CSCE 475/875 Multiagent Systems

Handout 18: Efficient Groves Mechanisms and Clarke Tax

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(Based on Shoham and Leyton-Brown 2011)

Groves Mechanisms

Efficiency (Definition 10.3.6) is often considered to be one of the most important properties for a mechanism to satisfy in the quasilinear setting. Consequently, a great deal of research has considered the design of mechanisms that are **guaranteed** to select efficient choices when agents follow dominant or equilibrium strategies.

The most important family of efficient mechanisms are the **Groves mechanisms**.

Definition 10.4.1 (Groves mechanisms) Groves mechanisms *are direct quasilinear mechanisms* $(\chi, \wp)$*, for which*

$$\chi(\hat{v}) = arg \max_x \sum_i \hat{v}_i(x),$$

$$\wp_i(\hat{v}) = h_i(\hat{v}_{-i}) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v})).$$

In other words, Groves mechanisms are direct mechanisms in which agents can declare any valuation function $\hat{v}$:

- The mechanism then optimizes its choice assuming that the agents disclosed their true utility function.
- An agent is made to pay an arbitrary amount $h_i(\hat{v}_{-i})$ which does ***not*** depend on its own declaration and is paid the ***sum of every other agent's declared valuation for the mechanism's choice***.

The fact that the mechanism designer has the freedom to choose the h_i functions explains why we refer to the *family* of Groves mechanisms rather than to a single mechanism.

The remarkable property of Groves mechanisms is that they provide a dominant strategy truthful implementation of a social-welfare-maximizing social choice function.

Theorem 10.4.2 *Truth telling is a dominant strategy under any Groves mechanism.*

Intuitively, the reason that Groves mechanisms are dominant-strategy truthful is that agents' externalities are internalized.

INTUITION!! Imagine a mechanism in which agents declared their valuations for the different choices $x \in X$and the mechanism selected the efficient choice, but in which the mechanism did not impose any payments on agents. Clearly, agents would be able to change the mechanism's choice to another that they preferred by overstating their valuation. Under Groves mechanisms, however, an agent's utility does not depend only on the selected choice, because payments *are* imposed. Since agents are paid the (reported) utility of all the other agents under the chosen allocation, each agent becomes just as interested in maximizing the other agents' utilities as in maximizing his or her own. Thus, once payments are taken into account, all agents have the same interests. (***Note: What a beautiful design! Elegant!)***

*(**Note:** The ultimate trick of mechanism design: How to "sneak in rules" or "plant the seeds" that lead to emergent coherence: Design it such that the external imposition translates into self-motivation!)*

The Vickrey-Clarke-Groves (VCG) Mechanism (aka Pivot Mechanism)

So far, we have said nothing about how to set the function h_i in a Groves mechanism's payment function. Here we will discuss the most popular answer, which is called the Clarke tax.

Definition 10.4.4 (Clarke tax) *The* Clarke tax *sets the h_i term in a Groves mechanism as $h_i(\hat{v}_{-i}) = \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}_{-i}))$, where x is the Groves mechanism allocation function.*

Definition 10.4.5 (Vickrey–Clarke–Groves (VCG) mechanism) *The* VCG mechanism *is a direct quasilinear mechanism $(\chi, \wp)$, where*

$$\chi(\hat{v}) = arg \max_{x} \sum_{i} \hat{v}_i(x),$$

$$\wp_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}_{-i})) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v})).$$

First, note that because the Clarke tax does ***not*** depend on an agent i's own declaration $\hat{v}_i$, our previous arguments that Groves mechanisms are dominant-strategy truthful and efficient carry over immediately to the VCG mechanism.

Now, we try to provide some intuition about the VCG payment rule. Assume that all agents follow their dominant strategies and declare their valuations truthfully. The second sum in the VCG payment rule pays each agent i the sum of every other agent $j \neq i$'s utility for the mechanism's choice. The first sum charges each agent i the sum of every other agent's utility for the choice that *would have been made* had i not participated in the mechanism. Thus, each agent is made to pay his or her *social cost*—the aggregate impact that his or her participation has on other agents' utilities. (*Note*: In a way (not entirely exact): If *i* does not play and the group still earns a lot of money, then for the *i* to play, then *i* must pay more. If *i* does not play and the group's gain is very small, then the system will PAY the agent *i* to play!)

What can we say about the amounts of different agents' payments to the mechanism? If some agent i does not change the mechanism's choice by his or her participation—that is, if $\chi(v) = \chi(v_{-i})$—then the two sums in the VCG payment function will cancel out. The social cost of i's participation is zero, and so he or she has to pay nothing.

In order for an agent i to be made to pay a nonzero amount, he or she must be *pivotal* in the sense that the mechanism's choice $\chi(v)$ is different from its choice without i, $\chi(v_{-i})$. This is why VCG is sometimes called the ***pivot*** mechanism—***only pivotal agents are made to pay***.

Of course, it is possible that some agents will *improve* other agents' utilities by participating; such agents will be made to pay a negative amount, or in other words will be paid by the mechanism.

Drawbacks of VCG

The VCG mechanism is one of the most powerful positive results in mechanism design: it gives us a general way of constructing dominant-strategy truthful mechanisms to implement social-welfare-maximizing social choice functions in quasilinear settings. We have seen that no fundamentally different mechanism could do the same job. Thus, it is not surprising that this mechanism has been enormously influential and continues to be widely studied.

However, despite these attractive properties, VCG also has some undesirable characteristics.

- **Agents must fully disclose private information.**
- **Susceptibility to collusion.**
- **VCG is not frugal.**
- **Dropping bidders can increase revenue.** *(Note: If we have agents that are not pivotal, then they don't have to pay …)*
- **Cannot return all revenue to the agents**
- **Computational intractability.** *(Note: Evaluating the $argmax$ can require solving an NP-hard problem in many practical domains.)*