

# Predicting the Expected Behavior of Agents that Learn About Agents: The CLRI Framework

Josè M. Vidal and Edmund H. Durfee (2003).  
Autonomous Agents and Multi-Agent Systems, 6(1):77-107

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# Intro

- The CLRI Framework: used to model and predict the behavior of multiagent systems with learning agents
- CLRI:
  - C - change rate
  - L - learning rate
  - R - retention rate
  - I - Impact
- Purpose: a way to model/predict the behavior of a MAS beyond observation-based experiment results

# Framework for modeling MASs

- $N$  – the set of all agents, where  $i$  is one particular agent
- $W$  – the set of possible states of the world, where  $w$  is one particular state
- $A_i$  – the set of all actions that agent  $i$  can take
- A decision function for agent  $i$  – a mapping which tells us which action agent  $i$  will take in each state
- A target function for agent  $i$  – a mapping which tells us what action agent  $i$  should take. Takes into account the actions other agents will take.
- An error probability function for agent  $i$  at time  $t$  – the probability that agent  $i$  will take an incorrect action (i.e., the decision function is not the same result as the target function for an agent  $i$  at time  $t$  in state  $w$ )
- Note: an action taken by an agent that is not the action indicated by the target function is “incorrect” – there is no continuum of “correctness”

# Framework for modeling MASs (continued)

- Change rate – the probability that the agent will change at least one of its incorrect mappings of a decision function at a given state  $w$  (i.e., the likelihood the agent will change an incorrect mapping to something else, but not necessarily into the correct mapping)
- $C = .5 = 50\%$  chance that an incorrect mapping of a decision function will be modified
- Learning rate – the probability that the agent changes an incorrect mapping to the correct action at at a given state  $w$  (note: different than learning rate in Q-learning; we care about whether the agent correctly changes an incorrect mapping)
- $L = .5 = 50\%$  chance that an incorrect mapping of a decision function will be changed to the correct mapping (i.e., to match the target function mapping)

# Framework for modeling MASs (continued)

- Retention rate – probability that a correct mapping will stay correct in the next iteration
- $R = 0$  = all correct mapping will be made incorrect.  $R = 1$  = all correct mapping will persist.
- Impact – given agent  $i$  and agent  $j$ , the impact that  $i$ 's changes in its decision function have on  $j$ 's target function
- The greater an agent  $i$ 's Impact on agent  $j$ , the greater the likelihood if  $i$  changes its decision function that it will change  $j$ 's target function
- Volatility - probability the target for learning will change from world state to world state
- $V = 1$  = an agent's target function will always change between iterations.  $V = 0$  = an agent's target function will always remain the same.

# Volatility and Impact

- Intuitively:
  - Volatility: odds that how an agent should react (the correct  $w \rightarrow a$  at time  $t$ , a.k.a. the target function) will change
  - Impact: how much influence one agent's decision has on how another agent should act: if agent  $i$  has a high impact on agent  $j$ , and agent  $i$ 's decision for  $w \rightarrow a$  changes, there's a good change how agent  $j$  should act  $w \rightarrow a$  will also change because of it

# Calculating the Agent's Error

- 2 conditions that determine the new error - the result of volatility and whether or not the current decision function matches the target
- This results in 4 different cases to consider
  - $a$  = an agent's target function remains the same between iterations
  - $b$  = an agent's decision function is the same as its target function
  - (1)  $a \ \& \ b$  (2)  $a \ \& \ !b$  (3)  $!a \ \& \ b$  (4)  $!a \ \& \ !b$
- (1) agent's target function does not change, decision function is a correct mapping  $w \rightarrow a$ : agent has a probability of changing this mapping with probability  $1 - r$  (i.e., the odds that it will change a correct mapping)
- (2) agent's target function does not change, but decision function is incorrect mapping  $w \rightarrow a$ : agent has a probability of changing this mapping to a correct mapping of  $1 - l$  (i.e., the odds that the agent will be incorrect in the next iteration -- odds that agent does not change the incorrect mapping)

# Calculating the Agent's Error (continued)

- (3) agent's mapping of  $w \rightarrow a$  is correct, but the target function changes: probability that a correct mapping does change  $1 - r$  (if  $r = 1$ , 0% chance it will change, agent will definitely be wrong in the next iteration)
- (4) agent's mapping of  $w \rightarrow a$  is incorrect, and the target function also changes
  - 1 - agent does not change its mapping ( $1 - c$ )
  - 2 - agent changes its mapping, and changes it to the correct mapping ( $l$ )
  - 3 - agent changes its mapping, does not change it to the correct mapping ( $c - l$ )



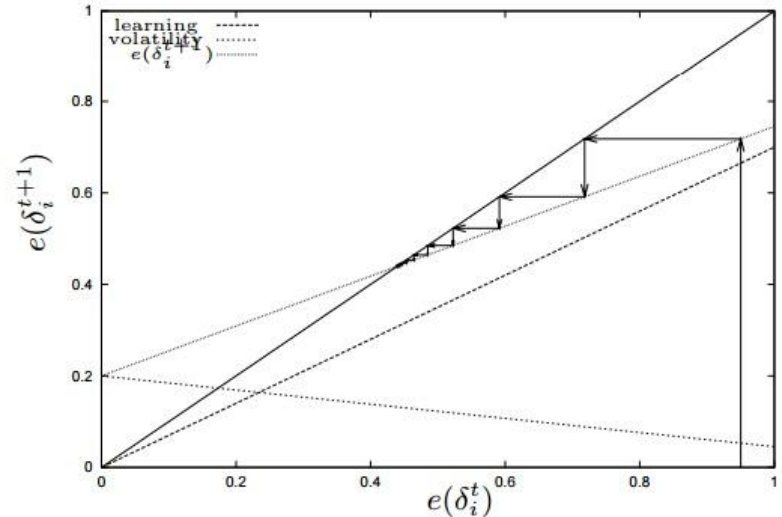
## Equation and Simplification of a Further nature

$$\begin{aligned} E[e(\delta_i^{t+1})] &= E\left[\sum_{w \in W} \mathcal{D}(w) \Pr[\delta_i^{t+1}(w) \neq \Delta_i^{t+1}(w)]\right] = \sum_{w \in W} \mathcal{D}(w) \left( \right. \\ &\quad \Pr[\Delta_i^{t+1}(w) = \Delta_i^t(w) | \delta_i^t(w) = \Delta_i^t(w)] \cdot \Pr[\delta_i^t(w) = \Delta_i^t(w)] \cdot (1 - r_i) \\ &\quad + \Pr[\Delta_i^{t+1}(w) = \Delta_i^t(w) | \delta_i^t(w) \neq \Delta_i^t(w)] \cdot \Pr[\delta_i^t(w) \neq \Delta_i^t(w)] \cdot (1 - l_i) \\ &\quad + \Pr[\Delta_i^{t+1}(w) \neq \Delta_i^t(w) | \delta_i^t(w) = \Delta_i^t(w)] \\ &\quad \cdot \Pr[\delta_i^t(w) = \Delta_i^t(w)] \cdot (r_i + (1 - r_i) \cdot B) \\ &\quad \left. + \Pr[\Delta_i^{t+1}(w) \neq \Delta_i^t(w) | \delta_i^t(w) \neq \Delta_i^t(w)] \cdot \Pr[\delta_i^t(w) \neq \Delta_i^t(w)] \right. \\ &\quad \left. \cdot (1 - c_i)D + l_i + (c_i - l_i)F. \right) \end{aligned}$$

# Equation and Simplification of a Further nature

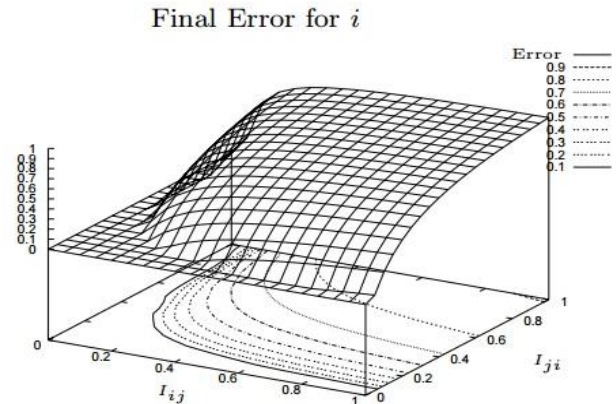
- 2 assumptions can be made to simplify the equation above
  - that actions chosen when either mapping changes are chosen from a flat probability distribution of actions
  - the probability that the target changes is irrelevant to the probability that an agent's mapping is correct (this was not the case in the matching game)

$$E[e(\delta_i^{t+1})] = 1 - r_i + v_i \left( \frac{|A_i|r_i - 1}{|A_i| - 1} \right) + e(\delta_i^t) \left( r_i - l_i + v_i \left( \frac{|A_i|(l_i - r_i) + l_i - c_i}{|A_i| - 1} \right) \right)$$



# An Example

- Example:  $l = .2$ ;  $c = 1$ ;  $r = 1$ ; 20 actions; values identical for  $i$  and  $j$
- As impact increases, the emergent error does as well
- Abrupt change from final errors of 0 and non-zero - this aspect is a result of systems that converge or diverge
- Graph is asymmetric!
- Agent  $j$ 's error makes it harder for  $i$  to come to a smaller emergent error



# An Example

- If the impact of  $i$  is high, then when  $i$ 's error is large,  $j$ 's will increase and  $j$  will start changing its decision function
- If the impact of  $i$  is low, then regardless of  $j$ 's impact, both agents will probably settle to a low emergent error
- Goal is to hit outer region, where error is 0

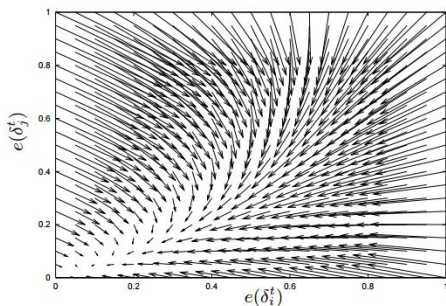
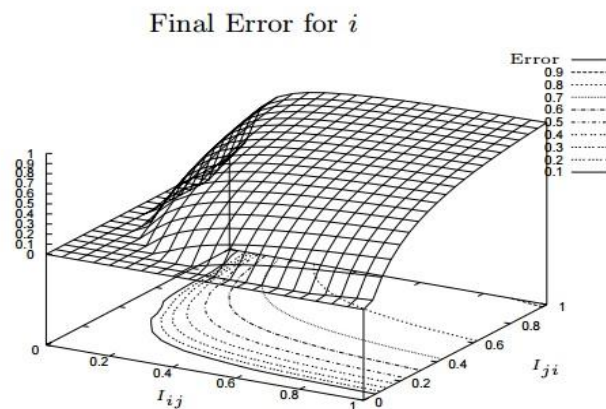
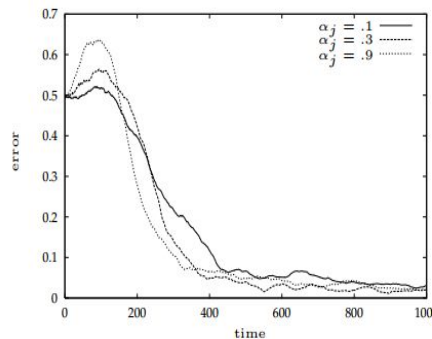


Figure 8: Vector plot for  $e(\delta_i^t)$  and  $e(\delta_j^t)$ , where  $|A_i| = |A_j| = 20$ ,  $l_i = l_j = .2$ ,  $r_i = r_j = 1$ ,  $c_i = .5$ ,  $c_j = 1$ ,  $I_{ij} = .1$ ,  $I_{ji} = .3$ . It shows the error progression for a pair agents  $i$  and  $j$ . For each pair of errors  $(e(\delta_i^t), e(\delta_j^t))$ , the arrows indicate the expected  $(e(\delta_i^{t+1}), e(\delta_j^{t+1}))$ .

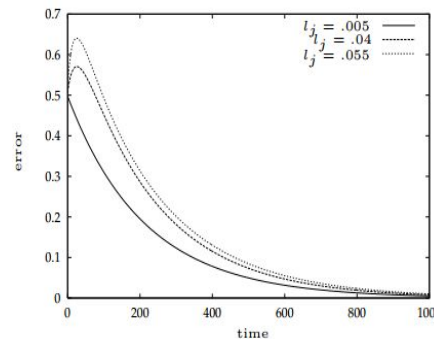


# An Application

- Market based MAS - 3 agents: 1 buyer, 2 sellers
- Reinforcement: profit earned each round; 20 possible bid values
- Vary  $j$ 's alpha value (the weight the algorithm assigns to the latest result)
- $i$ 's error rate shown below
- slight delay before decrease - due to high exploration rate



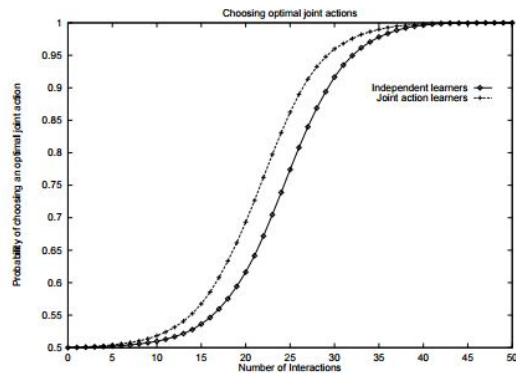
(a) Experiment



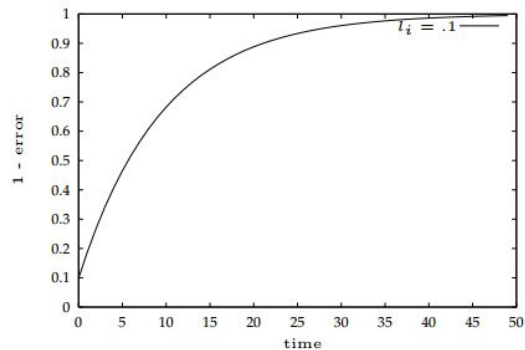
(b) Theory

# Applications of the Theory

- A comparison of CLRI framework application to experimental results from the matching game explored by Claus and Boutilier
- CLRI cannot account for initial exploration yet again



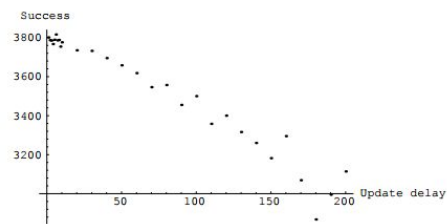
(a) Experiment



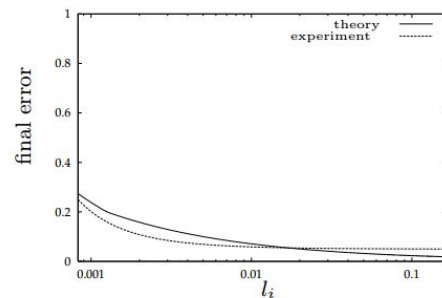
(b) Theory

# Applications of the Theory

- Shoham and Tennenholtz investigate an experiment similar to the matching game known as the coordination game; all agents use their custom algorithm: HCR (highest cumulative reward)
- at every  $t$ , each agent takes an action, the agents are paired up randomly with the goal of taking the same action
- the agents take their action after a delay, this delay is varied in the experiment



(a) Original Experiment



(b) Theory and Experiment

# Bounding the Learning rate with Sample Complexity

- Previous examples assume  $l$ ,  $c$ ,  $r$  can be found based on an agent's learning algorithm
- What if the algorithm is highly complex or unknown?
- Probably Approximately Correct (PAC) theory: loose upper bound on number of examples an agent must observe before arriving at a PAC hypothesis
- Determine  $l$  and then find  $c$  and  $r$
- With assumption that  $r = 1$  and  $v = 0$
- Simply put: can find the lower bound of an agent's learning rate ( $l$ ), and can be used to determine its  $c$  and  $r$



# Conclusions

- The CLRI framework can get fairly limited as the constraints imposed on the agents are critical for its function; additionally, there is no room for non-binary degrees of success, an agent is correct or incorrect
- The world states the framework must act upon must be chosen from a uniform probability distribution and must be episodic. There isn't much room for chaotic, shifting environments
- The description given by the CLRI is very high-level behavior, meaning it doesn't work well when describing specific systems, but works well in predicting agents emergent behaviors