Recitation 3

Created by Taylor Spangler, Adapted by Beau Christ

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- Rosen 1.3:15
  Show that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology

Step	**Sentence**	**Equivalence law**
0.	$\neg(\neg q \wedge (p \rightarrow q)) \vee \neg p$	
1.	$\equiv \neg(\neg q \wedge (\neg p \vee q)) \vee \neg p$	Implication law
2.	$\equiv (q \vee (p \wedge \neg q)) \vee \neg p$	DeMorgan's law
3.	$\equiv ((q \vee p) \wedge (q \vee \neg q)) \vee \neg p$	Distributive law
4.	$\equiv ((q \vee p)) \vee \neg p$	Identity law
5.	$\equiv (q \vee (p \vee \neg p))$	Associative law
6.	$\equiv q \vee T$	Identity law
7.	$\equiv T$	Identity law

- Rosen 1.3:23
  Show that $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

Step	**Sentence**	**Equivalence law**
0.	$(p \rightarrow r) \wedge (q \rightarrow r)$	
1.	$\equiv (\neg p \vee r) \wedge (\neg q \vee r)$	Implication law
2.	$\equiv (\neg p \wedge \neg q) \vee r$	Distributive law
3.	$\equiv \neg(\neg p \wedge \neg q) \rightarrow r$	Implication law
4.	$\equiv (p \vee q) \rightarrow r$	DeMorgan's law and double negation law

- Example from a url that no longer exists:[1]

  Given:

  1. $p \wedge q$
  2. $p \rightarrow \neg(q \wedge r)$
  3. $s \rightarrow r$

  Prove $\neg s$ using the rules of inference.

[1] `http://marauder.millersville.edu/~bikenaga/mathproof/rules-of-inference/rules-of-inference.html`

	Step	Reason
1.	$p \wedge q$	Premise
2.	$p \rightarrow \neg(q \wedge r)$	Premise
3.	$s \rightarrow r$	Premise
4.	p	Simplification of (1)
5.	q	Simplification of (1)
6.	$\neg(q \wedge r)$	Modus Ponens (2) and (4)
7.	$\neg q \vee \neg r$	DeMorgan's Law (6)
8.	$\neg r$	Disjunctive syllogism (5) and (7)
9.	$\neg s$	Modus Tollens (3) and (8)

- Transcribe in FOL the following English statement: "Politicians can fool some of the people all of the time, all of the people some of the time, but they cannot fool all of the people all of the time."

  1. First we define the predicates:
     $Fools(x, y, t)$: x fool y at time t.
     $P(x)$: x is a politician.
  2. Universe of discourse: x, y all human beings, t all time instants.
  3. The answer is:

$$\begin{aligned} \forall x[P(x) \rightarrow \quad [\quad & (\exists y \forall t Fools(x, y, t)) \\ & \wedge(\exists t \forall y Fools(x, y, t)) \\ & \wedge\neg(\forall y \forall t Fools(x, y, t))]] \end{aligned}$$

  The expression $\neg(\forall y \forall t Fools(x, y, t))$ can be difficult to spell out in English. We recommend that you use the expression "*it is not the case that*" whenever an expression starts with a negation. Thus, the above reads: "It is not the case that every x fools every y at every time t."

- Beware of errors:

  - Let's compare the meaning of the two expressions:
    * $\exists t \forall y Fools(x, y, t)$: There is at least one time (or one incident) where everyone was fooled by x.
    * $\forall y \exists t Fools(x, y, t)$: Everyone is fooled at some point in time by x, but the time is not necessarily the same for all human beings.

    Naturally, we intend the first meaning.
  - Also, compare the meaning of the following two expressions:
    * $\forall x[P(x) \rightarrow \neg(\forall y \forall t Fools(x, y, t))] \equiv \forall x[P(x) \rightarrow (\exists y \exists t \neg Fools(x, y, t))]$: It is not the case that every politician fools everyone all the time.
    * $\forall x[P(x) \rightarrow (\forall y \forall t \neg Fools(x, y, t))]$: Politicians do not ever fool anyone or politicians never fool anyone.

- Now let's look at Rosen 1.4:43
  Determine whether $\forall x(P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ are logically equivalent. Justify your answer.

  Do we think they are equivalent? *No, they are not.*

  Let $P(x)$ be a proposition that is sometimes true and sometimes false, and let $Q(x)$ be some proposition that is always false. For example:

$P(x)$:	x is even
$Q(x)$:	x is irrational
UoD:	$x \in Z$

  Where an even number is always divisible by 2 and a rational number is a number that can be written as the ratio of two integers, $\forall x(Q(x) \to \exists a, b \in Z$ such that $x = \frac{a}{b})$.

  In this situation, $\forall x(P(x) \to Q(x))$ is false whereas $\forall x P(x) \to \forall x Q(x)$ is true. Indeed:

  - The statement $\forall x(P(x) \to Q(x))$ states that every odd integer is irrational, which obviously does not hold. (Actually, it states, for every integer, if it is odd then it is irrational.)
  - The statement $(\forall x P(x)) \to (\forall x Q(x))$ is true because $(\forall x P(x))$ is always false (remember, $P(x)$ sometimes holds and sometimes does, so it cannot hold for every $x \in Z$) because it is says that every integer is even, which is not true. On the other hand, we know that every integer is rational so it cannot be an irrational, so $(\forall x Q(x))$ is always false. So, $(\forall x P(x)) \to (\forall x Q(x))$ is true becase $0 \to 0$ holds.

- Now let's look at Rosen 1.4:41 part a.
  Express the following English statement using predicates, quantifiers and logical connectives: *At least one mail message, among a nonempty set of messages, can be saved if there is a disk with more than ten kilobits of free space.*

  1. We define our predicates:
     $D(x, y)$: *Disk x has more than y kilobits free.*
     $S(z)$: *The message z can be saved.*
  2. Universe of discourse: x is a disk, y is an integer, z is a message.
  3. We can express the specification in the following way: $\exists x D(x, 10) \to \exists z S(z)$.

- Next, Rosen 1.5:31 part d.
  Express the negation of the following statement so that negation immediately precedes a predicate.

  $$\forall x \exists y(P(x, y) \to Q(x, y))$$

0.	$\neg\forall x\exists y(P(x,y) \rightarrow Q(x,y))$	Given
1.	$\equiv \exists x\forall y\neg(P(x,y) \rightarrow Q(x,y))$	Move negation onward
2.	$\equiv \exists x\forall y\neg(\neg P(x,y) \vee Q(x,y))$	Implication rule
3.	$\equiv \exists x\forall y(P(x,y) \wedge \neg Q(x,y))$	De Morgan's law

Q.E.D.

- (Last 10 minutes) Give quiz