1 Simple scheduling problem

Consider the problem of scheduling five tasks: $T_1$, $T_2$, $T_3$, $T_4$, and $T_5$, each of which takes one hour to complete. The tasks may start at 1:00, 2:00, 3:00. Any number of tasks can be executed simultaneously provided the following restrictions are satisfied.

- $T_1$ must start after $T_3$.
- $T_3$ must start before $T_4$ and after $T_5$.
- $T_2$ cannot execute at the same time as $T_1$.
- $T_2$ cannot execute at the same time as $T_4$.
- $T_4$ cannot start at 2:00.

1. Formulate the problem as a CSP by stating: the variables, their domain, and the applicable constraints.
   
   **Hints:** focus on the start time of a task.

2. Draw the constraint graph.

3. Apply arc-consistency to each constraint in the CSP until no values can be ruled out (i.e., the CSP becomes arc-consistent).

2 N-Queen Problem as a CSP

Consider the 4-queens problem where each queen is associated with a row and can be assigned to any column in the row.

1. Define this problem as a CSP. Specify the variables and their domain, and each binary constraint by ‘extension.’

2. Define a binary constraint $C_{Q_i,Q_j}$ between two variables $Q_i$ and $Q_j$ by ‘intension.’

3. What is the size of this CSP (which is the size of the search tree it may yield)?

4. Draw the constraint graph.
5. Arc-consistency of a binary constraint $C_{Q_i, Q_j}$ between two variables $Q_i$ and $Q_j$ ensures that every value for the variable $Q_i$ has a support (at least one consistent value) in the domain of $Q_j$ and vice-versa. Run manually arc-consistency on the 4-Queens problem. Can you remove any value? At the end of the operation the CSP is said to be arc-consistent.

6. Arc-consistency is also called 2-consistency because it considers all combinations of two variables at the same time. Let’s consider all combinations of 3 variables at the same time and let’s check whether or not every value in the domain of a given variable has a support in the domain of the two other variables (simultaneously). If it does not, the value can be removed. Can you remove any value? This consistency property is called (1,2)-consistency.