CSCE 990: *Real-Time Systems*

Dynamic-Priority Scheduling

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Dynamic-priority Scheduling

- The remaining scheduling disciplines we consider are **priority-based**.
  - » Each job is assigned a priority, and the highest-priority job executes at any time.
- We begin with **dynamic-priority scheduling**.
  - » Under dynamic-priority scheduling, different jobs of a task may be assigned different priorities.
  - » Can have the following: job $J_{i,k}$ of task T_i has higher priority than job $J_{j,m}$ of task T_j, but job $J_{i,\ell}$ of T_i has lower priority than job $J_{j,n}$ of T_j.
- We will consider static-priority scheduling later.

Outline

- We consider both **earliest-deadline-first (EDF)** and **least-laxity-first (LLF)** (called least-slack-time-first by Liu) scheduling.
- **Outline:**
  - » Optimality of EDF and LLF (Section 4.6 of Liu).
  - » Utilization-based schedulability test for EDF (Section 6.3 of Liu).
  - » Non-preemptive EDF from: Jeffay, Stanat, and Martel, "On Optimal, Non-preemptive Scheduling of Periodic nad Sporadic Tasks," 18th IEEE RTSS, 1991, pp. 129-139.

Optimality of EDF

Theorem 4-1: [Liu and Layland] When preemption is allowed and jobs do not contend for resources, the EDF algorithm can produce a feasible schedule of a set **J** of independent jobs with arbitrary release times and deadlines on a processor if and only if **J** has feasible schedules.

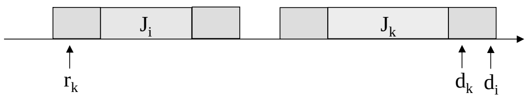
Notes:

- Applies even if tasks are not periodic.
- If periodic, a task's relative deadline can be less than its period, equal to its period, or greater than its period.

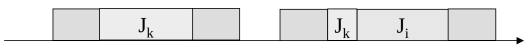
Proof of Theorem 4-1

We show that any feasible schedule of **J** can be systematically transformed into an EDF schedule.

Suppose parts of two jobs J_i and J_k are executed out of EDF order:



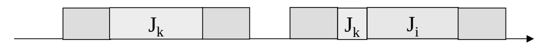
This situation can be corrected by performing a “swap”:



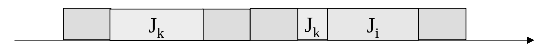
Proof (Continued)

If we inductively repeat this procedure, we can eliminate all out-of-order violations.

The resulting schedule may still fail to be an EDF schedule because it has idle intervals where some job is ready:

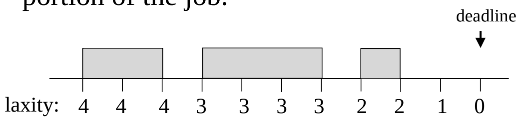


Such idle intervals can be eliminated by moving some jobs forward:



LLF Scheduling

- **Definition:** At any time t, the **slack** (or **laxity**) of a job with deadline d is equal to d – t minus the time required to complete the remaining portion of the job.



- **LLF Scheduling:** The job with the smallest laxity has highest priority at all times.

Optimality of LLF

Theorem 4-3: When preemption is allowed and jobs do not contend for resources, the LLF algorithm can produce a feasible schedule of a set **J** of independent jobs with arbitrary release times and deadlines on a processor if and only if **J** has feasible schedules.

The proof is similar to that for EDF and is left as an exercise.

Question: Which of EDF and LLF would be preferable in practice?

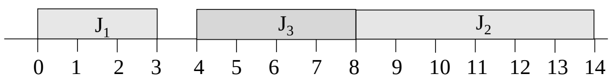
Preemptive vs. Nonpreemptive EDF

The rest of our discussion of dynamic-priority scheduling will focus preemptive and non-preemptive EDF. We first show the following:

Theorem: Non-preemptive EDF is not optimal.

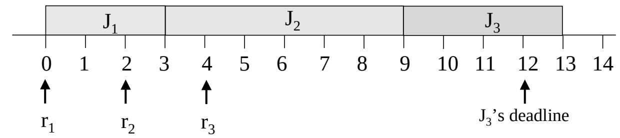
Proof: Consider a system of three jobs J_1, J_2, and J_3 such that $(r_1, e_1, d_1) = (0, 3, 10)$, $(r_2, e_2, d_2) = (2, 6, 14)$, $(r_3, e_3, d_3) = (4, 4, 12)$.

Here's a schedule:



Proof (Continued)

But under non-preemptive EDF, a deadline is missed!



Question: Should we conclude from this result that preemptive EDF is always better than non-preemptive EDF in practice?

- **Note:** The EDF optimality proof assumes there is no penalty due to preemption.
- Are there other practical issues we have ignored?

Utilization-based Schedulability Test for (Preemptive) EDF

Note: Whenever we say "EDF" from now on, we mean preemptive EDF, unless specified otherwise.

> **Theorem 6-1:** [Liu and Layland] A system **T** of independent, preemptable, periodic tasks with relative deadlines equal to their periods can be feasibly scheduled (under EDF) on one processor if and only if its total utilization U is at most one.

Proof: The "only if" part is obvious: If U > 1, then some task clearly must miss a deadline. So, we concentrate on the "if" part.

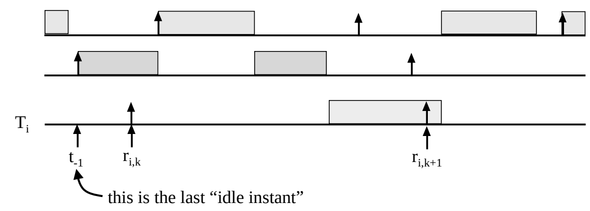
Setting Up the Proof

We wish to show: $U \le 1 \Rightarrow \mathbf{T}$ is schedulable.

We prove the contrapositive, i.e., T is not schedulable $\Rightarrow U > 1$.

Assume T is not schedulable.

Let $J_{i,k}$ be the first job to miss its deadline.



Proof (Continued)

Because $J_{i,k}$ missed its deadline…

the demand placed on the processor in $[t_{-1}, r_{i,k+1})$ by jobs with deadlines $\leq r_{i,k+1}$ is greater than the available processor time in $[t_{-1}, r_{i,k+1}]$.

Thus,

$$
\begin{aligned}
& r_{i,k+1} - t_{-1} \\
&= \text{available processor time in } [t_{-1}, r_{i,k+1}] \\
&< \text{demand placed on the processor in } [t_{-1}, r_{i,k+1}) \text{ by jobs with deadlines} \leq r_{i,k+1} \\
&= \sum_{j=1}^{N} (\text{the number of jobs of } T_j \text{ with deadlines} \leq r_{i,k+1} \text{ released in } [t_{-1}, r_{i,k+1})) \cdot e_j \\
&\leq \sum_{j=1}^{N} \left\lfloor \frac{r_{i,k+1} - t_{-1}}{p_j} \right\rfloor \cdot e_j \\
&\leq \sum_{j=1}^{N} \frac{r_{i,k+1} - t_{-1}}{p_j} \cdot e_j
\end{aligned}
$$

Proof (Continued)

Thus, we have

$$r_{i,k+1} - t_{-1} < \sum_{j=1}^{N} \frac{r_{i,k+1} - t_{-1}}{p_j} \cdot e_j.$$

Cancelling $r_{i,k+1} - t_{-1}$ yields

$$1 < \sum_{j=1}^{N} \frac{e_j}{p_j},$$

i.e.,

$$1 < U.$$

This completes the proof.

Note: This proof is actually still valid if **deadlines are larger than periods**.

EDF with Deadlines < Periods

If deadlines are less than periods than $U \leq 1$ is no longer a sufficient schedulability condition.

This is easy to see. Consider two tasks such that, for both, $e_i = 1$ and $p_i = 2$. If both have deadlines at 1.9, then the system is not schedulable, even though $U = 1$.

For these kinds of systems, we work with **densities** instead of utilizations.

Definition: The **density of task T_k** is defined to be $\delta_k = e_k/\min(D_k, p_k)$. The **density of the system** is defined to be $\Delta = \sum_{k=1,,,,,N} \delta_k$.

Deadlines < Periods (Continued)

Theorem 6-2: A system **T** of independent, preemptable, periodic tasks can be feasibly scheduled on one processor if its density is at most one.

The proof is similar to that for Theorem 6-1 and is left as an exercise.

Note: This theorem only gives **sufficient** condition.

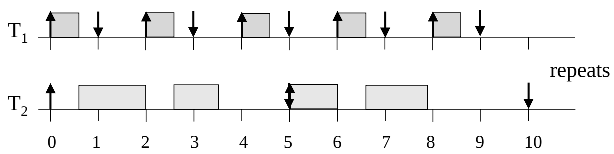
We refer to the following as the **schedulability condition for EDF**:

$$\boxed{\sum_{k=1}^{n} \frac{e_k}{\min(D_k, p_k)} \leq 1}$$

Proof of Non-tightness

To see that $\Delta > 1$ doesn't imply non-schedulability, consider the following example.

Example: We have two tasks $T_1 = (2, 0.6, 1)$ and $T_2 = (5, 2.3)$. $\Delta = 0.6/1 + 2.3/5 = 1.06$. Nonetheless, we can schedule this task set under EDF:



Non-preemptive EDF

(Jeffay *et al.*)

Theorem : Let $\mathbf{T} = \{T_1, T_2, \ldots, T_n\}$ be a system of independent, periodic tasks with relative deadlines equal to their periods such that the tasks in **T** are indexed in non-decreasing order by period (i.e., if $i < j$, then $p_i \leq p_j$). **T** can be scheduled by the non-preemptive EDF algorithm if:

1) $\sum_{i=1}^{n} \frac{e_i}{p_i} \leq 1$

2) $\left(\forall i : 1 \leq i \leq n :: \left(\forall L : p_1 < L < p_i :: L \geq e_i + \sum_{j=1}^{i-1} \left\lfloor \frac{L-1}{p_j} \right\rfloor \cdot e_j \right) \right)$

Note: This condition is actually necessary and sufficient for "real-world" sporadic tasks.

Explanation

The **first condition** is just a constraint on utilization.

In the **second condition**, the term

$$L \geq e_i + \sum_{j=1}^{i-1} \left\lfloor \frac{L-1}{p_j} \right\rfloor \cdot e_j$$

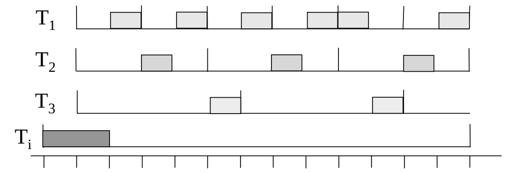
gives an upper bound on processor demand in an interval [t, t+L].

Intuition: The "worst-case" pattern of job releases occurs when a job of some T_i begins executing (**non-preemptively!**) one time unit before some tasks with smaller periods begin releasing some jobs.

These other jobs are **blocked** by the job of T_i.

Second Condition (Continued)

Here's an illustration:



For any L over the range $p_1 < L < p_i$, the total demand on the processor in [t, t + L] due to jobs with deadlines at or before t + L is:

$$e_i + \sum_{j=1}^{i-1} \left\lfloor \frac{L-1}{p_j} \right\rfloor \cdot e_j$$

For the system to be schedulable, this demand must not exceed the length of the interval (which is L).

Proof of Theorem

Suppose conditions (1) and (2) hold for **T** but a deadline is missed. Let t_d be the earliest point in time at which a deadline is missed.

There are two cases.

<u>Case 1:</u> No job with a deadline after time t_d is scheduled prior to time t_d. The analysis is just like with preemptive EDF.

As before, let t_{-1} be the last "idle instant".

As before, because a deadline is missed at t_d, demand over $[t_{-1}, t_d]$ exceeds $t_d - t_{-1}$. In addition, this demand is at most $\sum_{j=1,\ldots,n} \lfloor (t_d - t_{-1})/p_j \rfloor \cdot e_j$.

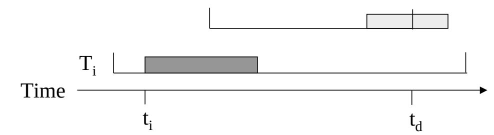
Thus, we have $t_d - t_{-1} < \sum_{j=1,\ldots,n} \lfloor (t_d - t_{-1})/p_j \rfloor \cdot e_j \leq \sum_{j=1,\ldots,n} [(t_d - t_{-1})/p_j] \cdot e_j$.

This implies utilization exceeds one, which contradicts condition (1).

Proof (Continued)

<u>Case 2:</u> Some job with a deadline after time t_d is scheduled prior to time t_d.

Let T_i be the task with the last job with deadline after t_d that is scheduled prior to t_d. Then, we have the following:



Let us bound the processor demand in $[t_i, t_d]$ …

Proof (Continued)

- Observe the following:
  - » $p_i > t_d - t_i$.
    - This follows from the fact that the job of task T_i scheduled at time t_i had a deadline after t_d.
  - » No task with index greater than i is scheduled in the interval $[t_i, t_d]$.
  - » Other than a job of task T_i, no job scheduled in $[t_i, t_d]$ could have been released at time t_i.
  - » There is no idle time in the interval $[t_i, t_d]$.
  - » There is a least one job that is released in $[t_i, t_d]$ with a deadline at or before time t_d.

Proof (Continued)

From these facts, we conclude that demand over $[t_i, t_d]$ is less than or equal to

$$e_i + \sum_{j=1}^{i-1} \left\lfloor \frac{t_d - (t_i + 1)}{p_j} \right\rfloor \cdot e_j.$$

Let $L = t_d - t_i$. Then,

$$L < e_i + \sum_{j=1}^{i-1} \left\lfloor \frac{L - 1}{p_j} \right\rfloor \cdot e_j.$$

This contradicts condition (2).

Notes

- Note that this scheduling condition requires pseudo-polynomial time to evaluate. **(Why?)**
- Using "real-world" terminology, this condition is necessary and sufficient for sporadic and non-concrete periodic task systems. **(Why?)**
  - "**Concrete**" = fixed release times (though maybe not all 0).
  - For a non-concrete task system to be feasible, it must be schedulable for any initial phasing.
- In the rest of the paper, it is shown that the feasibility problem for non-preemptive concrete periodic task systems is NP-hard in the strong sense.
  - Implies that a pseudo-polynomial-time feasibility test is unlikely for such systems. (We cover this result later when we consider intractability.)