Auctions: Properties

(Based on Shoham and Leyton-Brown (2008). *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*, Cambridge.)

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Auctions are mechanisms

Recall that MAS designers use mechanisms to “engineer” incentives, to motivate agents to “do the right thing”

Do the auction protocols motivate bidders to reveal their true valuations?

Independent Private Value (IPV) Setting

- To analyze properties of the various auction protocols, let's consider agents' valuations of goods: **their utilities for different allocations of the goods**
- One of the best-known and most extensively studied is the ***independent private value* (*IPV*)** setting
  - All agents' valuations are drawn ***independently*** from the **same (commonly known) distribution**, and
  - An agent's type (or "signal") consists only of its ***own*** valuation, giving itself no information about the valuations of the others
- An example where the IPV setting is appropriate is in auctions consisting of bidders with personal tastes who aim to buy a piece of art purely for their own enjoyment

Also think about the *common-value* assumption. Resale value!

Dominant Strategy: Second-Price Auctions

- **Theorem 11.1.1** *In a second-price auction where bidders have independent private values,* ***truth telling is a dominant strategy****.*
- **Proof.** Assume that all bidders other than *i* bid in some arbitrary way, and consider *i*'s best response.
- **Case 1: *i*'s valuation is larger than the highest of the other bidders' bids**
- If *i* bids honestly, *i* would win and would pay the next-highest bid amount
- Could *i* be better off by bidding **dishonestly** in this case?
  - If *i* bid higher, *i* would still win and would still pay the same amount (**same as above**)
  - If *i* bid lower, *i* would either (1) still win and still pay the same amount (**same as above**) or (2) **lose and pay zero**.
  - Since *i* gets **nonnegative utility** for receiving the good at a price less than or equal to its valuation, *i* ***cannot gain***, and would ***sometimes lose by bidding dishonestly*** in this case

Dominant Strategy: Second-Price Auctions 2

- **Theorem 11.1.1** *In a second-price auction where bidders have independent private values,* ***truth telling is a dominant strategy****.*
- **Proof.** Assume that all bidders other than *i* bid in some arbitrary way, and consider *i*'s best response.
- **Case 2: *i*'s valuation is less than at least one other bidder's bid**
- *i* would **lose and pay zero**
- Could *i* be better off by bidding **dishonestly** in this case?
  - If *i* bid less, it would still lose and pay zero (**same as above**)
  - If *i* bid more, either (1) it would still lose and pay zero (**same as above**) or (2) it would **win and pay more than its valuation**, achieving **negative utility**
  - Thus again, *i* ***cannot gain***, and would ***sometimes lose by bidding dishonestly*** in this case.

Dominant Strategy: Japanese Auctions

- **Are Japanese auctions different from Second-Price auctions?**
  - In both cases the bidder must select a number (Sealed-bid: the number is the one written down; Japanese: the price at which the agent will drop out)
  - The bidder with **highest amount wins**, and **pays the amount selected by the second-highest bidder**
  - Difference in the **information about other agents' bid amounts disclosed**
    - Seal-bid: Amount is decided ***without*** knowing anything about others' bids
    - Japanese: Amount ***can be updated based on the prices at which lower bidders are observed to drop out***
    - **In general, this difference can be important**—interdependent values, common values, etc.; **however, it makes no difference in the IPV case**
- Japanese auctions are ***also*** **dominant-strategy truthful** when agents have IPVs

Unlike the case of second-price auctions, we do ***not*** have dominant strategies in first-price auctions

WHY?

(Hint: At what price a winning agent has to pay?)

Also, in a first-price auction, an agent's risk attitude matters. E.g., a risk-averse agent would be willing to sacrifice some expected utility (by increasing its bid over what a risk-neutral agent would bid) in order to increase its probability of winning the auction

First-Price and Dutch Auctions

- The Dutch auction and the first-price auction, while quite different in appearance, are actually the same auction
  - ***strategically equivalent***
- In both, each agent must select an amount ***without*** knowing about the other agents' selections; the agent with ***the highest amount wins*** the auction, and must ***purchase the good for that amount***
- **Strategic equivalence is a very strong property**
  - Auctions are ***exactly*** the same regardless of agents' risk attitudes, and regardless of the valuation model describing their utility functions

Since they are strategically equivalent, why do we have both of them in practice?

First-Price and Dutch Auctions 2

- Tradeoffs between **time complexity** and **communication complexity**.
- **First-price auctions** require each bidder to send a message to the auctioneer, which could be **unwieldy with a large number of bidders**
- **First-price auctions** can be done **asynchronously**; while **Dutch auctions** require everybody to be at the same place **synchronously**
- Dutch auctions require ***only*** a single bit of information to be communicated to the auctioneer, but requires the **auctioneer to broadcast prices**

If you are an auctioneer, how would you decide between First-Price and Dutch?

If you are a bidder, how would you prefer between First-Price and Dutch?

Connection to MAS?

Auction protocols might be different in their processes; but under the IPV setting, some are actually strategically equivalent; and some have the same truth-telling dominant strategy

When deciding on which protocols to use, consider the objectives: Revenue maximization for the auctioneer? Economically efficient for the system? Time complexity? Communication complexity?

Silly Question: Suppose that an online bidding site has the following rule: (1) every bid costs the bidder $X, and (2) if the bidder ends up winning the item, then the total amount of costs spent on bidding is returned to that bidder. How would this rule affect the above four objectives? How would you bid?