Auctions: Collusion and Winner’s Curse

(Based on Shoham and Leyton-Brown (2008). *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*, Cambridge.)

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Collusion

- Cooperation between bidders to reduce their expected payments to the auctioneer by reducing competition among themselves is called ***collusion***
- Collusion is usually **illegal**; however, it is also very **difficult** for agents to pull off
  - **If an agent can collude to "cheat" in the first place, what would prevent the agent from colluding again with a subset of colluders to cheat other colluders?**
- **Which collusive protocols have the property that agents will gain by colluding while being unable to gain further by deviating from the protocol?**

Bidding Ring in Second-Price Auctions

- Consider second-price (or Japanese/English) auctions
- Assumptions
  - All agents are **risk neutral** and have **IPV** valuations
  - There is a set of agents is called a ***cartel*** or a ***bidding ring***
  - There is an agent who is ***not*** interested in the good being auctioned, but who ***serves to run the bidding ring***
    - Does *not* behave strategically (could be a simple computer program)
    - aka the ***Ring Center***
- Observe that there may be agents who participate in the **main auction** and **do not participate in the ring**; there may even be **multiple rings**

Bidding Ring in 2nd-Price Auctions | Protocol

- Each agent in the ring submits a bid to the ring center.
- The ring center
  - Identifies the **maximum bid** that it received, $\hat{v}_1^r$
  - Submits this bid in the **main auction**
  - Drops all the other bids and denotes the **highest dropped bid** as $\hat{v}_2^r$.
- If the ring center's bid wins in the main auction (at the second-highest price in that auction, $\hat{v}_2$), the ring center awards the good to the bidder who placed the maximum bid in the ring and **requires that bidder to pay $max(\hat{v}_2, \hat{v}_2^r)$**
- The ring center **gives every agent who participated in the bidding ring a payment of k**, regardless of the amount of that agent's bid and regardless of whether or not the ring's bid won the good in the main auction

What's in It for the Ring Center?

- Recall that

  If the ring center's bid wins in the main auction (at the second-highest price in that auction, $\hat{v}_2$), the ring center awards the good to the bidder who placed the maximum bid in the ring and **requires that bidder to pay $max(\hat{v}_2 , \hat{v}_2^r)$**

- What if $\hat{v}_2^r > \hat{v}_2$?
  - The ring center will pay $\hat{v}_2$ for the good in the main auction, but it will be paid $\hat{v}_2^r$ for it by the winning bidder, **gaining $\hat{v}_2^r - \hat{v}_2$**
- What if $\hat{v}_2^r \leq \hat{v}_2$?
  - The ring center will pay $\hat{v}_2$ for the good in the main auction, but it will be paid $\hat{v}_2$ for it by the winning bidder, **gaining nothing**

When would one want to be a ring leader? It depends on how confident one believes that the second highest bid in the ring is going to be higher than the second highest bid in the main auction!

But, Why Would Agents Want to Join?

- Recall that

The ring center **gives every agent who participated in the bidding ring a payment of k**, regardless of the amount of that agent's bid and regardless of whether or not the ring's bid won the good in the main auction

- Let $c = \hat{v}_2^r - \hat{v}_2 > 0$ denote the ring center's expected profit
- If there are n_r agents in the ring, then **the ring center could pay each agent**
  - up to $k = \frac{c}{n_r}$ and still **budget balance** on expectation
  - or $k < \frac{c}{n_r}$ but $k > 0$ and **profit** on expectation

What if Ringer Leader Does Not Pay *k*?

- The protocol is still strategically equivalent to a second-price auction in a world where the bidder's ring does not exist
  - The high bidder always wins, and always pays the globally second-highest price (the max of the second-highest prices in the ring and in the main auction
  - Thus the auction is still **dominant-strategy truthful**, and agents have no incentive to cheat each other in the bidding ring's "pre-auction."
- However, agents then do ***not*** gain by participating in the bidding ring
  - they would be just as happy if the ring disbanded and they had to bid directly in the main auction

Back to the Question: Which collusive protocols have the property that agents will gain by colluding while being unable to gain further by deviating from the protocol?

Bidding Ring in Second-Price Auction

Agents would still pay the same price to win an item but now always gain *k* from the ring center

Common Values & Winner's Curse

- In **interdependent values**, agents' valuations depend on ***both*** their own signals and other agents' signals
- In **common values,** all agents value the good at exactly the same amount
- The ***twist*** is that the agents do ***not*** know this amount, though they have (common) prior beliefs about its distribution
  - Each agent has a **private** signal about the value, which allows it to condition its prior beliefs to arrive at a posterior distribution over the good's value

Example

- Consider the problem of buying the rights to drill for oil in a particular oil field
  - The field contains some (uncertain but fixed) amount of oil, the cost of extraction is about the same no matter who buys the contract, and the value of the oil will be determined by the price of oil when it is extracted
  - Given **publicly available information** about these issues, all oil drilling companies have the same prior distribution over the value of the drilling rights
  - The **difference** between agents is that each has **different geologists** who estimate the amount of oil and how easy it will be to extract, and **different financial analysts** who estimate the way oil markets will perform in the future
  - These **signals** cause agents to arrive at **different posterior distributions** over the value of the drilling rights, based on which, **each agent i can determine an expected value v_i**

Example | How to Interpret Expected Value v_i

- One way of understanding it is to note that if a single agent i was selected at random and offered a take-it-or-leave-it offer to buy the drilling contract for price p
  - it would achieve **positive expected utility by accepting the offer if and only if $\boldsymbol{p < v_i}$**
- Now consider what would happen if these drilling rights were sold in a second-price auction among *k* risk-neutral agents
  - One might ***expect*** that each bidder i ought to bid v_i (**dominant strategy**!)
  - **However**, it turns out that bidders would achieve **negative *expected* utility** by following this strategy

Why negative expected utility?

Example | Winner's Curse

- ***Negative Expected Utility?!!?***
  - *Didn't we previously claim that i would be happy to pay any amount up to v_i for the rights?*
- The catch is that, since the value of the good to each bidder is the same, **each bidder cares as much about *other* bidders' signals as it does about its own**
- When it finds out that it won the second-price auction, the winning bidder also learns that it had the ***most optimistic signal***
- This information causes the winning bidder to ***downgrade his/her expectation about the value*** of the drilling rights, which can make him/her conclude that he/she paid ***too much***
- This phenomenon is called the ***winner's curse***

Of course, the winner's curse does not mean that in the common values (CV) setting the winner of a second-price auction always pays too much. Instead, it means that truth telling is no longer a dominant strategy of the second-price auction in this setting

Connection to MAS?

IPV vs. CV settings could bring about very different approaches to bidding or agent design

In MAS with CV settings, agents often are designed to collect data from the "market/environment" to better construct their bids

Data on market values allows an agent to estimate the common value (e.g., posterior probabilities) more accurately to avoid winner's curse, for example

How to collect, integrate, and use data become sensing and fusion problems in MAS research

Tidbits

- The ***unconditional*** payment that every agent receives from the ring center ***violates*** the second condition of the revenue equivalence theorem **that a bidder with the lowest possible valuation must receive zero expected utility**
  - An agent has an expected utility of k
- The construction of bidding ring protocols is **much more difficult** in the first-price auction setting
  - The second-highest bidder could trick the highest bidder into bidding lower by *offering to drop out*, and then could still win the good at less than its valuation