relations $\{r_1, r_2, r_3, \ldots, r_n\}$ where each $r_i \in C^{\mathrm{ar}(i)}$. The *schema of the database d* is $s(d) = \{R_1, R_2, R_3, \ldots, R_n\}$. The *schema of a well-formed formula* μ consists of the set of all predicate symbols occuring in μ, denoted by $s(\mu)$.

The *interpretations* of μ are those databases d for which $s(\mu) \subseteq s(d)$. The set of all interpretations is IN. The set of all databases is DB.

The *models* of μ are those interpretations d of μ, for which the following properties hold: if μ is in the form of

(i) $a_i = a_j$ then $i = j$;
(ii) $R_i(c_1, c_2, c_3, \ldots c_n)$ then $\langle c_1, c_2, c_3, \ldots c_n \rangle \in r_i$;
(iii) $\nu \wedge \varphi$ then d is a model of ν, and d is a model of φ;
(iv) $\nu \vee \varphi$ then d is a model of μ or φ;
(v) $\neg \nu$ then d is not a model of ν;
(vi) $\exists x \nu$ then d is a model of $\nu(x \mid c)$, $c \in C$, where $\nu(x \mid c)$ means the substitution c into all free occurences of x in the formula ν.

$\mathrm{Mod}(\mu)$ denotes the set of all models of μ. By a *knowledgebase k* we mean a finite set of databases with the same schema. The *schema of the knowledgebase* is equal to the schema of its components. For example, the set of models of a formula μ is a knowledgebase. The set of all knowledgebases is denoted by KB.

3.2. Updating First-Order Knowledgebases

According to Theorem 2.3.1, updating a knowledgebase k with respect to the formula f means finding for each database d of k the closest interpretation among the models of f with respect to a class of family of partial preordering, $\leq_d$. Then the updated knowledgebase is the union of these pointwise closest models. Since each database d corresponds to a propositional formula (see, e.g., [8]), the theorem can be immediately applied. Reference [4] defines a pointwise comparison among the databases in the following way. The database d_m is closer to the database d than d_n, iff

(i) $s(d_m) = s(d_n)$ and $s(d) \subseteq s(d_m)$.
(ii) $d_m \leq_d d_n$ iff for $r_i^m \in d_m$, $r_i^n \in d_n$, $r_i \in d$,
  (a) $r_i^m \oplus r_i \subseteq r_i^n \oplus r_i$ (where $\oplus$ means the symmetric difference: $A \oplus B = (A \setminus B) \cup (B \setminus A)$) for all relations whose schemas occur in each of d_m, d_n, and d.
  (b) $r_i^m \oplus \emptyset \subseteq r_i^n \oplus \emptyset$ for the remaining relations.

Clearly, $\leq_d$ is a partial ordering on DB. Similarly to the propositional case, the database d_m is minimal in $D \subseteq$ DB with respect to $\leq_d$ iff $d_m \in D$, and for each if $d_n \in D$ and $d_n \leq_d d_m$ implies $d_n = d_m$. We denote the set of minimal elements by $\mathrm{Min}\{D, \leq_d\}$. Now, the transformation function

$$u : \mathrm{KB} \times S \to \mathrm{KB}, \qquad u(k,\varphi) := \bigcup_{d \in k} \mathrm{Min}\{\mathrm{Mod}(\varphi), \leq_d\}$$

satisfies the update axioms (U1)–(U8) as it is shown in [4], so it is a real update function.

3.3. Revising First-Order Knowledgebases

The first-order revision can be carried out analoguosly to the update operator. That is, to find a revision operator we have to define a faithful function. Dalal's [6,7] distance for propositional interpretations can be extended also for first-order databases in the following way [9].

The distance between any two relations r_i, r_j with the same schema R is

$$\mathrm{dist}\,(r_i, r_j) := |r_i \oplus r_j|. \tag{3.3.1}$$

The distance between any two databases d_m, d_n is

$$\mathrm{dist}\,(d_m, d_n) := \sum_i \mathrm{dist}\,(r_i^m, r_i^n), \qquad \text{where } r_i^m \in d_m, \quad r_i^n \in d_n. \tag{3.3.2}$$

Then the distance between the knowledgebase k and the database d is:

$$k\_\mathrm{dist}\,(k, d) := \mathop{\mathrm{Min}}_{d_k \in k}\{\mathrm{dist}\,(d_k, d)\}. \tag{3.3.3}$$

Thus $d_m \leq_k d_n$ iff $k\_\mathrm{dist}\,(k, d_m) \leq k\_\mathrm{dist}\,(k, d_n)$. Now consider the following assignment: each knowledgebase k corresponds the preorder $\leq_k$ defined by (3.3.1)–(3.3.3). Clearly, $\leq_k$ is a total preorder, and the assignment is faithful. So the function

$$r : \mathrm{KB} \times S \to \mathrm{KB}, \qquad r(k,\varphi) := \mathrm{Min}\{\mathrm{Mod}(\varphi), \leq_k\}, \tag{3.3.4}$$

satisfies the Theorem 2.4.2 above, and the axioms (R1)–(R6).

Formula (3.3.4) means if k is the original knowledgebase and φ represents the new information about the world (described by k), and we want to revise k with respect to φ, then the result(s) is (are) the model(s) of φ being closest to k.

REMARK. Obviously in (3.3.1) for $\mathrm{dist}\,(r_i, r_j)$ instead of $|r_i \oplus r_j|$ we can take, e.g.,

1. $$\mathrm{dist}\,(r_i, r_j) := \begin{cases} \dfrac{|r_i \setminus r_j|}{|r_i|} + \dfrac{|r_j \setminus r_i|}{|r_j|} & \text{if } |r_i| \neq 0, \quad |r_j| \neq 0, \\ \dfrac{|r_i \setminus r_j|}{|r_i|} & \text{if } |r_i| \neq 0, \quad |r_j| = 0, \\ \dfrac{|r_j \setminus r_i|}{|r_j|} & \text{if } |r_i| = 0, \quad |r_j| \neq 0, \\ 0 & \text{if } |r_i| = 0, \quad |r_j| = 0. \end{cases}$$

2. $$\mathrm{dist}\,(r_i, r_j) := \frac{|r_i \oplus r_j|}{|r_i \cup r_j|}.$$

The distances 1. and 2. are better measurments of the similarity of the relations than (3.3.1) since they give information not only about the number of different rows in the relations but their proportion to the size of the relations.

Clearly, all the examples also satisfy axiom (R7).

4. WEIGHTED KNOWLEDGEBASES

4.1. Introduction

In this section we modify the notion of the weighted knowledgebases introduced in [3]. The aim is to extend propositional logic with the possibility of expressing the relative degree of importance of interpretations.

DEFINITION 4.1.1. *A weighted knowledgebase is the function* $\underline{\varphi} : \Im \to [0,1]$.

A weighted interpretation is the ordered pair $(I, \alpha) \in \Im \times [0,1]$.

The model of a weighted knowledgebase $\underline{\varphi}$ *is that interpretation for which* $\underline{\varphi}(I) \geq \alpha > 0$, *so the model set of* φ *is the following:*

$$\mathrm{Mod}(\underline{\varphi}) := \left\{(I,\alpha) \mid I \in \Im,\ \underline{\varphi}(I) \geq \alpha > 0\right\}.$$

It follows from this definition that the weighted knowledgebase $\underline{\varphi}$ is *unsatisfiable* iff $\underline{\varphi}(I) = 0$ for all $I \in \Im$.

The set of interpretations for which $\underline{\varphi}(I) > 0$ is denoted by $C\_\mathrm{Mod}(\underline{\varphi})$ (Classical Model). Clearly, $I \in C\_\mathrm{Mod}(\underline{\varphi})$, iff $(I,\alpha) \in \mathrm{Mod}(\underline{\varphi})$ for some $\alpha > 0$.

We say that the weighted knowledgebase $\underline{\varphi}$ implies the weighted knowledgebase $\underline{\mu}$, iff for all $I \in \Im$, $\underline{\varphi}(I) \leq \underline{\mu}(I)$. This fact is denoted by $\underline{\varphi} \to \underline{\mu}$. The definition of equivalence follows from the foregoing: $(\underline{\varphi} \to \underline{\mu}) \wedge (\underline{\mu} \to \underline{\varphi}) = \underline{\varphi} \leftrightarrow \underline{\mu}$; that is, the knowledgebases $\underline{\varphi}$ and $\underline{\mu}$ are equivalent if $\underline{\varphi}(I) = \underline{\mu}(I)$ for all $I \in \Im$.

The set of all weighted knowledgebases is denoted by $\underline{F}$.

We can define the disjunction, conjuntion and negation as follows.

DEFINITION 4.1.2.

$$\begin{aligned}
\underline{\varphi} \vee \underline{\mu}(I) &= \mathrm{Max}\left\{\underline{\varphi}(I), \underline{\mu}(I)\right\},\\
\underline{\varphi} \wedge \underline{\mu}(I) &= \mathrm{Min}\left\{\underline{\varphi}(I), \underline{\mu}(I)\right\},\\
\neg\underline{\varphi} &= 1 - \underline{\varphi}(I).
\end{aligned}$$

In [3], the weights are positive numbers. That is why the negation is not defined there. The disjunction of two weighted knowledgebases in [3] is defined as the sum of the corresponding weights.

In the following, we deal with the weighted knowledgebase transformations.

4.2. Revision for Weighted Knowledgebases

In this section, we define the revision operation for weighted knowledgebases. The axioms (R1)–(R6) should be valid for weighted knowledgebases as well. But because of the definition of the equivalence, we do not need the axiom (R4). So we say that the operator $\underline{\circ} : \underline{F} \times \underline{F} \to \underline{F}$ is a *weighted revision operator* iff it satisfies the following axioms.

(WR1) $\underline{\varphi} \,\underline{\circ}\, \underline{\mu}$ implies $\underline{\mu}$.
(WR2) If $\underline{\varphi} \wedge \underline{\mu}$ is satisfiable, then $\underline{\varphi} \,\underline{\circ}\, \underline{\mu} \leftrightarrow \underline{\varphi} \wedge \underline{\mu}$.
(WR3) If $\underline{\mu}$ is satisfiable, then $\underline{\varphi} \,\underline{\circ}\, \underline{\mu}$ is satisfiable as well.
(WR4) $(\underline{\varphi} \,\underline{\circ}\, \underline{\mu}) \wedge \underline{\nu}$ implies $\underline{\varphi} \,\underline{\circ}\, (\underline{\mu} \wedge \underline{\nu})$.
(WR5) If $(\underline{\varphi} \,\underline{\circ}\, \underline{\mu}) \wedge \underline{\nu}$ satisfiable then $\underline{\varphi} \,\underline{\circ}\, (\underline{\mu} \wedge \underline{\nu})$ implies $(\underline{\varphi} \,\underline{\circ}\, \underline{\varphi}) \wedge \underline{\nu}$.

To get the similar result to Theorem 2.4.2 we need a preordering among the weighted interpretations. Let us denote the set of the preorders over the set $\Im \times [0,1]$ by $\underline{\mathrm{PO}}$.

DEFINITION 4.2.1. *The function $f : \underline{F} \to \underline{\mathrm{PO}}$ is said to be faithful if it satisfies the following properties.*

(i) *The preorder is total with respect to the first element of the pairs.*
(ii) *If $I \in C\_\mathrm{Mod}(\varphi)$ and $I \notin C\_\mathrm{Mod}(\underline{\varphi})$ then $(I,\alpha) \leq_{\varphi} (J,\beta)$.*
(iii) *If $(I,\alpha),(J,\beta) \in \mathrm{Mod}(\underline{\varphi})$ then $(I,\alpha) \leq_{\underline{\varphi}} (J,\beta)$ and $(J,\beta) \leq_{\underline{\varphi}} (I,\alpha)$.*
(iv) *For all weighted knowledgebase $\underline{\varphi}$ and interpretation I there exists the constant $\alpha_{\underline{\varphi}}(I) \in]0,1]$ depending on $\underline{\varphi}$, for which $(I, \mathrm{Min}\{\alpha_{\underline{\varphi}}(I),\beta\}) \leq_{\underline{\varphi}} (I,\beta)$ and $\alpha_{\underline{\varphi}}(I) = \underline{\varphi}(I)$, whenever $I \in \mathrm{Mod}(\underline{\varphi})$.*

REMARK. Property (iii) means if $I, J \in C\_\mathrm{Mod}(\underline{\varphi})$ then $I =_{\mathrm{form}(C\_\mathrm{Mod}(\underline{\varphi}))} J$.

Using this definition the following theorem holds.

THEOREM 4.2.2. *The operator $\underline{\circ} : \underline{F} \times \underline{F} \to \underline{F}$ satisfies the axioms (WR1)–(WR5) iff there exists a faithful function f which maps each weighted knowledgebase $\underline{\varphi}$ to the preorder $\leq_{\underline{\varphi}}$, and*

$$\mathrm{Mod}\left(\underline{\varphi} \,\underline{\circ}\, \underline{\mu}\right) = \mathrm{Min}\left\{\mathrm{Mod}(\mu), \leq_{\underline{\varphi}}\right\}.$$

PROOF. PART I. Suppose that the operator $\underline{\circ}$ satisfies the axioms (WR1)–(WR5). The function $\underline{f}$ maps the weighted knowledgebase $\underline{\varphi}$ to the following relation $\leq_{\underline{\varphi}}$.

(i) $(I,\alpha) \leq_{\varphi} (J,\beta)$ iff $I \in C\_\mathrm{Mod}(\underline{\varphi}\,\underline{\circ}((I,1) \vee (J,1)))$, and $I \neq J$.
(ii) $(I, \mathrm{Min}\{\alpha_{\underline{\varphi}}(I),\beta\}) \leq_{\underline{\varphi}} (I,\beta)$, where $\alpha_{\underline{\varphi}}(I) = (\underline{\varphi}\,\underline{\circ}(I,1))(I)$.

We have to show that

(A) the function $\underline{f}$ is faithful;
(B) $\mathrm{Mod}(\underline{\varphi} \,\underline{\circ}\, \underline{\mu}) = \mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$.

PART IA. First we prove that the relation $\leq_{\underline{\varphi}}$ is a preorder, satisfying the requirement of totality with respect to the first elements of the pairs (the property (i) of the faithfulness).

The relation is total with respect to the first element of the pairs, since by the axioms (WR1) and (WR3) $\mathrm{Mod}(\underline{\varphi}\,\underline{\circ}((I,1) \vee (J,1)))$ is a nonempty subset of $\mathrm{Mod}((I,1) \vee (J,1))$, so any pair of interpretations are comparable.

The relation is reflexive by the definition of the relation $\leq_{\underline{\varphi}}$ itself.

The transitivity occurs only in case of different first elements. So the proof can be restricted for the unweighted case; see the detailed proof, e.g., in [3, p. 80].

Now we prove the property (ii) of faithfulness: if $I \in C\_\mathrm{Mod}(\underline{\varphi})$ and $J \notin C\_\mathrm{Mod}(\underline{\varphi})$, then $(I,\alpha) <_{\underline{\varphi}} (J,\beta)$. Because of the axiom (WR2), $C\_\mathrm{Mod}(\underline{\varphi}\,\underline{\circ}((I,1) \vee (J,1)) = C\_\mathrm{Mod}(\underline{\varphi} \wedge ((I,1) \vee (J,1))) = C\_\mathrm{Mod}(I,1)$; hence $I \in C\_\mathrm{Mod}(\underline{\varphi}\,\underline{\circ}\,((I,1) \wedge (J,1)))$. But J cannot be in $C\_\mathrm{Mod}(\underline{\varphi}\,\underline{\circ}((I,1) \vee (J,1)))$, that is, by the definition of $\leq_{\underline{\varphi}}; (I,\alpha) <_{\underline{\varphi}} (J,\beta)$.

The property $(I,\alpha),(J,\beta) \in \mathrm{Mod}(\underline{\varphi})$ then $(I,\alpha) \leq_{\underline{\varphi}} (J,\beta)$ and $(J,\beta) \leq_{\underline{\varphi}} (I,\alpha)$ will be shown (property (iii)). Applying the axiom (WR2), $\mathrm{Mod}(\underline{\varphi}\,\underline{\circ}((I,1) \vee (J,1))) = \mathrm{Mod}(\underline{\varphi} \wedge ((I,1) \vee (J,1))) = \mathrm{Mod}((I,1) \vee (J,1)) = \{(I,\alpha),(J,\beta) \mid 1 \geq \alpha > 0,\ 1 \geq \beta > 0\}$, and hence, $(I,\alpha),(J,\beta) \in \mathrm{Mod}(\underline{\varphi} \bullet ((I,1) \vee (J,1))$; that is, $(I,\alpha) \leq_{\underline{\varphi}} (J,\beta)$ and $(J,\beta) \leq_{\underline{\varphi}} (I,\alpha)$.

For the property (iv) of the faithfulness, the constant $\alpha_{\underline{\varphi}}(I)$ has been already given in the definition of the relation $\leq_{\underline{\varphi}}$, so $(I, \mathrm{Min}\{\alpha_{\underline{\varphi}}(I),\beta\}) \leq_{\underline{\varphi}} (I,\beta)$ follows directly from this definition. We have to prove that $\alpha_{\underline{\varphi}}(I) = \underline{\varphi}(I)$ whenever $I \in \mathrm{Mod}(\underline{\varphi})$. It follows from the axiom (WR2), because—as we will prove it for the point (ii)—$\underline{\varphi} \,\underline{\circ}\, \underline{\mu}(I) = \mathrm{Min}\{\alpha_{\underline{\varphi}}(I), \underline{\mu}(I)\}$ always holds. If $I \in \mathrm{Mod}(\underline{\varphi})$ and $I \in \mathrm{Mod}(\underline{\mu})$ (which is the case) then $\underline{\varphi} \,\underline{\circ}\, \underline{\mu}(I) = \underline{\varphi} \wedge \underline{\mu}(I) = \mathrm{Min}\{\underline{\varphi}(I), \underline{\mu}(I)\}$. Hence $\mathrm{Min}\{\alpha_{\underline{\varphi}}(I), \underline{\mu}(I)\} = \mathrm{Min}\{\underline{\varphi}(I), \underline{\mu}(I)\}$. But $\underline{\mu}(I)$ can be any number in $]0,1]$, so the equality holds only in case $\alpha_{\underline{\varphi}}(I) = \underline{\varphi}(I)$.

PART IB. First we prove that $C\_\mathrm{Mod}(\underline{\varphi} \,\underline{\circ}\, \underline{\mu}) = C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$. We need to show both the $\subseteq$ and the $\supseteq$ directions. If either $\underline{\varphi}$ or $\underline{\mu}$ are unsatisfiable, then $C\_\mathrm{Mod}(\underline{\varphi} \,\underline{\circ}\, \underline{\mu}) = \emptyset = C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$. Hence, assume that both are satisfiable, and $C\_\mathrm{Mod}(\underline{\varphi} \,\underline{\circ}\, \underline{\mu}) \subseteq$

$C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$. Assume that $I \in C\_\mathrm{Mod}(\underline{\varphi} \circ \underline{\mu})$, and $I \notin C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$. Since I is not a minimal model, according to the definition of minimal, there must be another model $(J,\beta) \in \mathrm{Mod}(\underline{\mu})$ such that $(J,\beta) <_{\underline{\varphi}} (I,\alpha)$; i.e., $(J,\beta) \leq_{\underline{\varphi}} (I,\alpha)$ and $(I,\alpha) \not\leq_{\underline{\varphi}} (J,\beta)$. It means that $(I,\alpha) \notin (\underline{\varphi} \circ((I,\alpha) \vee (J,\beta))$. Since both I and J are in $C\_\mathrm{Mod}(\underline{\mu})$, $C\_\mathrm{Mod}(\underline{\mu}) \cap \{I,J\} = \{I,J\}$. By the axiom (WR5), $C\_\mathrm{Mod}(\underline{\varphi} \circ \underline{\mu}) \cap \{I,J\} \subseteq C\_\mathrm{Mod}(\underline{\varphi} \circ(\underline{\mu} \wedge ((I,\alpha) \vee (J,\beta))) = C\_\mathrm{Mod}(\underline{\varphi} \circ((I,\alpha) \vee (J,\beta))) = \{J\}$; hence, I cannot be in $C\_\mathrm{Mod}(\underline{\varphi} \circ \underline{\mu})$, which is a contradiction.

To prove the other direction assume that $I \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$ and $I \notin C\_\mathrm{Mod}(\underline{\varphi} \circ \underline{\mu})$. By the axiom (WR3), there is a model (J,β) of $\underline{\varphi} \circ \underline{\mu}(J,\beta)$ which is also in $\mathrm{Mod}(\underline{\mu})$ by the axiom (WR1). Since both I and J are in $C\_\mathrm{Mod}(\underline{\mu})$, $C\_\mathrm{Mod}(\underline{\mu}) \cap \{I,J\} = \{I,J\}$. Applying the axioms (WR4), (WR5), $C\_\mathrm{Mod}((\underline{\varphi} \circ \underline{\mu}) \wedge ((I,\alpha) \vee (J,\beta))) \subseteq C\_\mathrm{Mod}(\underline{\varphi} \circ(\underline{\mu} \wedge ((I,\alpha) \vee (J,\beta))) = C\_\mathrm{Mod}(\underline{\varphi} \circ((I,\alpha) \vee (J,\beta)))$ and by the axioms (WR1), (WR3), $C\_\mathrm{Mod}(\underline{\varphi} \circ((I,\alpha) \vee (J,\beta))) \subseteq \{I,J\}$. Since I is not in $C\_\mathrm{Mod}(\underline{\varphi} \circ \underline{\mu})$, $I \notin C\_\mathrm{Mod}(\underline{\varphi} \circ((I,\alpha) \vee (J,\beta)))$ as well. That is, $(J,\beta) <_{\varphi} (I,\alpha)$, and hence, $I \notin C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$, which is a contradiction.

Furthermore, we have to prove that $\underline{\varphi} \circ \underline{\mu}(I) = \mathrm{Min}\{\alpha_{\underline{\varphi}}(I), \underline{\mu}(I)\}$. By the axioms (WR1) and (WR3), $0 < \alpha_{\underline{\varphi}}(I) \leq (\underline{\varphi} \circ(I,1))(I)$. Let $(I, \underline{\mu}(I))$ be a model of the weighted knowledgebase $\underline{\mu}$. In this case $\underline{\mu}(I) > 0$, so $(\underline{\varphi} \circ(I,1)) \wedge \underline{\mu}$ satisfiable, and by the axioms (WR4) and (WR5), $((\underline{\varphi} \circ(I,1)) \wedge \underline{\mu})(I) = (\underline{\varphi} \circ((I,1) \wedge \underline{\mu}))(I)$.

Supposing that $\alpha_{\underline{\varphi}}(I) \geq \underline{\mu}(I)$, we get $((\underline{\varphi} \circ(I,1)) \wedge \underline{\mu})(I) = \underline{\mu}(I) = \underline{\varphi} \circ((I,1) \wedge \underline{\mu})(I) = \underline{\varphi} \circ \underline{\mu}(I)$.

Now supposing that $\underline{\mu}(I) > \alpha_{\underline{\varphi}}(I)$, then $((\underline{\varphi} \circ(I,1)) \wedge \underline{\mu})(I) = \alpha_{\underline{\varphi}}(I)$. On the other hand $\underline{\varphi} \circ((I,1) \wedge \underline{\mu})(I) = \underline{\varphi} \circ \underline{\mu}(I)$, and hence $\underline{\varphi} \circ \underline{\mu}(I) = \alpha_{\underline{\varphi}}(I)$.

So the equality $\underline{\varphi} \circ \underline{\mu}(I) = \mathrm{Min}\{\alpha_{\underline{\varphi}}(I), \underline{\mu}(I)\}$ has been proved, which means that the operator $\underline{\circ}$ determines really the minimal elements of $\mathrm{Mod}(\underline{\mu})$.

PART II. Now the faithful function $\underline{f}$ is supposed. This function assigns to the weighted knowledgebase $\underline{\varphi}$ the preorder $\leq_{\underline{\varphi}}$, and the operator $\underline{\circ}$ is defined by the equality $\mathrm{Mod}(\underline{\varphi} \circ \underline{\mu}) = \mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$. We have to prove that $\underline{\circ}$ satisfies the axioms (WR1)–(WR5).

Axiom (WR1) holds, since the result is a subset of $\mathrm{Mod}(\underline{\mu})$.

We prove the axiom (WR2) in two steps. In the first the equality $C\_\mathrm{Mod}(\underline{\varphi} \wedge \underline{\mu}) = C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$ will be proved. The satisfiability of $\underline{\varphi} \wedge \underline{\mu}$ is supposed.

First we prove the $\subseteq$ direction: $C\_\mathrm{Mod}(\underline{\varphi} \wedge \underline{\mu}) \subseteq C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$. The faithfulness of the function $\underline{f}$ ensures that if $I \in C\_\mathrm{Mod}(\underline{\varphi})$, then $I <_{\underline{\varphi}} J$ for all interpretation J, such that $J \notin C\_\mathrm{Mod}(\underline{\varphi})$. The interpretation I is in $C\_\mathrm{Mod}(\underline{\mu})$ because $I \in C\_\mathrm{Mod}(\underline{\varphi} \wedge \underline{\mu})$. Hence $I \in \mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$.

The other direction is $C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\varphi}\} \subseteq C\_\mathrm{Mod}(\underline{\varphi} \wedge \underline{\mu})$. Suppose that there exists an interpretation I, such that $I \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\varphi}\}$ and $I \notin C\_\mathrm{Mod}(\underline{\varphi} \wedge \underline{\mu})$. Because $\underline{\varphi} \wedge \underline{\mu}$ is satisfiable, there is a model J in $C\_\mathrm{Mod}(\underline{\varphi} \wedge \underline{\mu})$. The faithful function $\underline{f}$ ensures that $(J,\beta) < (I,\alpha)$ since J is in $C\_\mathrm{Mod}(\underline{\varphi})$ and I is not in it. Then I cannot be a minimal element of $\mathrm{Mod}(\underline{\mu})$.

In the second step, we need to show that the weights are also correct with respect to the definitions. It is a straightforward corrollary of the following identity:

$$\mathrm{Min}\left\{\alpha_{\underline{\varphi}}(I), \underline{\mu}(I)\right\} = \mathrm{Min}\left\{\underline{\varphi}(I), \underline{\mu}(I)\right\} = \left(\underline{\varphi} \wedge \underline{\mu}\right)(I).$$

Axiom (WR3) clearly holds because of the definition of the operator $\underline{\circ}$.

Similarly to the proof of the axiom (WR2), the axioms (WR4) and (WR5) will be proved in two steps.

In the first step, we show that in case of the satisfiability of $(\underline{\varphi} \circ \underline{\mu}) \wedge \underline{\nu}$, the equality $C\_\mathrm{Mod}((\underline{\varphi} \circ \underline{\mu}) \wedge \underline{\nu}) = C\_\mathrm{Mod}(\underline{\varphi} \circ(\underline{\mu} \wedge \underline{\nu}))$ holds. (If $(\underline{\varphi} \circ \underline{\mu}) \wedge \underline{\nu}$ is not satisfiable, then the axiom (WR4) is trivially true.)

The first direction is $C\_\mathrm{Mod}((\underline{\varphi} \circ \underline{\mu}) \wedge \underline{\nu}) \subseteq C\_\mathrm{Mod}(\underline{\varphi} \circ(\underline{\mu} \wedge \underline{\nu}))$. That is, $C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\varphi}\} \cap C\_\mathrm{Mod}(\underline{\nu}) \subseteq C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu} \wedge \underline{\nu}), \leq_{\underline{\varphi}}\}$. Suppose that $I \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\} \cap C\_\mathrm{Mod}(\underline{\nu})$. In this case, I should be in $C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu} \wedge \underline{\nu}), \leq_{\underline{\varphi}}\}$, since if it did not hold, there then would be an interpretation $J \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu} \wedge \underline{\nu}), \leq_{\underline{\varphi}}\}$ for which $(J,\beta) <_{\underline{\varphi}} (I,\alpha)$. This contradicts the supposition $I \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$.

The proof of the other direction: $C\_\mathrm{Mod}(\underline{\varphi}\ \underline{\circ}(\underline{\mu} \wedge \underline{\nu})) \subseteq C\_\mathrm{Mod}((\underline{\varphi}\ \underline{\circ}\ \underline{\mu}) \wedge \underline{\nu})$ means that $C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu} \wedge \underline{\nu}), \leq_{\underline{\varphi}}\} \subseteq C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\} \cap C\_\mathrm{Mod}(\underline{\nu})$ holds. Suppose that $I \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu} \wedge \underline{\nu}), \leq_{\underline{\varphi}}\}$ and $I \notin C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\} \cap C\_\mathrm{Mod}(\underline{\nu})$. Since $I \in C\_\mathrm{Mod}(\underline{\nu})$, I is not in $C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$. Because of the satisfiability of the weighted knowledgebase $(\underline{\varphi}\underline{\circ}\underline{\mu}) \wedge \underline{\nu}$, there is an interpretation J, for which $J \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\} \cap C\_\mathrm{Mod}(\underline{\nu})$, which means that $J \in C\_\mathrm{Mod}(\underline{\mu} \wedge \underline{\nu})$. Because of $I \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu} \wedge \underline{\nu}), \leq_{\underline{\varphi}}\}$ the expression $(I,\alpha) \leq_{\underline{\varphi}} (J,\beta)$ holds. Since $J \in C\_\mathrm{Min}\{\mathrm{Mod}(\underline{\mu}), \leq_{\underline{\varphi}}\}$, $(J,\beta) \leq_{\underline{\varphi}} (I,\alpha)$. Therefore I is $C\_\mathrm{Min}\{\mathrm{Mod}(\mu), \leq_{\underline{\varphi}}\}$.

In the second step we show that the corresponding weights are also correct.

If the weighted knowledgebase $(\underline{\varphi}\ \underline{\circ}\ \underline{\mu}) \wedge \underline{\nu}$ is not satisfiable, then the axiom (WR4) holds, since for all interpretation I the weight is zero, and therefore $((\underline{\varphi}\ \underline{\circ}\ \underline{\mu}) \wedge \underline{\nu})(I) \leq \underline{\varphi}\ \underline{\circ}(\underline{\mu} \wedge \underline{\nu})(I)$ is true.

When $(\underline{\varphi}\ \underline{\circ}\ \underline{\mu}) \wedge \underline{\nu}$ is satisfiable, then the axioms (WR4) and (WR5) mean that $((\underline{\varphi}\ \underline{\circ}\ \underline{\mu}) \wedge \underline{\nu})(I) = \underline{\varphi}\ \underline{\circ}(\underline{\mu} \wedge \underline{\nu})(I)$. It is obvious, because

$$\left((\underline{\varphi}\underline{\circ}\underline{\mu}) \wedge \underline{\nu}\right)(I) = \mathrm{Min}\left\{\alpha_{\underline{\varphi}}(I), \underline{\mu}(I), \underline{\nu}(I)\right\} = \underline{\varphi}\underline{\circ}\left(\underline{\mu} \wedge \underline{\nu}\right)(I).$$

4.3. Weighted Model-Fitting

Similarly to the classical knowledgebases in Section 2.5, the operator $\underline{\nabla} : \underline{F} \times \underline{F} \to \underline{F}$ is a *weighted model-fitting operator*, iff it satisfies the following axioms (WM1)–(WM6):

(WM1) $\underline{\varphi}\underline{\nabla}\underline{\mu}$ implies $\underline{\mu}$.
(WM2) If $\underline{\varphi}$ is unsatisfiable, then $\underline{\varphi}\underline{\nabla}\underline{\mu}$ is unsatisfiable as well.
(WM3) If both $\underline{\varphi}$ and $\underline{\mu}$ are satisfiable, then $\underline{\varphi}\,\underline{\nabla}\,\underline{\mu}$ is also satisfiable.
(WM4) $(\underline{\varphi}\underline{\nabla}\underline{\mu}) \wedge \underline{\nu}$ implies $\underline{\varphi}\underline{\nabla}(\underline{\mu} \wedge \underline{\nu})$.
(WM5) If $(\underline{\varphi}\,\underline{\nabla}\,\underline{\mu}) \wedge \underline{\nu}$ is satisfiable then $\underline{\varphi}\underline{\nabla}(\underline{\mu} \wedge \underline{\nu})$ implies $(\underline{\varphi}\underline{\nabla}\underline{\mu}) \wedge \underline{\nu}$.
(WM6) $(\underline{\varphi}_1\underline{\nabla}\underline{\mu}) \wedge (\underline{\varphi}_2\underline{\nabla}\underline{\mu})$ implies $(\underline{\varphi}_1 \vee \underline{\varphi}_2)\underline{\nabla}\underline{\mu}$.

With aim of proving a similar theorem to Theorem 4.2.2 we need the notion of the loyality for weighted knowledgebases.

DEFINITION 4.3.1. *The function $\underline{wl} : \underline{F} \to \underline{PO}$ is loyal, if it assigns to each weighted knowledgebase $\underline{\varphi} \in D_{\underline{wl}}$ the preorder $\leq_{\underline{\varphi}}$, such that*

(i) *For all weighted knowledgebases $\underline{\varphi}$ and interpretation I there exists the constant $\alpha_{\underline{\varphi}}(I) \in]0,1]$ depending on $\underline{\varphi}$ for which $(I, \mathrm{Min}\{\alpha_{\underline{\varphi}}(I), \beta\}) \leq_{\underline{\varphi}} (I,\beta)$.*
(ii) *If $\underline{wl}(\underline{\varphi}_1) = \leq_{\underline{\varphi}_1}$, $\underline{wl}(\underline{\varphi}_2) = \leq_{\underline{\varphi}_2}$ and $(I,\alpha) \leq_{\underline{\varphi}_1} (J,\beta)$, $(I,\alpha) \leq_{\underline{\varphi}_2} (J,\beta)$ then $(I,\alpha) \leq_{\underline{\varphi}_1 \vee \underline{\varphi}_2} (J,\beta)$, where $\underline{wl}(\underline{\varphi}_1 \vee \underline{\varphi}_2) = \leq_{\underline{\varphi}_1 \vee \underline{\varphi}_2}$.*

The following theorem ensures that with the help of a loyal function and a special constant $\alpha_{\underline{\varphi}}(I)$ a model-fitting operator can be determined.

THEOREM 4.3.2. *Let $\underline{wl}$ be a loyal function assigning to the weighted knowledgebase $\underline{\varphi}$, the preorder $\leq_{\underline{\varphi}}$. The operator $\underline{\nabla} : \underline{F} \times \underline{F} \to \underline{F}$ defined as $\underline{\nabla} : \mathrm{Mod}(\underline{\varphi}\ \underline{\nabla}\ \underline{\mu}) := \mathrm{Min}\{\mathrm{Mod}(\mu), \leq_{\underline{\varphi}}\}$ is a weighted model-fitting operator if $\alpha_{\underline{\varphi}}(I)$ is equal to 1 for all interpretation I.*

PROOF. Because of $\alpha_{\underline{\varphi}}(I), \mathrm{Min}\{\alpha_{\underline{\varphi}}(I), \beta\} = \beta$. Hence the weight of each weighted interpretation I in $\mathrm{Min}\{\mathrm{Mod}\{\underline{\mu}, \leq_{\underline{\varphi}}\}\}$ is equal to $\underline{\mu}(I)$.

The proof of the axioms (SM1)–(SM6) consists of two steps, similarly to proof of Theorem 4.2.2. In the first step, the axioms should be proved for the unweighted case. Based on the proof of Theorem 4.2.2 this part of the proof can be easily done by the reader.

In the second step we show that the weights are correct as well.

Because the weights of the resulting interpretations are equal to the weights with respect to the weighted knowledgebase $\underline{\mu}$, the axioms (WM1), (WM3) hold.

Axiom (WM2) follows because if $\underline{\varphi}$ is unsatisfiable, then the minimal model with respect to $\underline{\varphi}$ is the empty set. Hence $\underline{\varphi}\ \underline{\nabla}\ \underline{\mu}$ is also unsatisfiable.

Axiom (WM4) follows from $\alpha_{\underline{\varphi}}(I) = 1$, since

$$\left(\left(\underline{\varphi}\underline{\nabla}\underline{\mu}\right) \wedge \underline{\nu}\right)(I) = \mathrm{Min}\left\{\underline{\mu}(I), \underline{\nu}(I)\right\} = \underline{\varphi}\underline{\nabla}\left(\underline{\mu} \wedge \underline{\nu}\right)(I).$$

Similarly to the proof of (WM4), if $(\underline{\varphi}\,\underline{\nabla}\,\underline{\mu})\vee\underline{\nu}$ is satisfiable, then $\underline{\varphi}\underline{\nabla}(\underline{\mu}\wedge\underline{\nu})(I) = \mathrm{Min}\{\underline{\mu}(I), \underline{\nu}(I)\} = ((\underline{\varphi}\underline{\nabla}\underline{\mu}) \wedge \underline{\nu})(I)$; therefore axiom (WM5) holds.

For the axiom (WM6), applying $\alpha_{\underline{\varphi}}(I) = 1$ again we get

$$\left(\left(\underline{\varphi}_1\underline{\nabla}\underline{\mu}\right) \wedge \left(\underline{\varphi}_2\underline{\nabla}\underline{\mu}\right)\right)(I) = \underline{\mu}(I) = \left(\left(\underline{\varphi}_1 \vee \underline{\varphi}_2\right)\underline{\nabla}\underline{\mu}\right)(I).$$

Analogously to the unweighted case, the symmetrical model-fitting can be defined as follows.

Definition 4.3.3. *The operator $\underline{\Delta} : \underline{F} \times \underline{F} \to \underline{F}$ is a symmetrical model-fitting operator, if*

$$\underline{\varphi}\underline{\Delta}\underline{\mu} := \left(\underline{\varphi} \vee \underline{\mu}\right)\underline{\nabla}\,\underline{M},$$

where $\underline{M}$ means that weighted knowledgebase, which assigns for each interpretation I, the weight 1.

Example 4.3.4. By Theorem 4.3.2 we have to define a loyal function. We define the overall distance o_dist between a weighted knowledgebase $\underline{\varphi}$ and an interpretation I as follows:

$$o\_\mathrm{dist}\left(\underline{\varphi}, (I, \alpha)\right) := \sum_{(J, \underline{\varphi}(J)) \in \mathrm{Mod}(\underline{\varphi})} \mathrm{dist}\,(I, J) * \underline{\varphi}(J).$$

Then the function f assigns to each weighted knowledgebase $\underline{\varphi}$ the preorder $\leq_{\underline{\varphi}}$ defined by $(I, \alpha) \leq_{\underline{\varphi}} (J, \beta)$ iff $o\_\mathrm{dist}\,(\underline{\varphi}, (I, \alpha)) \leq o\_\mathrm{dist}\,(\underline{\varphi}, (J, \beta))$.

5. OPEN PROBLEMS

According to Section 2.4, the first problem is how to restrict the family of revision operators to intuitively acceptable revisions, that is, to add more axioms (or equivalent to this, to give other properties for the corresponding function, which maps the knowledgebases to total preorders in the minimality theorem in 2.4).

It is interesting to consider extending the set of axioms by the reverse of axiom (R7), that is, by the following requirement:

(R8) If $(\varphi_1 \bullet \mu) \wedge (\varphi_2 \bullet \mu)$ is satisfiable, then $(\varphi_1 \vee \varphi_2) \bullet \mu$ implies $(\varphi_1 \bullet \mu) \wedge (\varphi_2 \bullet \mu)$.

Both of the axioms (R7)–(R8) were introduced in another system of axioms in [3]. It turns out that an operator satisfies both axioms (R7)–(R8) if and only if there is a strictly loyal function sl for which $\mathrm{Mod}(\varphi \bullet \mu) = \mathrm{Min}\{\mathrm{Mod}(\mu), sl(\varphi)\}$.

The function sl is said to be *strictly loyal*, if the following properties hold.

(i) If $\varphi_1 \leftrightarrow \varphi_2$ then $sl(\varphi_1) = sl(\varphi_2)$.
(ii) If $I <_{\varphi_1} J$ and $I \leq_{\varphi_2} J$ then $I <_{\varphi_1 \vee \varphi_2} J$.
(iii) If $I \leq_{\varphi_1} J$ and $I \leq_{\varphi_2} J$ then $I \leq_{\varphi_1 \vee \varphi_2 J}$.

If the function sl assigns to each knowledgebase the same preorder, then it is clearly strictly loyal. But unfortunately, the construction of a nontrivial strictly loyal function runs into difficulties. So the third task is to construct nontrivial loyal functions.

Remark. It is shown in [3] that the set of revision, update and model-fitting operators that also satisfy (R8) are pairwise disjoint. Since revision operators are characterized by faithful functions and model-fitting operators are characterized by strictly loyal functions, it follows that a function cannot be both faithful and strictly loyal. A more direct way to seeing this is the following lemma.

LEMMA 5.1. *The function f cannot be faithful and strictly loyal at the same time.*

PROOF. Consider the knowledgebases φ_1 and φ_2 such that $\mathrm{Mod}(\varphi_1) = I_1, I_2, \dots, I_k, J$ and $\mathrm{Mod}(\varphi_2) = I_1, I_2, \dots, I_k$. Suppose that there is a faithful and strictly loyal function f, which assigns to φ_i the preorder $\leq_{\varphi_i}$. Because of faithfulness, $I_{kl} =_{\varphi_1} J$ and $I_1 <_{\varphi_2} J$ hold for all $1 \leq l \leq k$. If the function was strictly loyal, then $I_l <_{\varphi_1 \vee \varphi_2} J$ should hold, which is a contradiction since $J \in \mathrm{Mod}(\varphi_1 \vee \varphi_2)$.

In 4.3 there is a solution for the weighted model-fitting. It is very special in the sense that $\alpha_{\varphi}(I) = 1$ for *all* interpretation I. It needs further analysis whether there is another more general solution for the weighted model-fitting or not.

Other questions may concern the complexity problem. Eitler and Gottlob dealt with the complexity of the revision and update for unweighted case in [10].

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