function GENERAL-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end

Essence of search: which node to expand first?

→ search strategy

A strategy is defined by picking the order of node expansion
Types of Search

**Uninformed:** use only information available in problem definition

**Heuristic:** exploits some knowledge of the domain

Uninformed search strategies

1. Breadth-first search
2. Uniform-cost search
3. Depth-first search
4. Depth-limited search
5. Iterative deepening depth-first search
6. Bidirectional search
**Search strategies**

**Criteria for evaluating search:**

1. Completeness: does it always find a solution if one exists?
2. Time complexity: number of nodes generated/expanded
3. Space complexity: maximum number of nodes in memory
4. Optimality: does it always find a least-cost solution?

**Time/space complexity measured in terms of:**

- $b$: maximum branching factor of the search tree
- $d$: depth of the least-cost solution
- $m$: maximum depth of the search space (may be $\infty$)
Breadth-first search (I)

→ Expand root node
→ Expand *all* children of root
→ Expand *each* child of root
→ Expand successors of each child of root, etc.

→ Expands nodes at depth $d$ before nodes at depth $d + 1$
→ Systematically considers all paths length 1, then length 2, etc.
→ Implement: put successors at end of queue.. FIFO
Breadth-first search (2)

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Breadth-first search (3)

→ One solution?

→ Many solutions? Finds shallowest goal first

1. Complete? Yes, if $b$ is finite

2. Optimal? provided cost increases monotonically with depth, not in general (e.g., actions have same cost)

3. Time? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$

   $$O(b^{d+1}) = \left\{ \begin{array}{c}
   \text{branching factor } b \\
   \text{depth } d
   \end{array} \right\}$$

4. Space? same, $O(b^{d+1})$, keeps every node in memory, big problem
   
   can easily generate nodes at 10MB/sec so 24hrs = 860GB
Uniform-cost search (I)

→ Breadth-first does not consider path cost \( g(x) \)
→ Uniform-cost expands first lowest-cost node on the fringe
→ Implement: sort queue in decreasing cost order

When \( g(x) = \text{Depth}(x) \) → Breadth-first \( \equiv \) Uniform-cost
Uniform-cost search (2)

1. Complete?
   Yes, if cost $\geq \epsilon$

2. Optimal?
   If the cost is a monotonically increasing function
   When cost is added up along path, an operator’s cost ........?

3. Time?
   $\#$ of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
   where $C^*$ is the cost of the optimal solution

4. Space?
   $\#$ of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
**Depth-first search** (I)

→ Expands nodes at deepest level in tree
→ When dead-end, goes back to shallower levels
→ Implement: put successors at front of queue.. LIFO

→ Little memory: path and unexpanded nodes

For $b$: branching factor, $m$: maximum depth, space .........?
Depth-first search (2)
**Depth-first search (3)**

Time complexity:

- We may need to expand all paths, $O(b^m)$
- When there are many solutions, DFS may be quicker than BFS
- When $m$ is big, much larger than $d$, $\infty$ (deep, loops), .. troubles

→ Major drawback of DFS: going deep where there is no solution..

**Properties:**

1. Complete? Not in infinite spaces, complete in finite spaces
2. Optimal?
3. Time? $O(b^m)$
   - terrible if $m$ is much larger than $d$, but if solutions are dense, may be much faster than breadth-first
4. Space? $O(bm)$, linear!
Depth-limited search (I)

→ DFS is going too deep, put a threshold on depth!
   For instance, 20 cities on map for Romania, any node deeper
   than 19 is cycling. Don’t expand deeper!
→ Implement: nodes at depth $l$ have no successor

Properties:

1. Complete?

2. Optimal?

3. Time? (given $l$ depth limit)

4. Space? (given $l$ depth limit)

Problem: how to choose $l$?
Iterative-deepening search (I)

→ DLS with depth = 0
→ DLS with depth = 1
→ DLS with depth = 2
→ DLS with depth = 3...

Limit = 0
Limit = 1
Limit = 2
Limit = 3

→ Combines benefits of DFS and BFS
Iterative-deepening search (2)

- Limit = 0
- Limit = 1
- Limit = 2
- Limit = 3
Iterative-deepening search (3)

→ combines benefits of DFS and BFS

Properties:

1. Time? \((d + 1).b^0 + (d).b + (d - 1).b^2 + \ldots + 1.b^d = O(b^d)\)
2. Space? \(O(bd)\), like DFS
3. Complete? like BFS
4. Optimal? like BFS (if step cost = 1)
Iterative-deepening search (4)

$\rightarrow$ Some nodes are expanded several times, wasteful?

$N(\text{BFS}) = b + b^2 + b^3 + \ldots + b^d + (b^{d+1} - b)$

$N(\text{IDS}) = (d)b + (d - 1)b^2 + \ldots + (1)b^d$

Numerical comparison for $b = 10$ and $d = 5$:

$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$

$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$

$\rightarrow$ IDS is preferred when search space is large and depth unknown
Bidirectional search (I)

→ Given initial state and the goal state, start search from both ends and meet in the middle

→ Assume same $b$ branching factor, $\exists$ solution at depth $d$, time:

\[ O(2b^{d/2}) = O(b^{d/2}) \]

\[ b = 10, \ d = 6, \ \text{DFS} = 1,111,111 \text{ nodes}, \ \text{BDS}=2,222 \text{ nodes!} \]
**Bidirectional search (2)**

**In practice:**

- Need to define predecessor operators to search backwards. If operators are invertible, no problem.
- What if there exist many goals (set state)?
  - Do as for multiple-state search.
- Need to check the 2 fringes to see how they match.
  - Need to check whether any node in one space appears in the other space (use hashing).
  - Need to keep all nodes in a half in memory $O(b^{d/2})$.
- What kind of search in each half space?
# Summary

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Breadth-First</th>
<th>Uniform-Cost</th>
<th>Depth-First</th>
<th>Depth-Limited</th>
<th>Iterative Deepening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete?</td>
<td>Yes*</td>
<td>Yes*</td>
<td>No</td>
<td>Yes, if $l \geq d$</td>
<td>Yes</td>
</tr>
<tr>
<td>Time</td>
<td>$b^{d+1}$</td>
<td>$b^{[C^*/\varepsilon]}$</td>
<td>$b^m$</td>
<td>$b^l$</td>
<td>$b^d$</td>
</tr>
<tr>
<td>Space</td>
<td>$b^{d+1}$</td>
<td>$b^{[C^*/\varepsilon]}$</td>
<td>$bm$</td>
<td>$bl$</td>
<td>$bd$</td>
</tr>
<tr>
<td>Optimal?</td>
<td>Yes*</td>
<td>Yes*</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

$b$ branching factor
$d$ solution depth
$m$ maximum depth of tree
$l$ depth limit
**Loops:** Avoid repeated states (I)

Avoid expanding states that have already been visited

Valid for both infinite and finite trees

\[
\begin{align*}
& m \text{ maximum depth} \\
& m + 1 \text{ states} \\
& 2^m \text{ possible branches (paths)}
\end{align*}
\]

Example:
**Loops: (2)**

Keep nodes in two lists:

- Open list: Fringe
- Closed list: Leaf and expanded nodes

Discard a current node that matches a node in the closed list

Tree-Search $\rightarrow$ Graph-Search

![Diagram of search algorithm with nodes and arrows]

**Issues:**

1. Implementation: hash table, access is constant time
   
   Trade-off cost of storing + checking vs. cost of searching

2. Losing optimality
   
   when new path is cheaper/shorter of the one stored

3. DFS and IDS now require exponential storage

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Summary

**Path**: sequence of actions leading from one state to another

**Partial solution**: a path from an initial state to another state

**Search**: develop a set of partial solutions

- Search tree & its components (node, root, leaves, fringe)
- Data structure for a search node
- Search space vs. state space
- Node expansion, queue order
- Search types: uninformed vs. heuristic
- 6 uninformed search strategies
- 4 criteria for evaluating & comparing search strategies