CSCE 936: Cyber-Physical Systems: Design a CPS - Part I (HW #2)

Assigned: 2017-08-31
Due: 2017-09-14 upload to Canvas before class

Homework Sequence Overview

This is really part 1 of a multi-part homework sequence. There are multiple objectives of this homework sequence:

- Inform me about your strengths and weaknesses in designing a CPS from beginning to end
- Discover for yourself your strengths and weaknesses in designing a CPS from beginning to end
- Practice the parts of design you are good at, and learn some about the parts of design you're not good at

Note that I **do not** expect you to have all the tools and background to do each part of the sequence. I want you to work hard and **attempt** each part of the sequence. If you cannot complete a question entirely tell me what process you used to try and figure it out and show me what you were able to accomplish. **The goal of this homework is to move away from the mathematical narrow focus of any individual area and think on a more "systems" level. Use your creativity!!!** Also, on this homework sequence you may collaborate with others if it will help. Also feel free to use Google, and any other resources at your disposal.

Problem

The CPS we will be designing in this homework is control of a Planar VTOL (vertical take-off and landing) or a multicopter. A sketch of the system can be seen in Figure 1.

In this design study we will explore the control design for a simplified planar version of a quadrotor following a ground target. In particular, we will constrain the dynamics to be in a two dimension plane comprising vertical and one dimension of horizontal, as shown in Figure 1. The planar vertical take-off and landing (VTOL) system is comprised of a center pod of mass m_c and inertia J_c, a right motor/rotor that is modeled as a point mass m_r that exerts a force f_r at a distance d from the center of mass, and a left motor/rotor that is modeled as a point mass m_l that exerts a force f_l at a distance $-d$ from the center of mass. The position of the center of mass of the planar VTOL system is given by horizontal position z_v and altitude h. The airflow through the rotor creates a change in the direction of flow of air and causes what is called "momentum drag." Momentum drag can be modeled as a viscous drag force that is proportional to the horizontal velocity $\dot{z}_v$. In other words, the drag force is $F_{drag} = -\mu \dot{z}_v$. The target on the ground will be modeled as an object with position z_t and altitude $h = 0$. We will not explicitly model the dynamics of the target.

Use the following physical parameters: $m_c = 1\,\text{kg}$, $J_c = 0.0042\,\text{kg}\,\text{m}^2$, $m_r = 0.25\,\text{kg}$, $m_l = 0.25\,\text{kg}$, $d = 0.3\,\text{m}$, $\mu = 0.1\,{}^{\text{kg}}/{}_{\text{s}}$, $g = 9.81\,{}^{\text{m}}/{}_{\text{s}^2}$.

HW #2 Problems

1. **(10 points)** Using the configuration variables z_v, h, and θ, write an expression for the kinetic energy of the system.

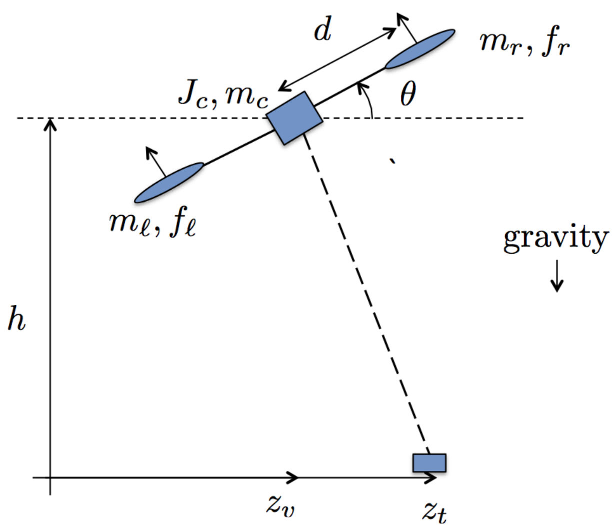

Figure 1: Planar VTOL

2. **(20 points)** Derive the equations of motion for the system by:
   (a) Find the potential energy for the system
   (b) Define the generalized coordinates and damping forces
   (c) Find the generalized forces. Note that the right and left forces are more easily modeled as a total force on the center of mass, and a torque about the center of mass
   (d) Derive the equations of motion for the planar VTOL system using the Euler-Lagrange equations

3. **(20 points)** Since f_r and f_l appear in the equations as either $f_r + f_l$ or $d\,(f_r - f_l)$, it is easier to think of the inputs as the total force $F \triangleq f_r + f_l$, and the torque $\tau = d\,(f_r - f_l)$. Note that since

$$\begin{bmatrix} F \\ \tau \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ d & -d \end{bmatrix} \begin{bmatrix} f_r \\ f_l \end{bmatrix},$$

that

$$\begin{bmatrix} f_r \\ f_l \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ d & -d \end{bmatrix}^{-1} \begin{bmatrix} F \\ \tau \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2\mathrm{d} \\ 1/2 & -1/2\mathrm{d} \end{bmatrix}^{-1} \begin{bmatrix} F \\ \tau \end{bmatrix}.$$

Therefore, in subsequent exercises, if we determine F and τ, then the right and left forces are given by

$$f_r = \frac{1}{2}F + \frac{1}{2d}\tau$$
$$f_l = \frac{1}{2}F - \frac{1}{2d}\tau.$$

Now:
   (a) Find the equilibria of the system
   (b) Linearize the equations about the equilibria using Jacobian linearization

4. **(20 points)** Form a state-space representation of the linear system:
   (a) Defining the longitudinal states as $\tilde{x}_{lon} = \left(\tilde{h}, \dot{\tilde{h}}\right)^\top$ and the longitudinal input as $\tilde{u}_{lon} = \tilde{F}$, find the linear state space equations in the form

$$\dot{\tilde{x}}_{lon} = A\tilde{x}_{lon} + B\tilde{u}_{lon}.$$

(b) Defining the lateral states as $\tilde{x}_{lat} = \left(\tilde{z}, \tilde{\theta}, \dot{\tilde{z}}, \dot{\tilde{\theta}}\right)^\top$ and the lateral input as $\tilde{u}_{lat} = \tilde{\tau}$, find the linear state space equations of the form

$$\dot{\tilde{x}}_{lat} = A\tilde{x}_{lat} + B\tilde{u}_{lat}.$$

5. **(20 points)** Simulate the linear system responding to some initial conditions and/or possibly some (open-loop) arbitrary control input

6. **(10 points)** Write a couple paragraphs on which parts were easy, which parts were hard and what you need to learn going forward to design this system well.

What to Submit

Please submit the following, on Canvas, by the specified time above:

1. **(100 points)** A PDF document with your answer to the questions above. I understand that many of you have not been exposed to this type of material, so this may be difficult. I expect you to make a reasonable effort for full credit. So convince me. Please observe the following:

   - Homework must be typed