Auctions: Revenue Equivalence

(Based on Shoham and Leyton-Brown (2008). *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*, Cambridge.)

Leen-Kiat Soh

Auctions are mechanisms

Recall that MAS designers use mechanisms to “engineer” incentives, to motivate agents to “do the right thing”

If ”doing the right thing” allows a designer to maximize the revenue for the auctioneer, how do these auction protocols differ?

Revenue Equivalence

- **Revenue Equivalence Theorem**. The theorem states that any allocation mechanism/auction in which
  - the bidder with the highest type/signal/valuation always wins,
  - the bidder with the lowest possible type/signal/valuation expects zero surplus,
  - all bidders are risk neutral, and
  - all bidders are drawn from a strictly increasing and atomless distribution will lead to the **same expected revenue for the seller**
- When bidders are ***risk neutral*** and have ***independent private valuations***, **English, Japanese, Dutch, and all sealed bid auction protocols are revenue equivalent**

Atomless distribution is any distribution where the probability of any particular value is zero

Risk Attitudes

- One of the key assumptions of the revenue equivalence theorem is that agents are **risk neutral**
- It turns out that many auctions ***cease to be revenue-equivalent when agents' risk attitudes change***
- **Risk averse agents prefer the sure thing**
- **Risk-neutral agents are indifferent**
- **Risk-seeking agents prefer to gamble**

Recall the expected utility discussion related to risk attitudes

Risk Attitudes 2

- Consider an auction environment involving n bidders with IPV valuations drawn uniformly from $[0, 1]$.
  - Bidder i, having valuation v_i, must decide whether it would prefer to engage in a first-price auction or a second-price auction
  - Regardless of which auction it chooses (presuming that the bidder, along with the other bidders, follows the chosen auction's equilibrium strategy), i knows that it will gain positive utility ***only*** if it has the highest utility
- **Case 1 (First-Price)**: i will ***always*** gain $\frac{1}{n} v_i$ when it has the highest valuation.
- **Case 2 (Second-Price)**: i's ***expected*** gain will be $\frac{1}{n} v_i$
  - Expected because he or she will pay the second-highest actual bid → the amount of i's gain will *vary* based on the other bidders' valuations
- Thus, i is presented with the choice between ***a sure payment and a risky payment with the same expected value***

Why $\frac{1}{n} v_i$ Gain?

- **Proposition 11.1.2** *In a first-price auction with two risk-neutral bidders whose valuations are drawn independently and uniformly at random from the interval* $[0, 1]$, $(\frac{1}{2} v_1, \frac{1}{2} v_2)$ *is a **Bayes–Nash equilibrium** strategy profile.*
  - Bidder 1's best response to bidder 2's strategy is $\frac{1}{2} v_1$
- **Theorem 11.1.3** *In a first-price sealed-bid auction with n risk-neutral agents whose valuations are independently drawn from a uniform distribution on the same bounded interval of the real numbers, the **unique symmetric equilibrium** is given by the strategy profile* $\left(\frac{n-1}{n} v_1, \ldots, \frac{n-1}{n} v_n\right)$.
  - The unique equilibrium occurs when each player bids $\frac{n-1}{n}$ of its valuation
- Thus the **gain** is the **utility** (or valuation of the good) **minus the amount paid** for the good: $\boldsymbol{v_i - \frac{n-1}{n} v_i = \frac{1}{n} v_i}$

Risk Attitudes 3

- Thus, i is presented with the choice between ***a sure payment and a risky payment with the same expected value***
- If i is ***risk averse***, it will value the sure payment more highly than the risky payment, and hence will ***bid more aggressively in the first-price auction***, causing it to ***yield the auctioneer a higher revenue than the second-price auction***
  - it is i's ***behavior in the first-price auction that will change***; the second-price auction has the ***same*** dominant strategy regardless of i's risk attitude
- If i is ***risk seeking*** it will bid ***less aggressively in the first-price auction***, and the ***auctioneer will derive greater profit from holding a second-price auction***

Strategic Equivalence

Risk-neutral, IPV	Japanese	=	English	=	2nd	=	1st	=	Dutch
Risk-averse, IPV		=		=		<		=	
Risk-seeking, IPV		=		=		>		=	

Table 11.1: Relationships between revenues of various single-good auction protocols. ('>' = more money for auctioneer)

- Dutch and first-price auctions are strategically equivalent regardless of risk attitudes
- The (weaker) equivalence of Japanese, English, and second-price auctions continues to hold as long as bidders have IPV valuations

As a MAS designer, if your goal is to maximize revenue for the auctioneer, find out about the risk attitudes of the potential bidders, and then decide on the protocols

Connection to MAS?

Auction protocols might be different in their processes; but under the IPV setting, risk attitudes can increase or decrease expected revenues for the auctioneer