# Efficiency and Fairness in Team Search with Self-interested Agents

Rochlin, I., Aumann Y., Sarne D., and Golosman L. (2016). Autonomous Agents and Multi-Agent Systems, 30:526–552

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# Overview

- Introduction and Background
- Problem Formulation
- Efficiency and Fairness
- Strategies
- Conclusions
- Related Works

# Introduction and Background

- Multiagent systems are designed to use teamwork in order to increase efficiency.
    - Using teamwork reduces costs to both the whole team and the individual agent.
- By partitioning the workload, the concept of fairness also plays a role in teamwork.
    - Those who are doing more work for the team should expect to be reimbursed for that extra work.

# Introduction and Background

- The efficiency and the fairness of teamwork can be at odds with one another.
    - *Example: One student agrees to purchase an expensive textbook so that the entire class can use the book. The other students will split the cost of the book and reimburse the buyer. If all students are going to split the cost evenly, the buyer has no real incentive to spend time and resources looking for a cheaper book because no one is going to give the buyer extra money for these search costs.*
- This paper analyzes the relationship between efficiency and fairness in MAS similar to this example.

# Breaking Down Efficiency and Fairness

- *Efficiency*
  - Minimizing costs and maximizing productivity
- *Fairness* - not so straightforward
  - Concerns the difference in costs between each team member
  - Expected costs differ from actual costs
    - Expected costs of splitting a textbook that costs $100 between five people is $20 per person.
    - Actual costs include search time, resources to purchase the book, etc. for the buyer.
  - *Ex-Ante Fairness* - Difference in the agent's expected costs
  - *Ex-Post Fairness* - Difference in the agent's actual costs

# Goal

- Goal is to find teamwork policies that are both efficient and fair to the extent possible.
- In doing so, it is assumed ex-post fairness cannot be accurately tracked, thus full ex-post fairness cannot be achieved.
  - Note: It can be reduced in some cases discussed later on.

# Problem Formulation

- There is a team of $n$ people who designate one team member to be the buyer.
- The buyer can visit $m$ stores where the price of the item can only be seen when the store is visited but prices are pulled from a probability distribution function that is known to the agents.
- The reimbursement policy ($P$) is how the team will reimburse the buyer.
    - Search costs are assumed to be unprovable in which case they are not part of the reimbursement policy.
- The buyer must then choose when to stop searching and purchase the item.
    - Continuing search will incur more search costs.
    - The buyer can also purchase an item from any of the previously visited stores after visiting (called recall)
    - The choice is called the stopping rule.

# The Efficient Strategy

- The Efficiency of a policy $P$ is defined as : $efficiency(\mathcal{P}) = \frac{\min_{\mathcal{Q}}\{E(expense^{\mathcal{Q}}(\text{Buyer}))\}}{E(expense^{\mathcal{P}}(\text{Buyer}))}$
- Main goal is to minimize expected expense
  - In the most efficient strategy it is assumed that the buyer will not be reimbursed, thus the buyer will want to save the most amount of money. This is considered policy $P_{\emptyset}$.
- The Buyer's goal is to minimize its overall expected expense.

$$E\left(expense^{\mathcal{P}_{\emptyset}}(Buyer)\right) = \min_{\mathcal{Q}}\left\{E\left(expense^{\mathcal{Q}}(Buyer)\right)\right\}$$

# The Efficient Strategy

- This is done by setting a threshold value , ‘*r*’
  - This is the value where the buyer becomes indifferent.
    - I.e. The expected marginal benefit of visiting one more store equals the cost incurred up to that point.
- The threshold depends on the search cost ‘*c*’, not the number of stores ‘*y*’.

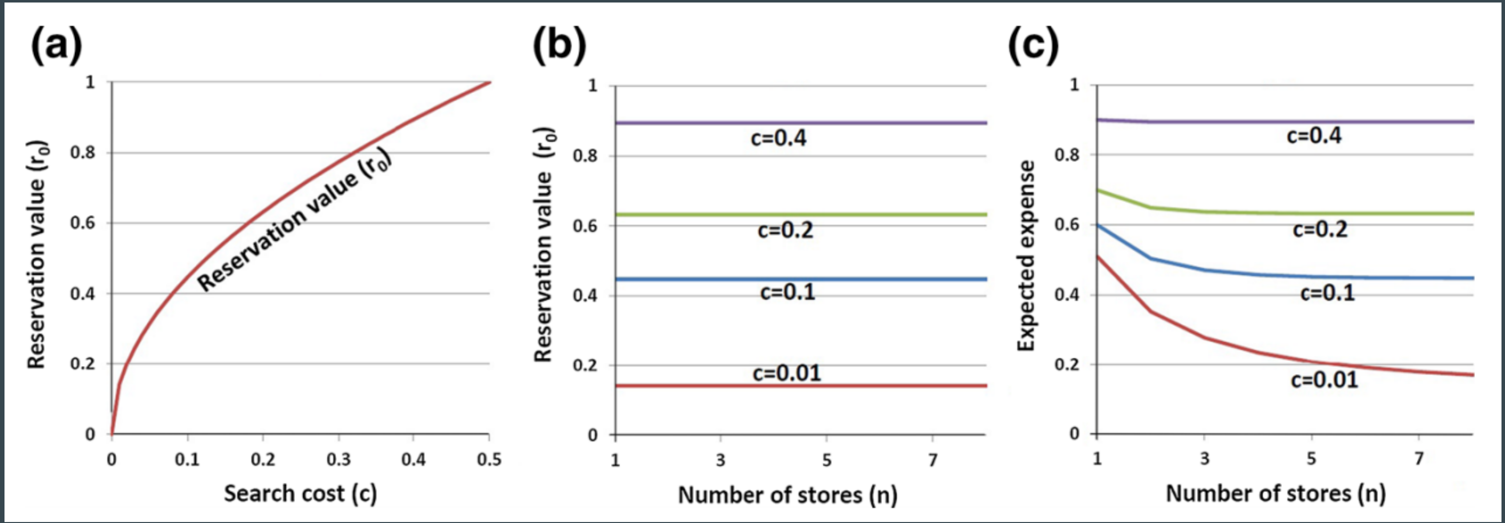
$$c = \int_{y=-\infty}^{r_{\emptyset}} (r_{\emptyset} - y) f(y) dy$$

Reservation Value

Number of stores

Probability of finding the cheaper price at the next store

# The Efficient Strategy



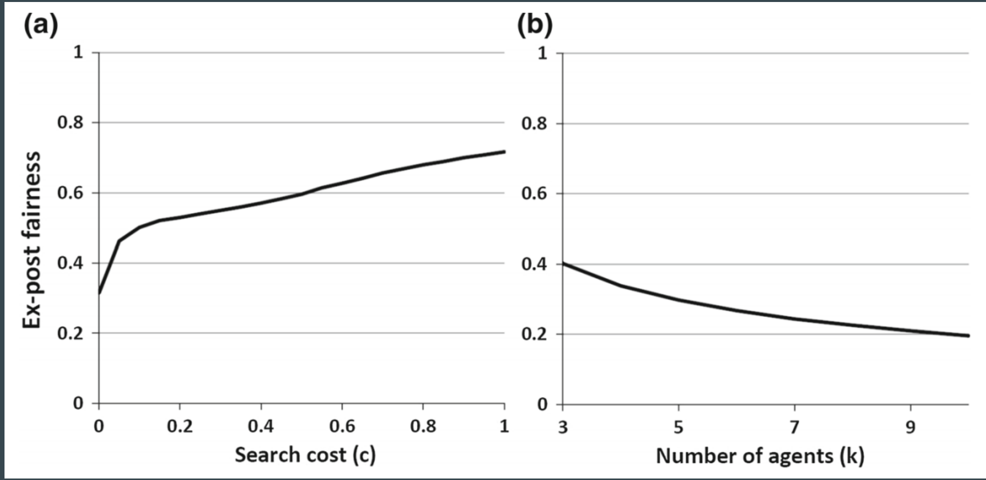
# The Receipt Splitting Strategy

- The receipt splitting policy is to split the cost equally among all team members
  - (as is the method most commonly used in these scenarios.)
  - This results in neither ***optimal efficiency*** nor ***optimal ex-post fairness.***
  - Why?
    - This method is not efficient as the buyer cares less about getting the best price. They will be getting reimbursed, so they will spend less time trying to get the best price in effort of trying to keep their search cost lower.
    - Also, it is not ex-post fair, as the buyer, even by visiting just one store, would still accumulate some search cost, which isn't reimbursed by the reimbursement policy.
- The buyers optimal strategy is to use a reservation-value based strategy:

$$c = \frac{1}{k} \cdot \int_{y=-\infty}^{r} (r - y) f(y) dy$$

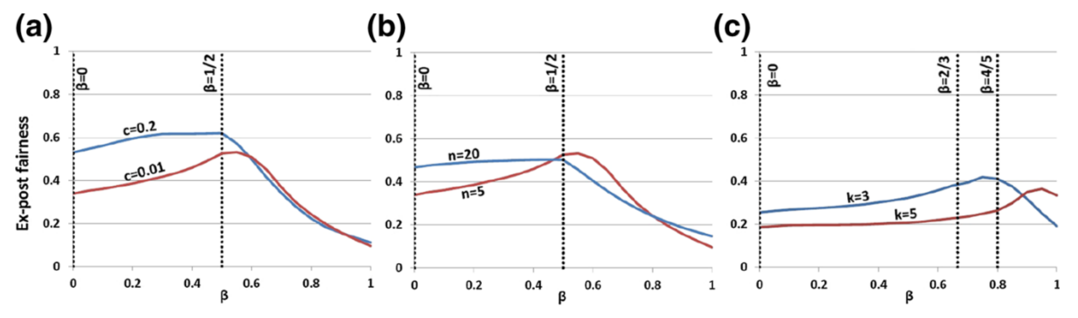
# Optimal Ex-Ante Fairness Policy 1

- To achieve the optimal ex-ante fairness, while also maintaining optimal efficiency, one can give the buyer a fixed amount '*s*' .
  - To find *s*, take the expected expense if the purchase was not being reimbursed at all, then multiple by *(k-1)/k*, where *k* is the number of agents in the team.



# Optimal Ex-Ante Fairness Policy 2

- Another method to achieve optimal ex-ante fairness, while also maintaining optimal efficiency, is to use a multiple part reimbursement policy.
  - The Buyer is reimbursed :
    - Fixed : a fixed amount $s$.
    - Sharing : the buyer gets a $\beta$ fraction of the actual amount paid for the product.
    - Bonus : an amount $b$ is added if the actual price paid for the good is no more than the reservation price as determined .



# Trading Efficiency for Fairness

- Remember: ex-post fairness is considered to be unachievable.
    - This is due to fact that the buyer can only supply evidence for the actual expense of the purchase (i.e. the receipt amount).
    - Given the probabilistic nature of the search, each receipt amount can be associated with a range of possible accumulated search costs, thus a reimbursement policy that relies on the receipt amount as a decision parameter can't necessarily result in identical cost for the buyer and the other agents.
- We can make improvements in the ex-post fairness, but this will be at the expense of efficiency.
- The challenge in design and analysis of reimbursement  strategies without max efficiency guarantee is how to determine the buyer's optimal search strategy.

# Theorem 2

For a function $g : \mathbb{R} \rightarrow \mathbb{R}$, *let* $\mathcal{P}_{function}(g)$ be the reimbursement policy wherein the buyer is reimbursed *g(x)* upon presenting a purchase receipt of x. Provided that

*dg(x)/dx <= 1 , i.e. ,* the lower the price found, then the greater the absolute reimbursement received. The buyer's optimal search strategy is reservation value based, wherein the optimal reservation value r satisfies:

$$c = \int_{y=-\infty}^{r} (r - g(r) - (y - g(y)))\, f(y)dy$$

Reservation Value

Reimbursement of the reservation value

Number of stores

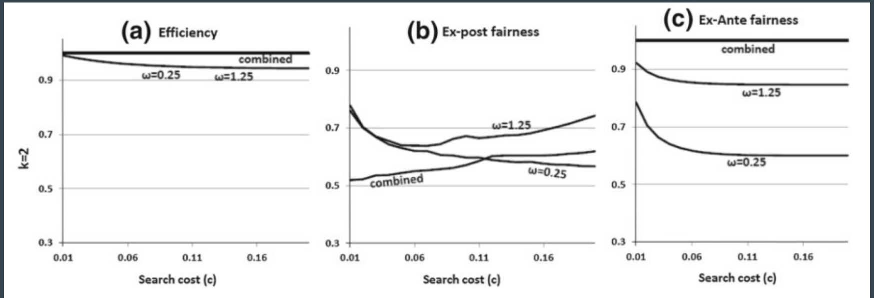
Reimbursement of searching the store

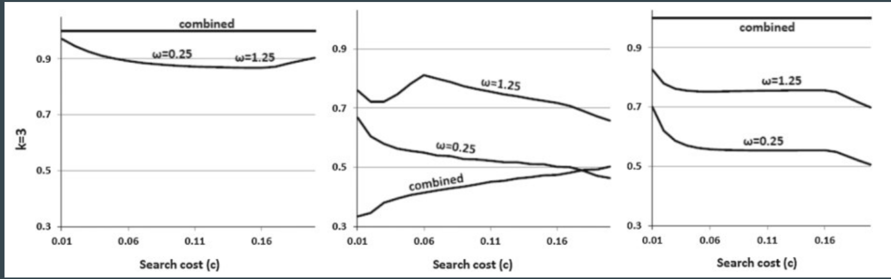
Probability of finding the cheaper price at the next store

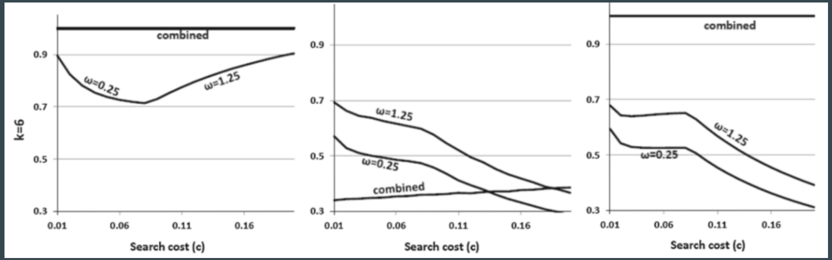
# Trading Efficiency for Fairness

- The reimbursement policy is the following :

$$g(x) = \frac{(k-1)x}{k} + \frac{w}{k-1} \cdot \left| E\left(cost^{\mathcal{P}_{split}}(\text{Buyer})\right) - E\left(cost^{\mathcal{P}_{split}}(A_i \neq \text{Buyer})\right) \right|$$



# Summary

- There are two families of reimbursement policies
    - 1.) Guarantees full efficiency and full ex-ante fairness, but lacks ex-post fairness.
        - Uses a mix of a fixed sum and receipt sharing reimbursement policies with a possible bonus if a certain price is achieved.
            - By choosing the right parameters for the bonus a system can incentivise the buyer to spend the time and resources (search costs)
    - The other family uses more complex reimbursement policies, including variable percentage sharing of the purchase price and multiple bonus levels to improve ex-post fairness, at the expense of efficiency.
- Thus, the MAS designer must decide between a tradeoff of efficiency and ex-post fairness.

# Their Conclusions

- In a MAS, not only efficiency but also fairness come into play
- Although common, the policy of splitting the receipt, results in degradation of efficiency
- Assignment of buyer can come into play when team members are heterogeneous
    - Search costs and price distributions may differ from agent to agent

# Their Conclusions

- Multiple goods complicate the problem of search allocation even when the buyers are homogeneous
  - Ex-post fairness could be vastly unfair based on the random probability functions as well as inefficient due to the receipt splitting at the end.
  - Better method - Make a single team member the buyer to buy all goods and use policies discussed

# Our Conclusions

1. Since the total number of stores is known, the search could be evenly distributed among all agents so that ex-post fairness is achieved. However, coordination/communication becomes the issue.
2. Often times agents may have different valuations of different costs. For example, the buyer may value money over search cost (time) or vice versa. Therefore, ex-post fairness will be prioritized based on buyer preference, and the most efficient strategy may change accordingly. This may also make costs much harder to determine.
3. When including humans in the MAS, fairness becomes a higher priority. Humans continually acting as the buyer or paying more than others will counteract the system to try and achieve their desired level of fairness. This will change the most efficient strategy as agent fairness changes.

# Related Works

- Many pieces of work in this area use the “cake cutting” example as a model for considering fairness in MAS.
  - This model uses a continuously-divisible good to be divided among a group of agents.
  - The agents may place different values on different parts of the good, and the goal is to divide it fairly.
  - To study fairness in the cake cutter model the “price of fairness” is used
    - the ratio between the maximal possible social welfare if no fairness is required and maximal possible welfare when fairness is also required

# Related Work

- Other works consider mediated negotiation procedures
  - allow agents to find the most desirable solution according to certain definitions of fairness or optimality
- The difference between this paper's research and these others is that this research focuses on fairness not on the way resources are allocated, but rather how the amount of effort invested in the search process.
  - Relies heavily on understanding the optimal search strategy that will be used under different reimbursement strategies.

Questions?