Clause Learning and Intelligent Backtracking in MiniSAT

CSCE 235H Introduction to Discrete Structures (Honors)
Spring 2019

URL: cse.unl.edu/~cse235h

All questions: Piazza

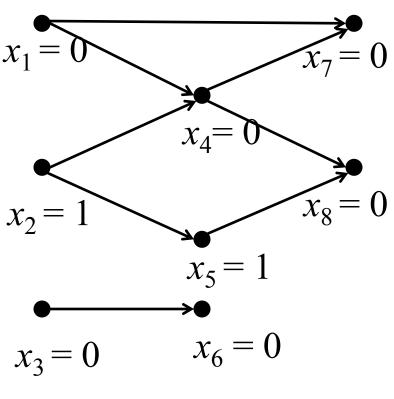
Clause Learning

- At every conflict, determine the cause of the conflict
- Create a new clause to prevent the conflict from being reached in the future
- Tools
 - Implication graph to determine cause of conflicts
 - Added clause is a "learnt" no-good

Implication Graph (1)

- Nodes correspond to assignments
- Nodes with no incoming edges are decision variables (assignments)
- Nodes with incoming edges were assigned through propagation

$$x_{1} \leftarrow 0 \qquad (x_{1} \vee \neg x_{2} \vee \neg x_{4}) \wedge \\ x_{2} \leftarrow 1 \qquad (x_{4} \vee \neg x_{5} \vee \neg x_{8}) \wedge \\ x_{3} \leftarrow 0 \qquad (x_{1} \vee x_{4} \vee \neg x_{7}) \wedge \\ (x_{3} \vee \neg x_{6}) \wedge \\ (\neg x_{2} \vee x_{5})$$

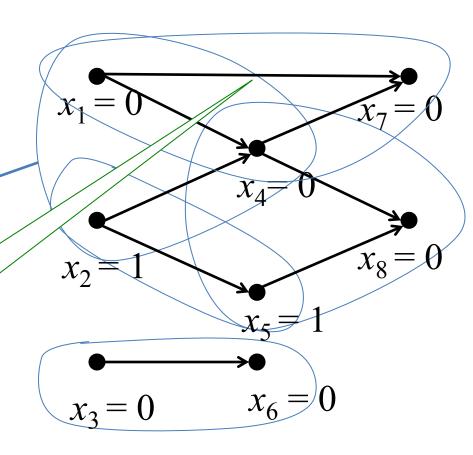


Implication Graph (2)

 A node and its immediate predecessors correspond to the propagating clause

$$(x_1 \vee \neg x_2 \vee \neg x_4)$$

Note: transitive links are required to show which clause caused propagation



Trail

- Series of assignments made up to current point in search
- Broken up by 'decision levels'
- Each decision level includes propagations

Decision level	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_4 = 0$
	$x_5 = 1$
	$x_7 = 0$
	$x_8 = 0$
3	$x_3 = 0$
	$x_6 = 0$

Clause Learning Example (1)

DL	Assignment	$(x_5 \lor x_6 \lor x_{10} \lor \neg x_{12}) \land$
1	$x_1 = 0$	$(\neg x_3 \lor x_5 \lor \neg x_{11} \lor x_{12}) \land$
2	$x_2 = 1$	$(\neg x_3 \lor x_6 \lor \neg x_{10} \lor \neg x_{11}) \land$
	$x_3 = 1$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
3	$x_4 = 0$	$(\neg x_7 \lor x_9) \land$
	$x_5 = 0$	$(\neg x_7 \vee \neg x_8) \wedge$
	$x_6 = 0$	$(x_4 \vee \neg x_5) \wedge$
4	$x_7 = 1$	$(x_4 \vee \neg x_6) \wedge$
	$x_8 = 0$	$(x_{10} \vee x_{11}) \wedge$
	$x_9 = 1$	$- (\neg x_{10} \lor x_{11})$
5	$x_{10} = 0$	(22 10
	$x_{11} = 1$	
235H	$x_{12} = 0$	Clause Learning and Intelligent

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Clause Learning Example (2)

D]	L	Assignment	$x_9 = 1$
1		$x_1 = 0$	$x_7 = 1$
2		$x_2 = 1$	
		$x_3 = 1$	$x_8 = 0$
3		$x_4 = 0$	x_1
		$x_5 = 0$	\bullet $x_3 = 1$
		$x_6 = 0$	$x_2 = 1$
4	,	$x_7 = 1$	$\chi_5 = Q$
		$x_8 = 0$	$x_4 = 0$
		$x_9 = 1$	$x_c = 0$
5		$x_{10} = 0$	$x_{10} = 0$
		$x_{11} = 1$	**10
CE 235H		$x_{12} = 0$	Clause Learning and Intelligent $x_{11} = 1$ Backtracking

Clause Learning Example (3)

Decision variables

Conflict caused by assignment

$$(\neg x_1 \wedge x_2 \wedge \neg x_4 \wedge \neg x_{10})$$

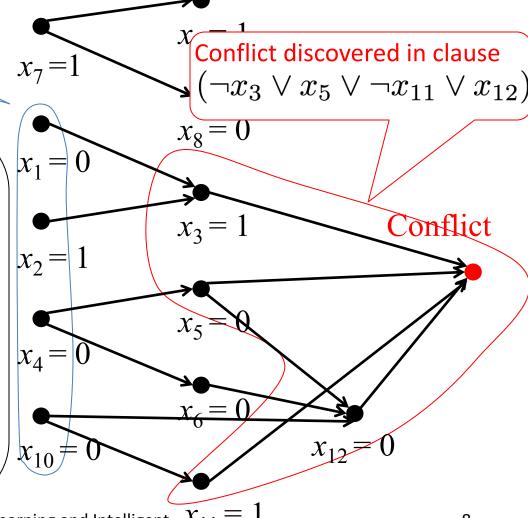
So, we negate it

$$\neg(\neg x_1 \land x_2 \land \neg x_4 \land \neg x_{10})$$

.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$$

to the formula to prevent this conflict in the future



Clause Learning and Intelligent $x_{11} = 1$ Backtracking

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Intelligent Backtracking

- When reaching a conflict, we
 - Consider conflicted clauses
 - Draw the implication graph
 - Identify the decision variables
 - Generate the learnt no-good
 - Add learned clause to the formula
- Undo assignments until the learned clause becomes a unit clause

Intelligent Backtracking Example (1)

	ı					
	DL	Assignment	$(x_5 \lor x_6 \lor x_{10} \lor \neg x_{12}) \land$			
	1	$x_1 = 0$	$(\neg x_3 \lor x_5 \lor \neg x_{11} \lor x_{12}) \land$			
	2	$x_2 = 1$	$(\neg x_3 \lor x_6 \lor \neg x_{10} \lor \neg x_{11}) \land$			
		$x_3 = 1$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$			
	3	$x_4 = 0$	$(\neg x_7 \lor x_9) \land$			
		$x_5 = 0$	$(\neg x_7 \vee \neg x_8) \wedge$			
		$x_6 = 0$	$(x_4 \vee \neg x_5) \wedge \bigcirc$			
	4	$x_7 = 1$	$(x_4 \lor \neg x_6) \land \qquad x_{10} \operatorname{deg}$			
		$x_8 = 0$	$(x_1 \lor x_0) \land (x_{10} \lor x_{11}) \land $ decision			
		$x_9 = 1$	$ (\neg x_{10} \lor x_{11}) \land$			
	5	$x_{10} = 0$	$(x_1 \lor \neg x_2 \lor x_4 \lor x_{10})$			
		$x_{11} = 1$	$(\omega_1 \vee \omega_2 \vee \omega_4 \vee \omega_{10})$			
CSCE 2	235H	$x_{12} = 0$	Clause Learning and Intelligent			

 x_{10} deepest cision variable

Intelligent Backtracking Example (2)

	DL	Assgn	DL	Accon	→
_		Assgii	<u> </u>	Assgn	$x_9 = 1$
_	1	$x_1 = 0$	1	$x_1 = 0$	$x_7=1$
	2	$x_2 = 1$	2	$x_2 = 1$	x = 0
_		$x_3 = 1$		$x_3 = 1$	$x_1 = 0$
	3	$x_4 = 0$	3	$x_4 = 0$	
		$x_5 = 0$		$x_5 = 0$	$x_3 = 1$
_		$x_6 = 0$		$x_6 = 0$	$x_2 = 1$
	4	$x_7 = 1$	4	$x_7 = 1$	$x_5 = 0$
		$x_8 = 0$		$x_8 = 0$	$x_4 = 0$
_		$x_9 = 1$		$x_9 = 1$	$x_6 = 0$
	5	$x_{10} = 0$		$x_{10} = 1$	
		$x_{11} = 1$			$x_{10} = 1$
	CSCE	235H = 0		Clau	se Learning and Intelligent Backtracking

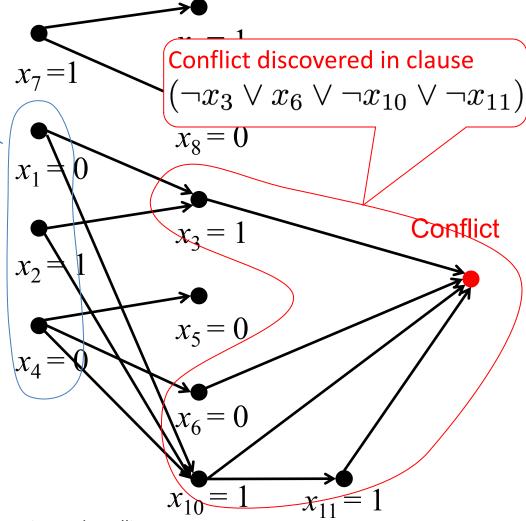
Intelligent Backtracking Example (3)

Conflict caused by these decision variables

.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4)$$

to the formula to prevent this conflict in the future



Intelligent Backtracking Example (4)

DL	Assgn	$(x_5 \lor x_6 \lor x_{10} \lor \neg x_{12}) \land $	
1	$x_1 = 0$	$(\neg x_3 \lor x_5 \lor \neg x_{11} \lor x_{12}) \land $	
2	$x_2 = 1$	$(\neg x_3 \lor x_6 \lor \neg x_{10} \lor \neg x_{11}) \land$	
	$x_3 = 1$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$	
3	$x_4 = 0$	$(\neg x_7 \lor x_9) \land$	
	$x_5 = 0$	$(\neg x_7 \lor \neg x_8) \land$	
	$x_6 = 0$	$(x_4 \vee \neg x_5) \wedge$	
4	$x_7 = 1$	$(x_4 \vee \neg x_6) \wedge$	
	$x_8 = 0$	$(x_{10} \vee x_{11}) \wedge$	
	$x_9 = 1$	$(\neg x_{10} \lor x_{11}) \land$	
	$x_{10} = 1$	$(x_1 \lor \neg x_2 \lor x_4 \lor x_{10}) \land$	
		$(x_1 \vee \neg x_2 \vee x_4)$	
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	I	Backtracking	

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Intelligent Backtracking Example (5)

DL	Assgn	DL	Assgn	$(x_5 \lor x_6 \lor x_{10} \lor \neg x_{12}) \land \\$
1	$x_1 = 0$	1	$x_1 = 0$	$(\neg x_3 \lor x_5 \lor \neg x_{11} \lor x_{12}) \land$
2	$x_2 = 1$	2	$x_2 = 1$	$(\neg x_3 \lor x_6 \lor \neg x_{10} \lor \neg x_{11}) \land$
	$x_3 = 1$		$x_3 = 1$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
3	$x_4 = 0$	3	$x_4 = 1$	$(\neg x_7 \lor x_9) \land$
	$x_5 = 0$			$(\neg x_7 \lor \neg x_8) \land$
	$x_6 = 0$			$(x_4 \vee \neg x_5) \wedge$
4	$x_7 = 1$			$(x_4 \vee \neg x_6) \wedge$
	$x_8 = 0$			$(x_{10} \vee x_{11}) \wedge$
	$x_9 = 1$			$(\neg x_{10} \lor x_{11}) \land$
	$x_{10} = 1$			$(x_1 \lor \neg x_2 \lor x_4 \lor x_{10}) \land$
				$(x_1 \vee \neg x_2 \vee x_4)$
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Summary

- Search
 - Assign variable, Propagate
 - Detect conflict? Intelligent backtracking
- Intelligent backtracking
 - Identify decision variables source of conflict
 - Add no-good clause so conflict cannot arise in the future
 - Backtrack the deepest variables in the learnt clause
 - Flip assignment of deepest variable in learnt clause
 - Proceed
- Do you see any problem in this strategy?