SPATIALLY CONSTRAINED WIENER FILTER WITH MARKOV AUTOCORRELATION MODELING FOR IMAGE RESOLUTION ENHANCEMENT

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ABSTRACT

This paper develops a practical method for image resolution enhancement. The method optimizes the spatially constrained Wiener filter for an efficiently parameterized model of the image autocorrelation based on a Markov random field (MRF) with affine transformation. The paper presents a closed-form solution to parameterize the model for an image. The enhancement method is computationally efficient, because it is formulated as convolution with a small kernel. Because the kernel is small, it can be optimized efficiently and only a small portion of the MRF autocorrelation model is required. Because the autocorrelation model parameters and optimal filter can be computed quickly, the method can be optimized locally for adaptive processing. Experimental results indicate that the new method can balance the error-budget tradeoff between signal error and aliasing error.

Keywords: Image processing, Interpolation, Wiener Filter, Markov Process.

1. INTRODUCTION

Image resolution enhancement is the process of defining a high-resolution (HR) image from a low-resolution (LR) image(s). It is fundamental in many biomedical imaging applications, such as image registration and scaling. Resolution enhancement frequently uses image interpolation methods. Convolution-based methods are most typical, including nearest neighbor, bilinear, bicubic, and B-spline interpolation [1, 2, 3]. These traditional convolution-based methods fix interpolation kernels without considering statistical properties of the image, such as image autocorrelation. In many imaging applications, interpolation accuracy can be improved by accounting for image statistics.

The Wiener filter minimizes expected mean square error (MSE) [4] based on the statistical autocorrelation, but it is computationally expensive due to its unconstrained spatial support and its derivation requires the entire autocorrelation function. It is recognized that typically the optimal Wiener filter is determined mainly by the local behavior of the semivariogram of a presumed random field [5] and that a few centrally located elements can account for most of the Wiener filter response [6].

This paper proposes a new, practical method for image resolution enhancement, based on small-kernel interpolation and efficient modeling of local autocorrelation. The spatially constrained Wiener filter can perform interpolation nearly as accurately as the spatially unconstrained Wiener filter, but it is computationally efficient. Moreover, unlike the unconstrained Wiener filter, which requires estimation of the entire autocorrelation function, the constrained kernel can be optimized with only a portion of the autocorrelation function.

This paper generalizes a Markov random field (MRF) with affine transformation to model image autocorrelation and presents a closed-form solution to fit the model for an image. Two-dimensional MRF models are popular for both images and geo-statistical quantities [8]. The MRF model developed in this paper accurately fits the autocorrelation function of a wide range of images. The model can be linearized with logarithms to derive a closed-form solution for a best fit to an image. Because the interpolation kernel is spatially constrained, only small portion of the MRF autocorrelation model is required and the model parameters can be computed quickly. The model can be fit locally to provide the basis for adaptive filtering.

2. FORMULATION

Image resolution enhancement uses a LR image(s) to produce a HR image(s). In this paper, the new method attempts to reconstruct a HR image from only one LR image by efficient interpolation. Let $X[m,n], (0 \leq m \leq M-1, 0 \leq n \leq N-1)$, be a LR image, R be the resolution magnification factor (which could be different for m-axis and n-axis), $Y[m,n], (0 \leq m \leq MR-1$ and $0 \leq n \leq NR-1)$, denote

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a HR image such that X and Y are equal at common points:

$$X[m,n] = Y[mR, nR]. \tag{1}$$

In the Fourier frequency domain:

$$\hat{X}[\mu,\nu] = \frac{1}{R^2}\sum_{r_1=0}^{R-1}\sum_{r_2=0}^{R-1}\hat{Y}(\mu + r_1 M, \nu + r_2 N), \tag{2}$$

where $\hat{X}$ and $\hat{Y}$ are the discrete Fourier transforms (DFTs) of X and Y respectively.

Interpolation typically is implemented by convolving the LR image X with a HR interpolation kernel $F[m,n]$, $(0 \leq m \leq MR-1$ and $0 \leq n \leq NR-1)$ as:

$$Y_a[m,n] \;=\; F[m-m', n-n'] \otimes X\left[\left\lfloor\frac{m'}{R}\right\rfloor, \left\lfloor\frac{n'}{R}\right\rfloor\right]. \tag{3}$$

In the frequency domain:

$$\hat{Y}_a[\mu,\nu] \;=\; \hat{F}[\mu,\nu]\hat{X}[\mu,\nu], \tag{4}$$

where $\hat{Y}_a$ and $\hat{F}$ is the DFTs of the resolution-enhanced image Y_a and the interpolation kernel F respectively. This section formulates the optimal Wiener filter and spatially constrained Wiener filter for image resolution enhancement and presents the MRF model for image autocorrelation.

2.1. Optimal Wiener Interpolation Filter

The optimal Wiener filter is derived for minimal MSE linear interpolation. By Rayleigh's theorem, the expected MSE ϵ^2 for interpolation of an ensemble of HR images can be analyzed in either the spatial or frequency domain:

$$\epsilon^2 \;=\; E\left\{\sum_{\mu=0}^{MR-1}\sum_{\nu=0}^{NR-1}\left|\hat{Y}_a[\mu,\nu] - \hat{Y}[\mu,\nu]\right|^2\right\}. \tag{5}$$

If co-aliased components of the sampled image are uncorrelated [7], the expected MSE can be expressed in terms of image power spectra and the interpolation kernel:

$$\epsilon^2 \;=\; \sum_{\mu=0}^{MR-1}\sum_{\nu=0}^{NR-1}\Big(\hat{\Phi}_Y[\mu,\nu] - \frac{2}{R^2}\Re\left\{\hat{F}[\mu,\nu]\right\}\hat{\Phi}_Y[\mu,\nu] + \frac{1}{R^2}\left|\hat{F}[\mu,\nu]\right|^2\hat{\Phi}_X[\mu,\nu]\Big), \tag{6}$$

where $\Re\{.\}$ denotes the real part of a complex number, $\hat{\Phi}_Y$ is the HR image power spectrum, and $\hat{\Phi}_X$ is the LR image power spectrum (with aliasing). Without loss of generality, images are normalized so that the mean and variance are zero and one respectively [5].

Minimizing ϵ^2 with respect to $\hat{F}$ yields the optimal Wiener filter for image resolution enhancement:

$$\hat{F}_w[\mu,\nu] \;=\; \frac{\hat{\Phi}_Y[\mu,\nu]}{\hat{\Phi}_X[\mu,\nu]}. \tag{7}$$

The optimal Wiener filter is a good benchmark for optimal linear image resolution enhancement, but it has unconstrained spatial support and requires an estimation of entire image autocorrelation. It is desirable to derive a kernel that can be implemented efficiently in the spatial domain and which requires only a local estimation of the autocorrelation function.

2.2. Spatially Constrained Wiener Filter

Let $C, (C \subseteq [0..MR-1] \times [0..NR-1])$ be the discrete spatial support for the filter on the LR image grid, such that:

$$F_c[m,n] = 0, \text{ if } [m,n] \notin C. \tag{8}$$

In order to derive the spatially constrained Wiener filter, the MSE ϵ^2 is rewritten equivalently as:

$$\begin{aligned}\epsilon^2 \;=\; \sum_{\mu=0}^{MR-1}\sum_{\nu=0}^{NR-1}\Bigg(&\hat{\Phi}_Y[\mu,\nu] \\ &-\Re\left\{\frac{2}{MN}\sum_{[m,n]\in C}F_c[m,n]W_{MR}^{m\mu}W_{NR}^{n\nu}\right\}\hat{\Phi}_Y[\mu,\nu] \\ &+\left|\frac{1}{MN}\sum_{[m,n]\in C}F_c[m,n]W_{MR}^{m\mu}W_{NR}^{n\nu}\right|^2\hat{\Phi}_X[\mu,\nu]\Bigg),\end{aligned} \tag{9}$$

where $W_{MR} = exp^{\frac{-j2\pi}{MR}}$ and $W_{NR} = exp^{\frac{-j2\pi}{NR}}$.

Minimizing ϵ^2 with respect to each value of F_c requires:

$$\begin{aligned}&\frac{\partial\epsilon^2}{\partial F_c[m,n]} \\ &= -\Re\left\{\frac{2}{MN}\sum_{\mu=0}^{MR-1}\sum_{\nu=0}^{NR-1}W_{MR}^{m\mu}W_{NR}^{n\nu}\hat{\Phi}_Y[\mu,\nu]\right\} \\ &\quad+\frac{2}{M^2N^2}\sum_{[m',n']\in C}F_c[m',n'] \\ &\quad\times\left(\sum_{\mu=0}^{MR-1}\sum_{\nu=0}^{NR-1}W_{MR}^{(m'-m)\mu}W_{NR}^{(n'-n)\nu}\hat{\Phi}_X[\mu,\nu]\right) \\ &= 0, \quad \forall[m,n]\in C,\end{aligned} \tag{10}$$

which yields:

$$\begin{aligned}&\frac{1}{MN}\sum_{[m',n']\in C}F_c[m',n']\Phi_X\left[\frac{m-m'}{R}, \frac{n-n'}{R}\right] \\ &= \Phi_Y[m,n], \quad \forall[m,n]\in C.\end{aligned} \tag{11}$$

The optimal constrained kernel F_c is given by the solution of the $|C|$ linearly independent equations in the $|C|$ unknown kernel values in (11).

The spatially constrained Wiener filter F_c requires an estimate of the image autocorrelation function, but the limited spatial support C facilitates efficient model fitting. The image autocorrelation function can be estimated from similar HR images or fitted to a LR image. The next subsection develops a MRF model and efficient method for fitting the model to an image.

2.3. MRF Model for Estimating Autocorrelation

The autocorrelation function $\Phi_Y(\tau)$ of an isotropic MRF is an appropriate model both for images and geo-statistical quantities [8]:

$$\Phi_Y[m,n] \;=\; e^{-\sqrt{m^2+n^2}/d}, \tag{12}$$

where d is the mean-spatial-detail (MSD). MSD can be interpreted as the average size of details in images, *i.e.*, images with larger objects (relative to the sampling interval) have larger MSD . This autocorrelation model is isotropic.

A more general autocorrelation model without rotational symmetry can be generated by affine transformation (without translation) of the isotropic MRF . The affine transformation for scaling and rotation is defined as:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} m' \\ n' \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}. \tag{13}$$

Substituting (13) into (12) and using the substitutions:

$$d_m = \frac{d}{\sqrt{a_{11}^2 + a_{21}^2}}, \quad d_n = \frac{d}{\sqrt{a_{12}^2 + a_{22}^2}}, \quad \text{and}$$
$$d_c = \frac{2(a_{11}a_{12} + a_{21}a_{22})}{d^2}, \tag{14}$$

gives a more general autocorrelation with three parameters:

$$\Phi_Y[m,n] \;=\; e^{-\sqrt{\frac{m^2}{d_m^2}+\frac{n^2}{d_n^2}+d_c mn}}. \tag{15}$$

For $d_c = 0$, d_m and d_n can be understood as the mean spatial details along the m-axis and n-axis respectively. The parameter d_c allows rotational orientation.

2.4. Autocorrelation Estimation Based on MRF

The MRF autocorrelation function in (15) can be fit to the observed image autocorrelation function by iterative numerical methods, but a closed-form fit is possible if the model is linearized in terms of its three parameters. First, (15) can be rewritten as:

$$\Phi_Y[m,n] \;=\; e^{-\sqrt{am^2+bn^2+cmn}} \tag{16}$$

where $a = 1/d_m^2$, $b = 1/d_n^2$, and $c = d_c$. Let the spatial support C be limited to $[-SR, SR-1] \times [-SR, SR-1]$. In (11), the image autocorrelation is required at the LR pixel locations $|m| < 2S, |n| < 2S$. The MSE of the linearized fit of the model to the observed autocorrelation function of the LR image $\bar{\Phi}_X$ is:

$$J(a,b,c) = \sum_{|m|,|n|<2S} \left|\log^2 \bar{\Phi}_X[m,n] - \log^2 \Phi_Y[mR,nR]\right|^2. \tag{17}$$

Computing the partial derivatives of J with respect to a, b and c, and solving for the simultaneous equality with zero yields a model fit:

$$a \;=\; \frac{1}{R^2(\alpha^2-\beta^2)} \sum_{|m|,|n|<2S} (\alpha m^2 - \beta n^2)\log^2 \bar{\Phi}_X[m,n]$$

$$b \;=\; \frac{1}{R^2(\alpha^2-\beta^2)} \sum_{|m|,|n|<2S} (\alpha n^2 - \beta m^2)\log^2 \bar{\Phi}_X[m,n]$$

$$c \;=\; \frac{1}{\beta} \sum_{|m|,|n|<2S} mn \log^2 \bar{\Phi}_X[m,n], \tag{18}$$

where:

$$\begin{aligned} \alpha &= \frac{256}{5}S^6 - \frac{384}{5}S^5 + \frac{112}{3}S^4 - \frac{16}{3}S^3 - \frac{8}{15}S^2 + \frac{2}{15}S \\ \beta &= \frac{256}{9}S^6 - \frac{128}{3}S^5 + \frac{208}{9}S^4 - \frac{16}{3}S^3 + \frac{4}{9}S^2. \end{aligned} \tag{19}$$

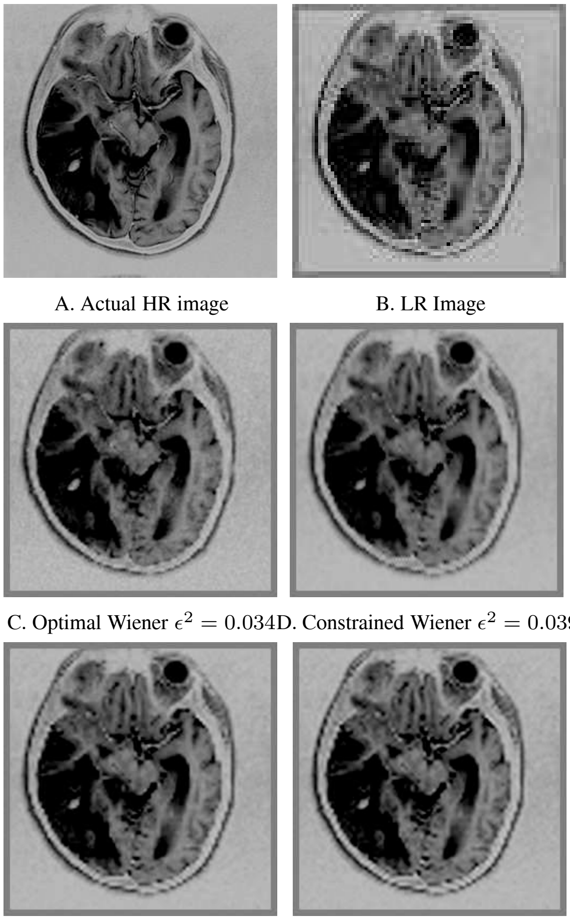
The equations in (18) can be computed globally for a global filter or locally for an adaptive filter.

3. EXPERIMENTAL RESULTS

The experiment begins with an ideal HR image, a 255×255 MRI pictured in Fig. 1(A). The LR image, created by downsampling the ideal HR image to 85×85 ($R = 3$), is pictured in Fig. 1(B) (with nearest neighbor interpolation to clearly illustrate the pixel resolution). Then, the LR image is interpolated back to 255×255 by various resolution enhancement methods and compared to the ideal HR image. To limit the boundary effects of convolution, the outer edges of all interpolated images are cleared.

The experiment compares four interpolation methods: unconstrained Wiener, constrained Wiener, bicubic, and cubic B-spline. The optimal Wiener filter $\hat{F}_w$ is derived with the actual HR image power spectrum (even though it typically is not known) in order to present the mathematically optimal result and is applied in the frequency domain. The constrained Wiener filter F_c is derived with the MRF autocorrelation model (15) using parameters estimated from the image (as typically would be necessary) and is constrained to width $S = 2$ (the same spatial support as the standard bicubic). Fig. 2 illustrates: (A) the actual autocorrelation of the original HR MRI, (B) the MRF autocorrelation model fit to the LR image, and (C) the model error. As can be seen, the model error is relatively small, less than 4% of the peak. The standard bicubic (with parameter value -0.5) and cubic B-spline interpolation are implemented by MATLAB's "INTERP2" function.

The interpolated images are shown in Fig. 1(C)–(F). The unconstrained Wiener filter (based on the HR autocorrelation) has the smallest MSE ($\epsilon^2 = 0.034$) and least aliasing artifacts. The other methods have similar MSE , but the constrained Wiener filter produces smaller aliasing artifacts (*e.g.*, ringing). The Wiener filters (including spatially



A. Actual HR image B. LR Image

C. Optimal Wiener $\epsilon^2 = 0.034$ D. Constrained Wiener $\epsilon^2 = 0.039$

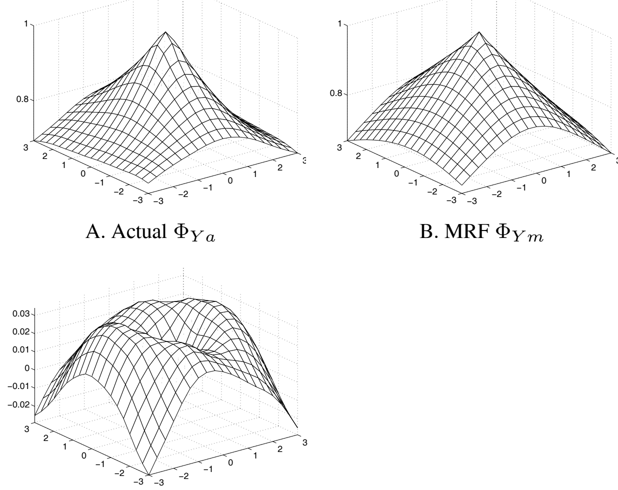
E. Bicubic ϵ^2 =0.039 F. Cubic B-spline $\epsilon^2 = 0.041$

Fig. 1. A HR 255×255 MRI downsampled to 85×85, then interpolated to 255×255 by various algorithms.

constrained and unconstrained) are superior to the cubic B-spline and bicubic interpolators, because they can balance the error budget tradeoff between attenuation of signal components (if the filter transfer function is less than 1) and propogation of aliasing (if the filter transfer function is greater than 0). This balance is important because aliasing typically is inherent in imaging system designs, especially in biomedical imaging applications where anti-aliasing filters may not be practically be applied on physical matter [9].

4. CONCLUSIONS

This paper proposes a new method for image resolution enhancement, based on optimal constrained linear interpolation and efficient estimation of image autocorrelation. The spatially constrained Wiener filter with MRF autocorrelation model fit to the image has nearly the interpolation accuracy of the optimal Wiener filter, but requires far less computation. The proposed MRF autocorrelation model works



A. Actual Φ_{Ya} B. MRF Φ_{Ym}

C. $\Phi_{Ym} - \Phi_{Ya}$

Fig. 2. The actual autocorrelation of the HR MRI, the MRF model fit to the LR image, and their difference.

well for a range of images. In experimental results, the method produces images with smaller artifacts than traditional convolution-based interpolation methods that do not account for image statistics. The autocorrelation model parameters and optimal constrained filter can be computed quickly, so the method can be optimized locally for adaptive processing — a problem for future research.

5. REFERENCES

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