

CSCE 475/875 Multiagent Systems
Handout 15: Voting Paradoxes

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(Based on Shoham and Leyton-Brown 2011)

Introduction

Even when a voting scheme makes sense, it can still fail, resulting in unexpected (undesired) emergent behavior!

Consider a situation in which there are 1,000 agents with three different sorts of preferences.

499 agents: $a > b > c$
3 agents: $b > c > a$
498 agents: $c > b > a$

Observe that 501 people out of 1,000 prefer b to a , and 502 prefer b to c ; this makes b the Condorcet winner. However, many of our voting methods would fail to select b as the winner.

Plurality would pick a , as it has the largest number of first-place votes.

Plurality with elimination would first eliminate b and would subsequently pick c as the winner.

In this example Borda voting would select b .

(*Note*: There are other cases where Borda voting fails to select the Condorcet winner—can you construct one?)

- **Ranking voting systems can be quite ambiguous. Non-ranking voting is much less ambiguous.**

Sensitivity to a Losing Candidate

Consider the following preferences by 100 agents.

35 agents: $a > c > b$
33 agents: $b > a > c$
32 agents: $c > b > a$

Plurality would pick candidate a as the winner, as would Borda. (*Note*: To confirm the latter claim, observe that Borda assigns a , b , and c the scores 103, 98, and 99 respectively.)

However, if the candidate c did not exist, then plurality would pick b , as would Borda. (*Note*: With only two candidates, Borda is equivalent to plurality.)

A third candidate who stands no chance of being selected can thus act as a “spoiler,” changing the selected outcome.

Another example demonstrates that the inclusion of a least-preferred candidate can even cause the Borda method to *reverse* its ordering on the other candidates.

3 agents: $a > b > c > d$
2 agents: $b > c > d > a$
2 agents: $c > d > a > b$

Given these preferences, the Borda method ranks the candidates $c > b > a > d$, with scores of 13, 12, 11, and 6 respectively. If the lowest-ranked candidate d is dropped, however, the Borda ranking is $a > b > c$ with scores of 8, 7, and 6.

Sensitivity to the Agenda Setter

Finally, we examine the *pairwise elimination method*, and consider the influence that the *agenda setter* can have on the selected outcome. Consider the following preferences, which we discussed previously.

35 agents: $a > c > b$

33 agents: $b > a > c$

32 agents: $c > b > a$

First, consider the order a, b, c . a is eliminated in the pairing between a and b ; then c is chosen in the pairing between b and c .

Second, consider the order a, c, b . a is chosen in the pairing between a and c ; then b is chosen in the pairing between a and b .

Finally, under the order b, c, a , we first eliminate b and ultimately choose a .

Thus, given these preferences, the agenda setter can select whichever outcome he or she wants by selecting the appropriate elimination order!

Next, consider the following preferences.

1 agent: $b > d > c > a$

1 agent: $a > b > d > c$

1 agent: $c > a > b > d$

Consider the elimination ordering a, b, c, d . In the pairing between a and b , a is preferred; c is preferred to a and then d is preferred to c , leaving d as the winner.

However, all of the agents prefer b to d —the selected candidate is Pareto dominated by another candidate!

Fundamental Difference between Borda and Pairwise Elimination

Last, we give an example showing that Borda is fundamentally different from pairwise elimination, *regardless* of the elimination ordering. Consider the following preferences.

3 agents: $a > b > c$

2 agents: $b > c > a$

1 agent: $b > a > c$

1 agent: $c > a > b$

Regardless of the elimination ordering, pairwise elimination will select the candidate a . The Borda method, on the other hand, selects candidate b .