Clause Learning and Intelligent Backtracking in MiniSAT

CSCE 235H Introduction to Discrete Structures (Honors)
Spring 2022
URL: cse.unl.edu/~cse235h
All questions: Piazza

Clause Learning

- At every conflict, determine the cause of the conflict
- Create a new clause to prevent the conflict from being reached in the future
- Tools
  - Implication graph to determine cause of conflicts
  - Added clause is a “learnt” no-good

Implication Graph (1)

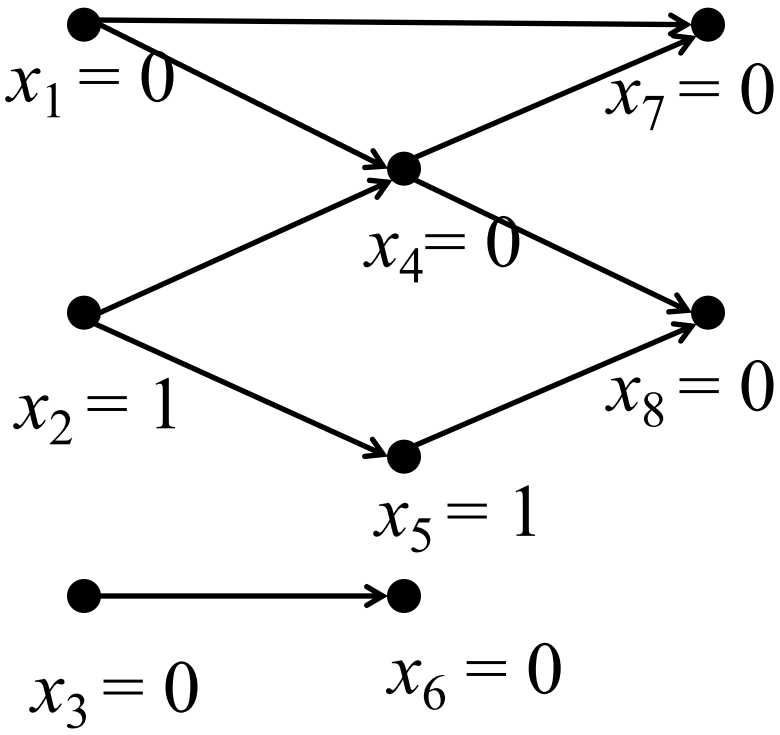
- Nodes correspond to assignments
- Nodes with no incoming edges are decision variables (assignments)
- Nodes with incoming edges were assigned through propagation

$x_1 \leftarrow 0$

$x_2 \leftarrow 1$

$x_3 \leftarrow 0$

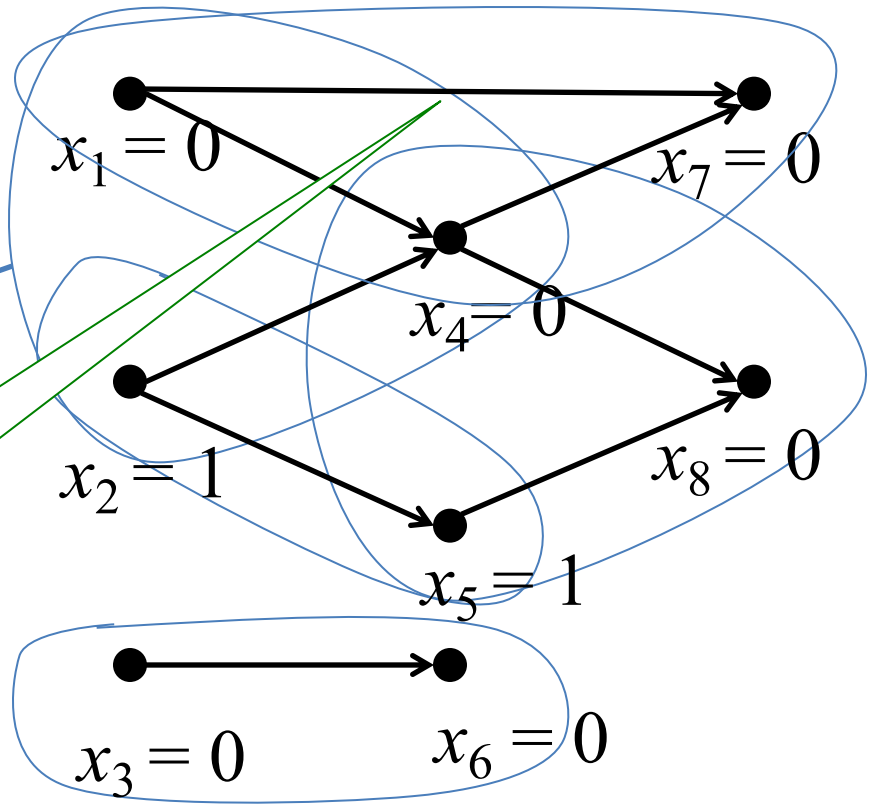
$$(x_1 \vee \neg x_2 \vee \neg x_4) \wedge$$
$$(x_4 \vee \neg x_5 \vee \neg x_8) \wedge$$
$$(x_1 \vee x_4 \vee \neg x_7) \wedge$$
$$(x_3 \vee \neg x_6) \wedge$$
$$(\neg x_2 \vee x_5)$$



Implication Graph (2)

- A node and its immediate predecessors correspond to the propagating clause

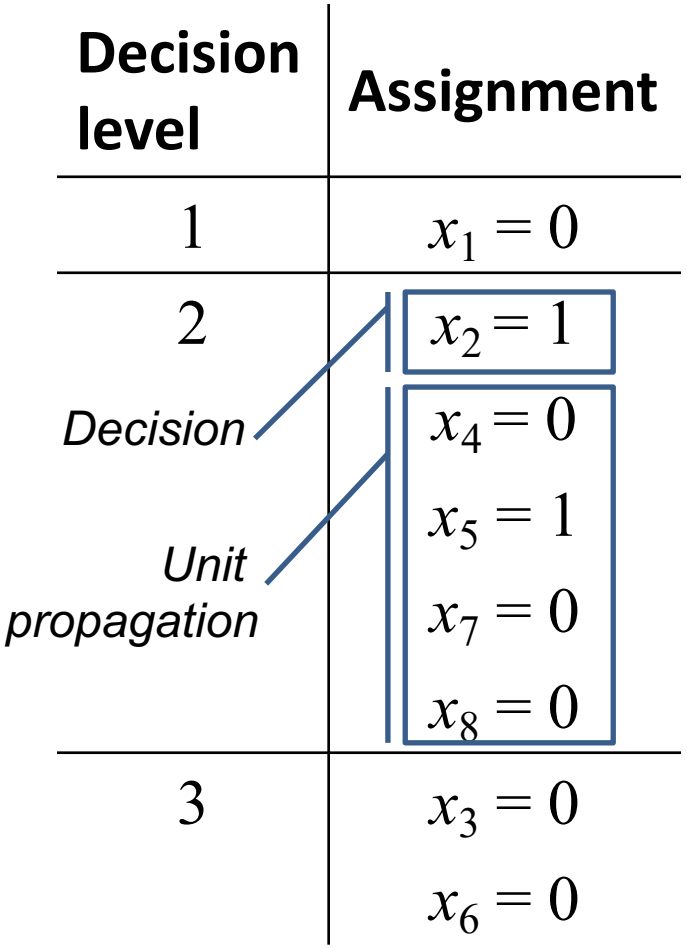
$$(x_1 \vee \neg x_2 \vee \neg x_4)$$



Note: transitive links are required to show which clause caused propagation

Trail

- Series of assignments made up to current point in search
- Broken up by ‘decision levels’
- Each decision level includes propagations



Clause Learning: UNSAT clause

DL	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$x_{10} = 0$
	$x_{11} = 1$
	$x_{12} = 0$

✓ $(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$

✗ $(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$

✓ $(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$

✓ $(x_1 \vee \neg x_2 \vee x_3) \wedge$

✓ $(\neg x_7 \vee x_9) \wedge$

✓ $(\neg x_7 \vee \neg x_8) \wedge$

✓ $(x_4 \vee \neg x_5) \wedge$

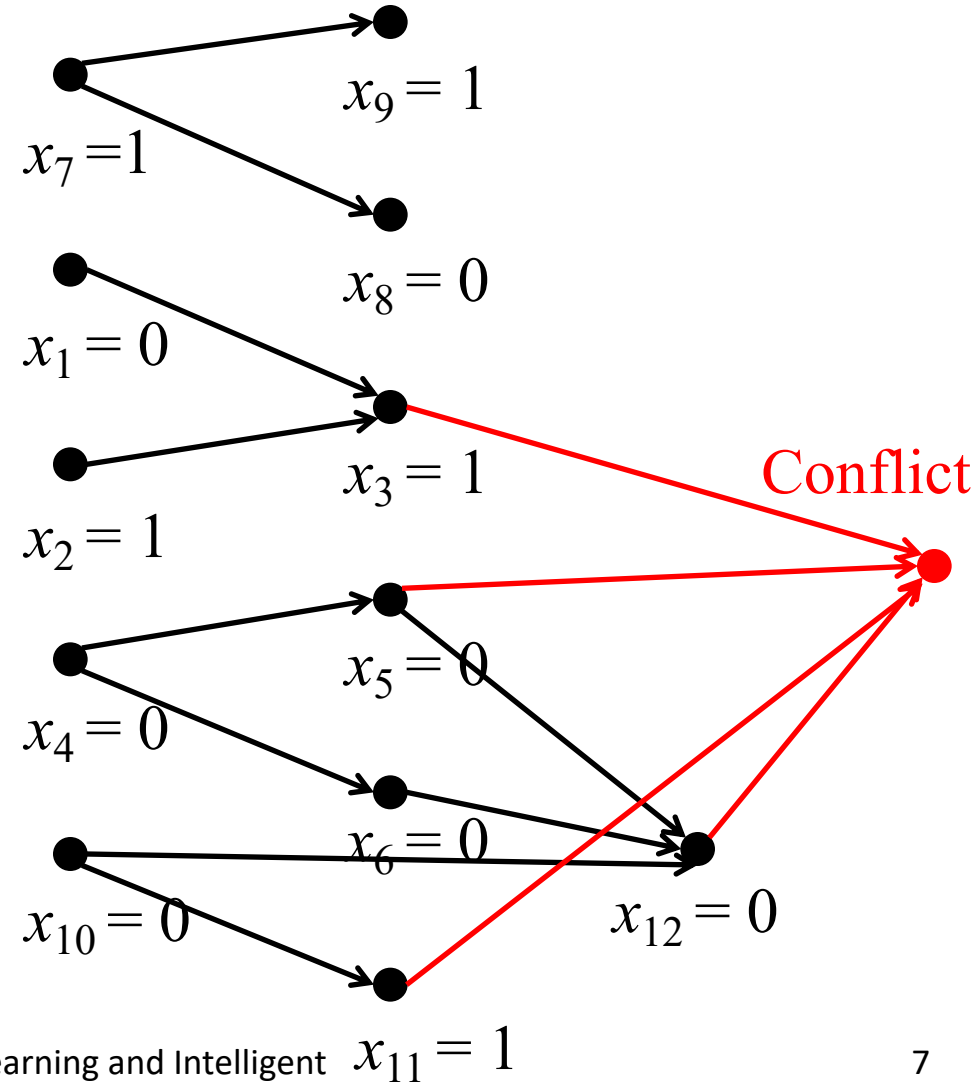
✓ $(x_4 \vee \neg x_6) \wedge$

✓ $(x_{10} \vee x_{11}) \wedge$

✓ $(\neg x_{10} \vee x_{11})$

Clause Learning: UNSAT clause in implication graph

DL	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$x_{10} = 0$
	$x_{11} = 1$
	$x_{12} = 0$



Clause Learning: Identify no-good

Decision variables

Conflict caused by assignment

$$(\neg x_1 \wedge x_2 \wedge \neg x_4 \wedge \neg x_{10})$$

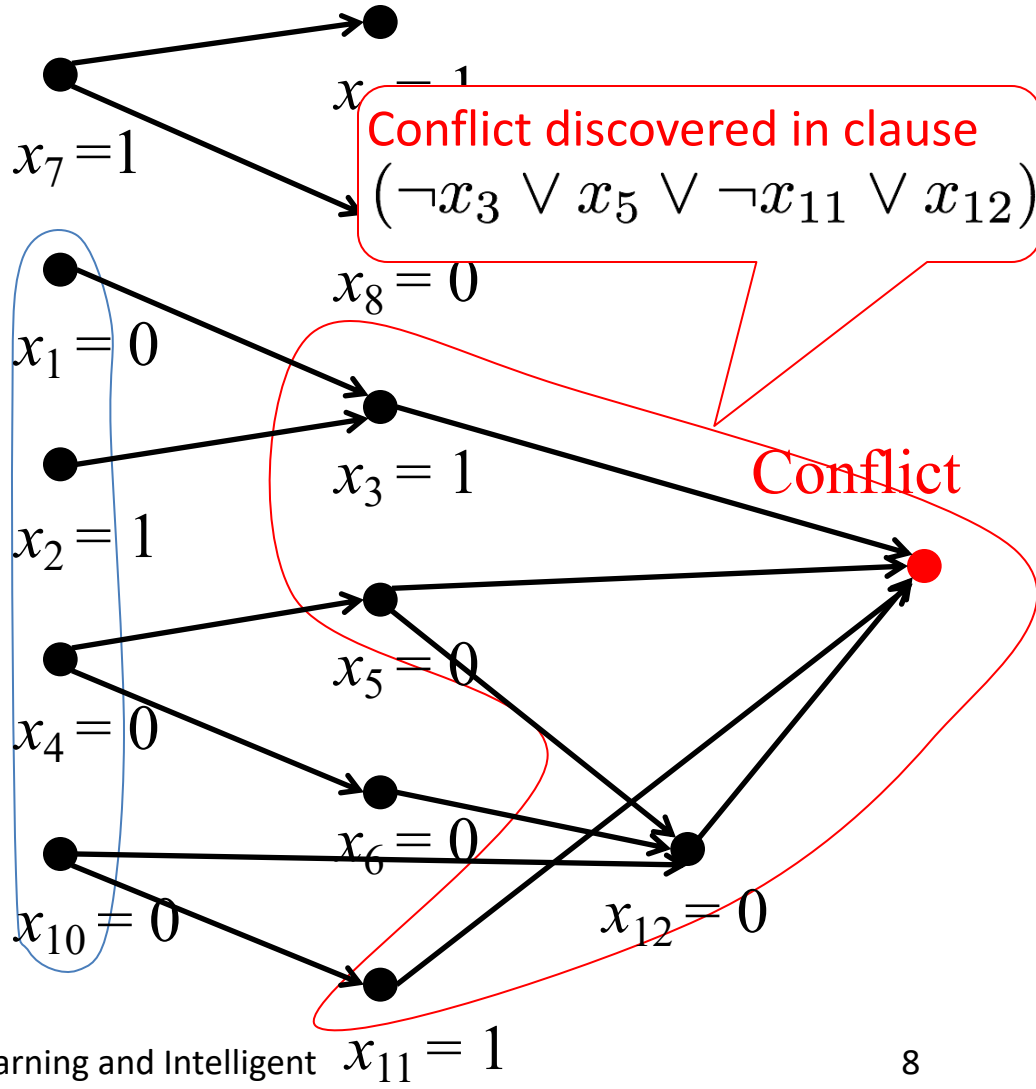
So, we negate it

$$\neg(\neg x_1 \wedge x_2 \wedge \neg x_4 \wedge \neg x_{10})$$

.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$$

to the formula to prevent this conflict in the future



Intelligent Backtracking

- When reaching a conflict, we
  - Consider conflicted clauses
  - Draw the implication graph
  - Identify the decision variables
  - Generate the learnt no-good
  - Add learned clause to the formula
- Undo assignments until the learned clause becomes a unit clause
- … Then propagate this new (unit clause)

Intelligent Backtracking: Undo Deepest Decision in Learned Clause

DL	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$\boldsymbol{x_{10} = 0}$
	$x_{11} = 1$
	$x_{12} = 0$

$$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$$
$$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$$
$$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_3) \wedge$$
$$(\neg x_7 \vee x_9) \wedge$$
$$(\neg x_7 \vee \neg x_8) \wedge$$
$$(x_4 \vee \neg x_5) \wedge$$
$$(x_4 \vee \neg x_6) \wedge$$
$$(x_{10} \vee x_{11}) \wedge$$
$$(\neg x_{10} \vee x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$$

x_{10} deepest decision variable

Intelligent Backtracking: Propagate New Clause

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$x_{10} = 0$
	$x_{11} = 1$
	$x_{12} = 0$

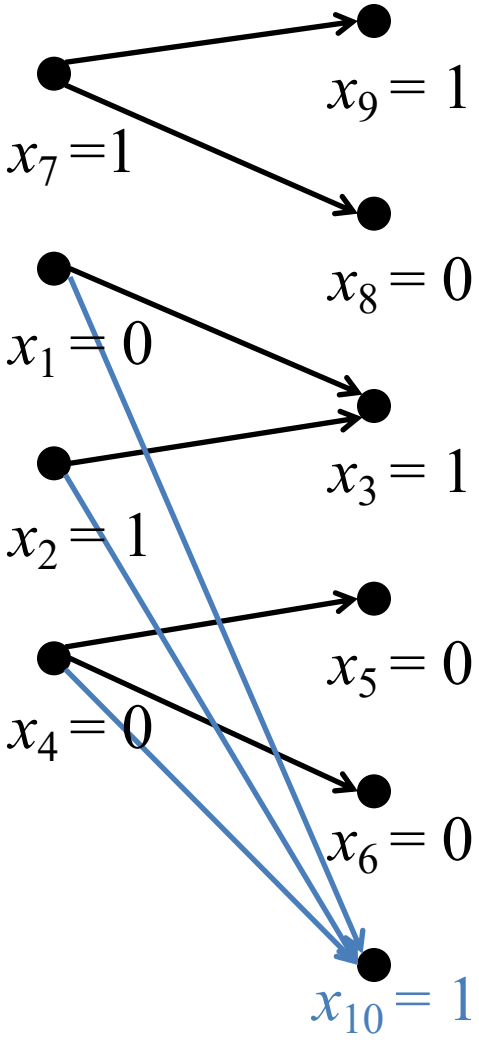
DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
	$\boldsymbol{x_{10} = 1}$

$$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$$
$$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$$
$$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_3) \wedge$$
$$(\neg x_7 \vee x_9) \wedge$$
$$(\neg x_7 \vee \neg x_8) \wedge$$
$$(x_4 \vee \neg x_5) \wedge$$
$$(x_4 \vee \neg x_6) \wedge$$
$$(x_{10} \vee x_{11}) \wedge$$
$$(\neg x_{10} \vee x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$$

Intelligent Backtracking: New Implication Graph

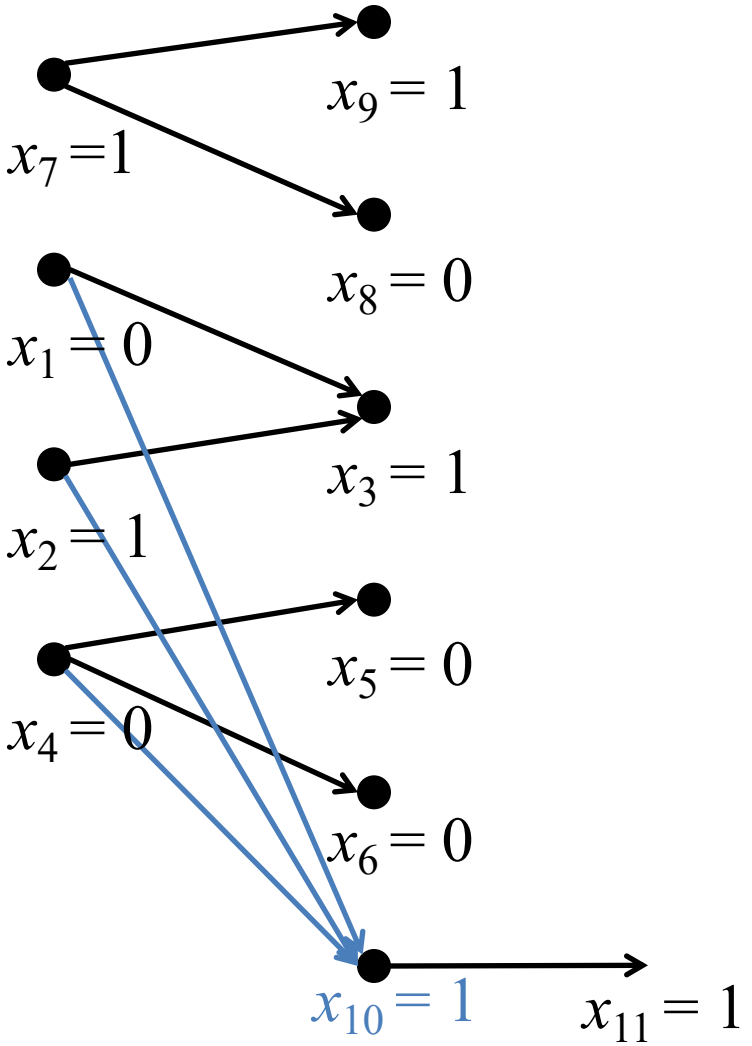
DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
5	$x_{10} = 0$
	$x_{11} = 1$
	$x_{12} = 0$

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
	$\boldsymbol{x_{10} = 1}$



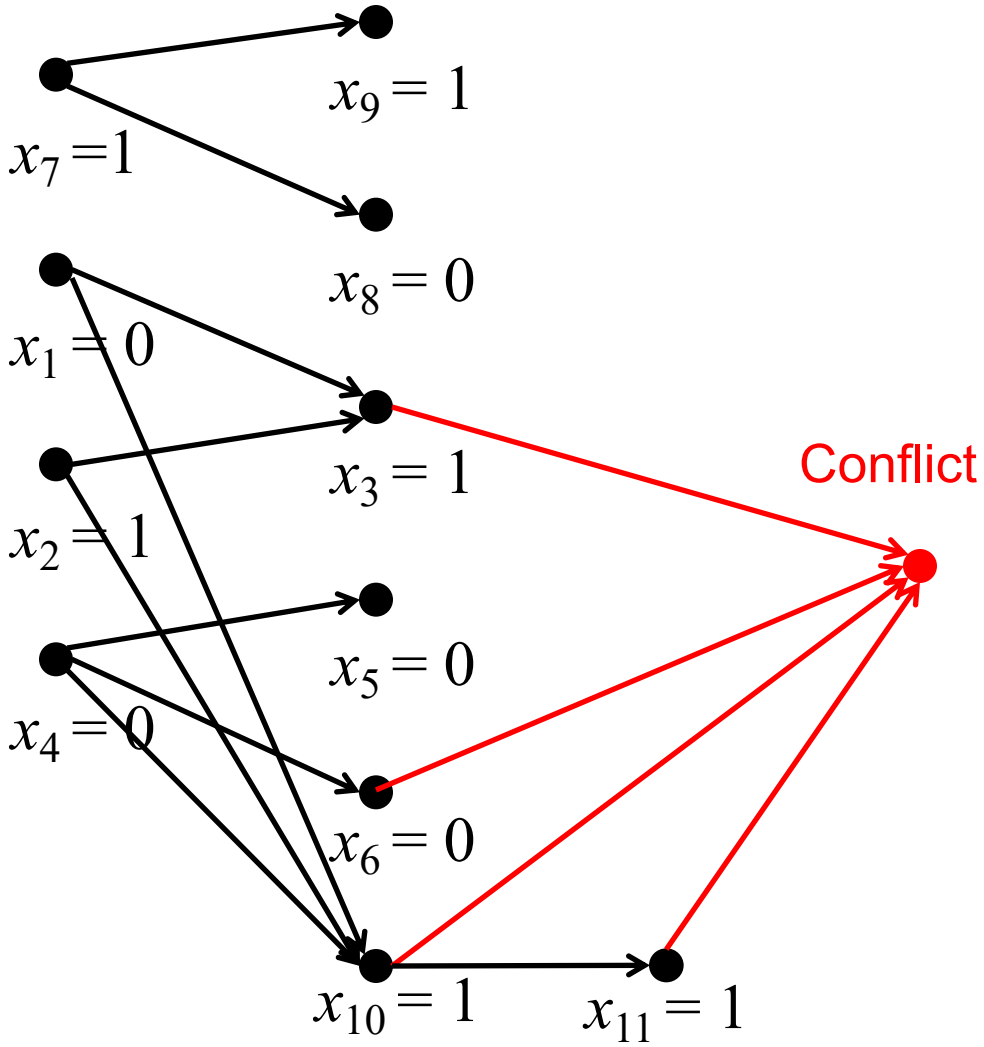
Intelligent Backtracking: More Unit Clause Propagation, New Conflicting Clause

- ✓ $(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$
- ✓ $(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$
- ✗ $(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$
- ✓ $(x_1 \vee \neg x_2 \vee x_3) \wedge$
- ✓ $(\neg x_7 \vee x_9) \wedge$
- ✓ $(\neg x_7 \vee \neg x_8) \wedge$
- ✓ $(x_4 \vee \neg x_5) \wedge$
- ✓ $(x_4 \vee \neg x_6) \wedge$
- ✓ $(x_{10} \vee x_{11}) \wedge$
- ✓ $(\neg x_{10} \vee x_{11}) \wedge$
- ✓ $(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$

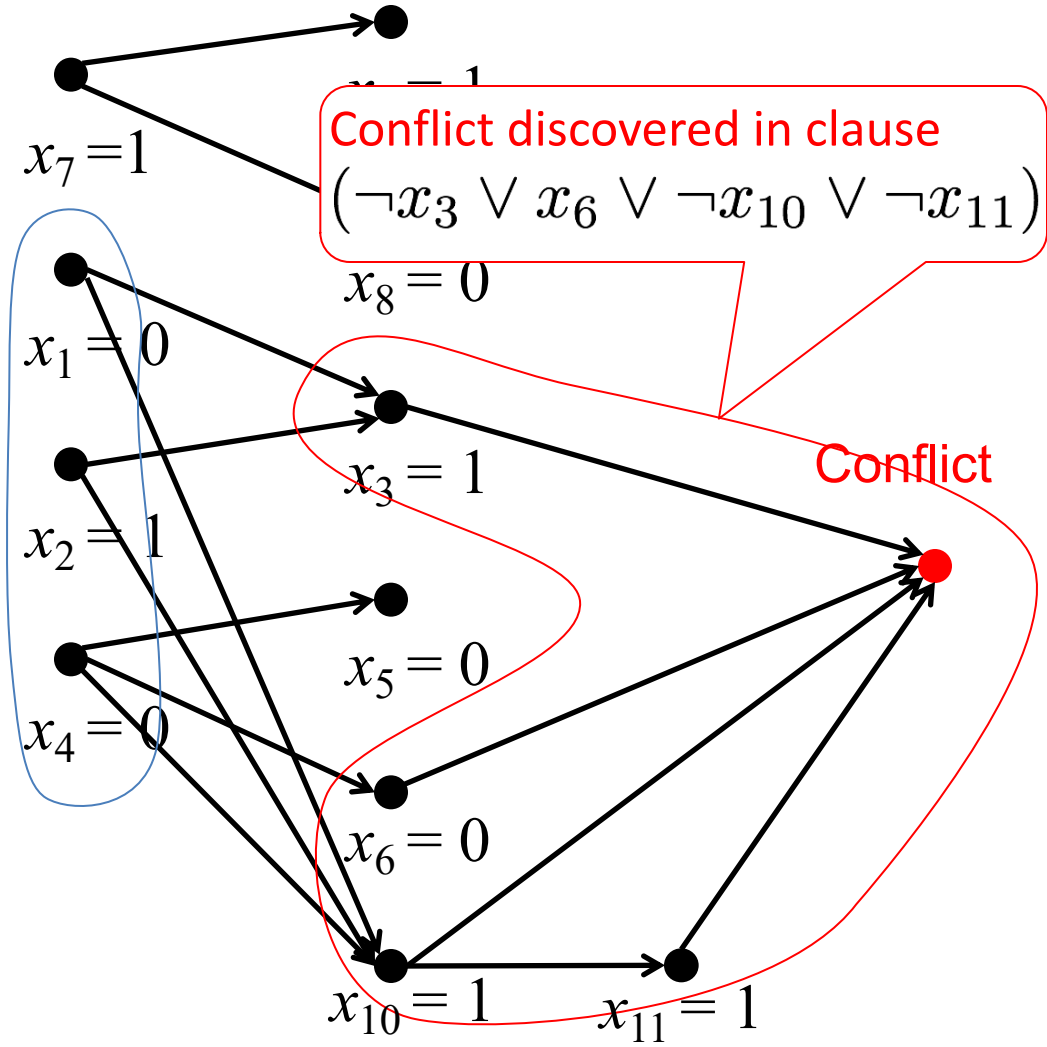


Intelligent Backtracking: Update Implication Graph

- ✓ $(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$
- ✓ $(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$
- ✗ $(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$
- ✓ $(x_1 \vee \neg x_2 \vee x_3) \wedge$
- ✓ $(\neg x_7 \vee x_9) \wedge$
- ✓ $(\neg x_7 \vee \neg x_8) \wedge$
- ✓ $(x_4 \vee \neg x_5) \wedge$
- ✓ $(x_4 \vee \neg x_6) \wedge$
- ✓ $(x_{10} \vee x_{11}) \wedge$
- ✓ $(\neg x_{10} \vee x_{11}) \wedge$
- ✓ $(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$



Intelligent Backtracking: Learn No-Good



.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4)$$

to the formula to prevent this conflict in the future

Intelligent Backtracking: Identify Deepest Decision in Learned Clause

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
	$\mathbf{x_{10} = 1}$
	$\mathbf{x_{11} = 1}$

$$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$$
$$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$$
$$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_3) \wedge$$
$$(\neg x_7 \vee x_9) \wedge$$
$$(\neg x_7 \vee \neg x_8) \wedge$$
$$(x_4 \vee \neg x_5) \wedge$$
$$(x_4 \vee \neg x_6) \wedge$$
$$(x_{10} \vee x_{11}) \wedge$$
$$(\neg x_{10} \vee x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4)$$

Intelligent Backtracking: Backtrack and Propagate New Clause

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
3	$x_4 = 0$
	$x_5 = 0$
	$x_6 = 0$
4	$x_7 = 1$
	$x_8 = 0$
	$x_9 = 1$
	$x_{10} = 1$

DL	Assgn
1	$x_1 = 0$
2	$x_2 = 1$
	$x_3 = 1$
	$\boldsymbol{x_4 = 1}$

$$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$$
$$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$$
$$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_3) \wedge$$
$$(\neg x_7 \vee x_9) \wedge$$
$$(\neg x_7 \vee \neg x_8) \wedge$$
$$(x_4 \vee \neg x_5) \wedge$$
$$(x_4 \vee \neg x_6) \wedge$$
$$(x_{10} \vee x_{11}) \wedge$$
$$(\neg x_{10} \vee x_{11}) \wedge$$
$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10}) \wedge$$
$$\boxed{(x_1 \vee \neg x_2 \vee x_4)}$$

Summary

- Search
  - Assign variable, Propagate
  - Detect conflict? Intelligent backtracking
- Intelligent backtracking
  - Identify decision variables source of conflict
  - Add no-good clause so conflict cannot arise in the future
  - Backtrack the deepest variables in the learnt clause
  - Propagate new learnt clause by flipping assignment of deepest variable in learnt clause
  - Proceed
- Do you see any problem with this strategy?

Problem with Adding New Clauses

- At every conflict, we learn a new clause
- Even when backtrack, we keep all new clauses
- Number of learned clauses quickly grows
- Several strategies exist
  - Every so often, remove inactive/un-used learned clauses
  - Remove subsumed clauses
    - Assume we learn $(x_1 \vee x_2 \vee x_3)$
    - Then we learn $(x_1 \vee x_2)$
    - We can remove the first learnt clause because it is subsumed by the smaller one
  - Etc.