The complexity of contract negotiation

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Agenda

- Abstract
- Introduction
- Preliminary Definitions
- Complexity Results
- Conclusion

Abstract

- Use of software agents for automatic contract negotiation
- Attempt to construct mutually beneficial optimal resource allocation by exchanging resources
- This paper examines the computational complexity of decision problem in this setting

Introduction

- Sandholm's[1] negotiation setting
  - Initial distribution of resources
  - Negotiation and exchange of resources (mutual benefit)
  - Optimal allocation of resources
- Given a particular negotiation setting, is a particular outcome is feasible
- Given some initial allocation Ps and an optimal allocation Pt, is it possible to realize Pt
  - Irrational agents
  - Yes/No

Introduction

- The paper shows that given an allocation, determining if it is pareto optimal is NP-Hard

Preliminary Definition

- Definition of Resource allocation setting is defined by a triple <A , R , U> where
  - A = { A1,A2,...,An };
  - R = { r1,r2,...,rm }
  - U is <u1,u2,...,un>
  - P of R to A is a partition <P1,P2,...,Pn> of R
  - ui(Pi) is called the utility of the resources assigned to Ai
  -

Preliminary Definitions

- Notion of pay-off function is used
- In change of allocation of resources from Pi to Qi, one of the following could result
  - ui(Pi) < ui(Qi) Ai values the allocation Qi as superior to Pi;
  - ui(Pi) = ui(Qi) Ai is indifferent between Pi and Qi; and
  - ui(Pi) > ui(Qi) Ai is worse off after the exchange

Preliminary Definitions

- A **deal** is a pair $< P,Q >$ where $P = < P1,...,Pn >$ and $Q = < Q1,...,Qn >$ are distinct partitions of R
- A deal $< P,Q >$ is said to be **individually rational** (IR) if there is a pay-off vector $\pi =< \pi1,\pi2,...,\pi n>$ satisfying,
  - i=1 to n $\Sigma \pi i= 0$.
  - (b) $ui(Qi) - ui(Pi) > \pi i$, for each agent Ai , except that πi is allowed to be 0 if $Pi = Qi$

Preliminary Definitions

- Let P be an allocation of R among A . The utilitarian social welfare resultingfrom P , denoted $\sigma u(P)$, is given by
  - for $i = 1$ to n, $\sum ui(Pi)$
- $\sigma u(P)$ gives global utility for the allocation
- Pareto optimal allocation
- A deal $< P,Q >$ is IR if and only if $\sigma u(Q) > \sigma u(P)$

Preliminary Definitions

- Let δ = < P,Q > be a deal involving an allocation of R among A . We say that δ is a **cluster contract** (C-contract) if there are distinct agents Ai and Aj for which
  - (C1) Pk = Qk if and only if k / ∈ { i,j }
  - (C2) There is a unique (non-empty) set S for which Qi = Pi ∪ S and Qj = Pj \ S (with S ⊆ Pj) or Qj = Pj ∪ S and Qi = Pi \ S (with S ⊆ Pi).
- One agent transferring a subset of its resources to another agent without receiving any resources in return

Preliminary Definitions

- For a resource allocation setting <A , R , U> and value k <= m = |R| , we say that δ is a **k-bounded cluster contract**, (C(k)-contract) if δ is a C-contract in which S—the set of resources transferred—contains at most k elements
- When k = 1, we use the term one contract (**O-contract**)

Preliminary Definitions

- Let P0 be any initial allocation of R to A and Pt be any other allocation.
  - The deal < P0,Pt > can always be realised by a contract path in which every deal is an O-contract.
  - There are resource allocation settings, <A , R , U> within which there are IR deals < P0,Pt > that cannot be realised by any IR C-contract path.

Preliminary Definitions

- The decision problem IR-k-path (IRk) is given by A 5-tuple <A , R , U ,Ps, Pt > in which <A , R , U> is a resource allocation setting,P(s) and P(t) are allocations of R to A in which σu(P(t)) > σu(P(s)).
- Question : Is there an IR C(k)-contract path that realises the deal (Ps, Pt)?

Preliminary Definitions

- Dealing with cautious agents

Complexity Analysis

- Resource allocat
- Welfare Improvement (WI)
  Instance: A tuple <A , R , U ,P > where A , R , and U are as before, and P is an allocation.
  Question: Is there an allocation Q for which σu(Q) > σu(P)?

Complexity Analysis

- Welfare Optimisation (WO)
  Instance: A tuple <A , R , U ,K > where A , R , and U are as before, and K is a rational number.
  - Question: Is there an allocation P for which σu(P) <= K?
- Pareto Optimal (PO)
  Instance: A tuple <A , R , U ,P > as for WI.
  Question: Is the allocation P Pareto optimal?

Complexity Analysis

- Theorem 11. Even if $|A| = 2$ and the utility functions are monotone
  (a) WI is NP-complete.
  (b) WO is NP-complete.
  (c) PO is CO-NP-complete.

Complexity Analysis

- Theorem 12. For all constant, k, IRk is NP-hard.
- Corollary 13. IRO is NP-hard in resource allocation settings for which all utility functions are monotone.
- Theorem 14. For k : N → N satisfying k(m) <= m/3, IR
  k(m) is NP-hard.
- Theorem 14. For k : N → N satisfying k(m) <= m/3, IRk(m)is NP-hard.

Conclusion

- If agents are rational and we place limit on the number of resources that can be transferred,
  - Constructing a suitable path to optimal allocation may fail
  - Determining if such a path exists is intractable
- Relax the conditions and permit ‘irrational’ deals
  Short term loss for long term gain

Comments

- Given that the problem of determining such a path is NP-hard, the we may have to rely on heuristics
- The same applies to termination condition for negotiation protocols, as determining if current allocation is optimal is intractable