CSCE 475/875 Multiagent Systems

Handout 15: Voting Paradoxes

October 5, 2017

(Based on Shoham and Leyton-Brown 2011)

Introduction

Even when a voting scheme makes sense, it can still fail, resulting in unexpected (undesired) emergent behavior!

Consider a situation in which there are 1,000 agents with three different sorts of preferences.

$$499\ agents: a \succ b \succ c$$
$$3\ agents: b \succ c \succ a$$
$$498\ agents: c \succ b \succ a$$

Observe that 501 people out of 1,000 prefer b to a, and 502 prefer b to c; this makes b the Condorcet winner. However, many of our voting methods would fail to select b as the winner.

Plurality would pick a, as it has the largest number of first-place votes.

Plurality with elimination would first eliminate b and would subsequently pick c as the winner.

In this example Borda voting would select b.

(***Note***: There are other cases where Borda voting fails to select the Condorcet winner—can you construct one?)

- **Ranking voting systems can be quite ambiguous. Non-ranking voting is much less ambiguous.**

Sensitivity to a Losing Candidate

Consider the following preferences by 100 agents.

$$35\ agents: a \succ c \succ b$$
$$33\ agents: b \succ a \succ c$$
$$32\ agents: c \succ b \succ a$$

Plurality would pick candidate a as the winner, as would Borda. (***Note***: To confirm the latter claim, observe that Borda assigns a, b, and c the scores 103, 98, and 99 respectively.)

However, if the candidate c did not exist, then plurality would pick b, as would Borda. (***Note***: With only two candidates, Borda is equivalent to plurality.)

A third candidate who stands no chance of being selected can thus act as a "spoiler," changing the selected outcome.

Another example demonstrates that the inclusion of a least-preferred candidate can even cause the Borda method to *reverse* its ordering on the other candidates.

$$3\ agents: a \succ b \succ c \succ d$$
$$2\ agents: b \succ c \succ d \succ a$$
$$2\ agents: c \succ d \succ a \succ b$$

Given these preferences, the Borda method ranks the candidates $c \succ b \succ a \succ d$, with scores of 13, 12, 11, and 6 respectively. If the lowest-ranked candidate d is dropped, however, the Borda ranking is $a \succ b \succ c$ with scores of 8, 7, and 6.

Sensitivity to the Agenda Setter

Finally, we examine the ***pairwise elimination method***, and consider the influence that the ***agenda setter*** can have on the selected outcome. Consider the following preferences, which we discussed previously.

$$35\ agents: a \succ c \succ b$$
$$33\ agents: b \succ a \succ c$$
$$32\ agents: c \succ b \succ a$$

First, consider the order a, b, c. a is eliminated in the pairing between a and b; then c is chosen in the pairing between b and c.

Second, consider the order a, c, b. a is chosen in the pairing between a and c; then b is chosen in the pairing between a and b.

Finally, under the order b, c, a, we first eliminate b and ultimately choose a.

Thus, given these preferences, the agenda setter can select whichever outcome he or she wants by selecting the appropriate elimination order!

Next, consider the following preferences.

$$1\ agent: b \succ d \succ c \succ a$$
$$1\ agent: a \succ b \succ d \succ c$$
$$1\ agent: c \succ a \succ b \succ d$$

Consider the elimination ordering a, b, c, d. In the pairing between a and b, a is preferred; c is preferred to a and then d is preferred to c, leaving d as the winner.

However, *all* of the agents prefer *b* to *d*—the selected candidate is Pareto dominated by another candidate!

Fundamental Difference between Borda and Pairwise Elimination

Last, we give an example showing that Borda is fundamentally different from pairwise elimination, *regardless* of the elimination ordering. Consider the following preferences.

$$3\ agents: a \succ b \succ c$$
$$2\ agents: b \succ c \succ a$$
$$1\ agent: b \succ a \succ c$$
$$1\ agent: c \succ a \succ b$$

Regardless of the elimination ordering, pairwise elimination will select the candidate a. The Borda method, on the other hand, selects candidate b.