Voting: Paradoxes

(Based on Shoham and Leyton-Brown (2008). *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*, Cambridge.)

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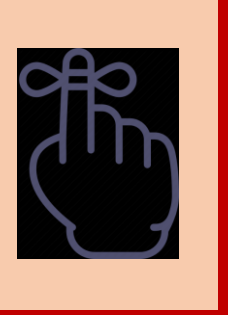

Social choice is *NOT* a straightforward matter

Which one is less ambiguous, ranking or non-ranking voting?

Introduction

- ***Even when a voting scheme makes sense, it can still fail, resulting in unexpected (undesired) emergent behavior!***
- Consider a situation in which there are 1,000 agents with three different sorts of preferences:

$$499\ agents\colon\ a \succ b \succ c$$
$$3\ agents\colon\ b \succ c \succ a$$
$$498\ agents\colon\ c \succ b \succ a$$

- Observe that 501 people out of 1,000 prefer b to a, and 502 prefer b to c

Condorcet Winner: b | **Plurality**: a | **Plurality with Elimination**: c | **Borda**: b

Sensitivity to Losing Candidates

- Consider the following preferences by 100 agents:

$$35\ agents\colon\ a \succ c \succ b$$
$$33\ agents\colon\ b \succ a \succ c$$
$$32\ agents\colon\ c \succ b \succ a$$

- Plurality would pick candidate a as the winner, as would Borda
  - ***Note***: Observe that Borda assigns a, b, and c the scores 103, 98, and 99 respectively
- However, if candidate c did *not* exist, then
  - Plurality would pick b, as would Borda
  - ***Note***: With only two candidates, Borda is equivalent to plurality
- **A third candidate who stands *no* chance of being selected can thus act as a "spoiler," changing the selected outcome**

Sensitivity to Losing Candidates 2

- Another example demonstrates that the inclusion of a least-preferred candidate can even cause the Borda method to ***reverse*** its ordering on the other candidates

$$3\ agents\colon\ a \succ b \succ c \succ d$$
$$2\ agents\colon\ b \succ c \succ d \succ a$$
$$2\ agents\colon\ c \succ d \succ a \succ b$$

- Using Borda:
  - $\boldsymbol{c \succ b \succ a \succ d}$, with scores of 13, 12, 11, and 6, respectively
  - But, If the lowest-ranked candidate d is dropped, $\boldsymbol{a \succ b \succ c}$ with scores of 8, 7, and 6.

Sensitivity to Agenda Setter

- Consider the ***pairwise elimination method***, and the following preferences:

$$35\ agents\colon\ a \succ c \succ b$$
$$33\ agents\colon\ b \succ a \succ c$$
$$32\ agents\colon\ c \succ b \succ a$$

- Consider the order a, b, c
  - a is eliminated in the pairing between a and b; then c is chosen in the pairing between b and c
- Consider the order a,c,b
  - a is chosen in the pairing between a and c; then b is chosen in the pairing between a and b
- Consider the order b,c,a
  - we first eliminate b and ultimately choose a.
- Thus, given these preferences, **the agenda setter can select *whichever* outcome he or she wants by *selecting the appropriate elimination order***

Difference between Borda & Pairwise Elimination

- An example showing that Borda is fundamentally different from pairwise elimination, *regardless* of the elimination ordering. Consider the following preferences:

$$3\ agents\colon\ a \succ b \succ c$$
$$2\ agents\colon\ b \succ c \succ a$$
$$1\ agent\colon\ b \succ a \succ c$$
$$1\ agent\colon\ c \succ a \succ b$$

- *Regardless* of the elimination ordering
  - pairwise elimination will select the candidate *a*.
- The Borda method
  - on the other hand, selects candidate *b*.

Exercise

Voter	Candidate1	Candidate2	Candidate3	Candidate4	Candidate5
1	4	0	2	1	3
2	0	1	3	2	4
3	1	2	3	4	0
Borda Count	**5**	**3**	**8**	**7**	**7**

* 4 is the highest rank, 0 is the lowest rank

- Plurality winner?
- Borda winner?
- Pairwise Elimination with order: 1, 2, 3, 4, 5?
  - Candidate 1 vs. Candidate 2, who wins?
- *Is there a Condorcet Winner?*
- *Is there a situation where Candidate A Pareto dominates Candidate B yet A is ranked lower than B? Pareto domination: at least one voter prefers A to B, and all the remaining voters weakly prefer A to B*

Connection to MAS?

Which is less ambiguous?
Non-ranking voting

Think about your goal: social choice or social welfare?
If just to select the top pick, perhaps don't ask for preference ordering at all