

Two Sudoku Rules in Conjunctive Normal Form

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February 22, 2018

This document presents the model of two rules of the Sudoku puzzle in CNF, first in a compact, semi-expanded, and fully expanded manner.

Rule: Every number is in every row

$$\bigwedge_{i=1}^9 \bigwedge_{n=1}^9 \bigvee_{j=1}^9 p(i, j, n) = \begin{array}{l} (p(1, 1, 1) \vee p(1, 2, 1) \vee \dots \vee p(1, 9, 1)) \quad \wedge \\ (p(1, 1, 2) \vee p(1, 2, 2) \vee \dots \vee p(1, 9, 2)) \quad \wedge \\ \dots \quad \wedge \\ (p(i, 1, n) \vee p(i, 2, n) \vee \dots \vee p(i, 9, n)) \quad \wedge \\ \dots \quad \wedge \\ (p(9, 1, 9) \vee p(9, 2, 9) \vee \dots \vee p(9, 9, 9)) \quad \wedge \end{array}$$

$$\begin{aligned}
& \wedge(p(9, 1, 1) \vee p(9, 2, 1) \vee p(9, 3, 1) \vee p(9, 4, 1) \vee p(9, 5, 1) \vee p(9, 6, 1) \vee p(9, 7, 1) \vee p(9, 8, 1) \vee p(9, 9, 1)) \\
& \wedge(p(9, 1, 2) \vee p(9, 2, 2) \vee p(9, 3, 2) \vee p(9, 4, 2) \vee p(9, 5, 2) \vee p(9, 6, 2) \vee p(9, 7, 2) \vee p(9, 8, 2) \vee p(9, 9, 2)) \\
& \wedge(p(9, 1, 3) \vee p(9, 2, 3) \vee p(9, 3, 3) \vee p(9, 4, 3) \vee p(9, 5, 3) \vee p(9, 6, 3) \vee p(9, 7, 3) \vee p(9, 8, 3) \vee p(9, 9, 3)) \\
& \wedge(p(9, 1, 4) \vee p(9, 2, 4) \vee p(9, 3, 4) \vee p(9, 4, 4) \vee p(9, 5, 4) \vee p(9, 6, 4) \vee p(9, 7, 4) \vee p(9, 8, 4) \vee p(9, 9, 4)) \\
& \wedge(p(9, 1, 5) \vee p(9, 2, 5) \vee p(9, 3, 5) \vee p(9, 4, 5) \vee p(9, 5, 5) \vee p(9, 6, 5) \vee p(9, 7, 5) \vee p(9, 8, 5) \vee p(9, 9, 5)) \\
& \wedge(p(9, 1, 6) \vee p(9, 2, 6) \vee p(9, 3, 6) \vee p(9, 4, 6) \vee p(9, 5, 6) \vee p(9, 6, 6) \vee p(9, 7, 6) \vee p(9, 8, 6) \vee p(9, 9, 6)) \\
& \wedge(p(9, 1, 7) \vee p(9, 2, 7) \vee p(9, 3, 7) \vee p(9, 4, 7) \vee p(9, 5, 7) \vee p(9, 6, 7) \vee p(9, 7, 7) \vee p(9, 8, 7) \vee p(9, 9, 7)) \\
& \wedge(p(9, 1, 8) \vee p(9, 2, 8) \vee p(9, 3, 8) \vee p(9, 4, 8) \vee p(9, 5, 8) \vee p(9, 6, 8) \vee p(9, 7, 8) \vee p(9, 8, 8) \vee p(9, 9, 8)) \\
& \wedge(p(9, 1, 9) \vee p(9, 2, 9) \vee p(9, 3, 9) \vee p(9, 4, 9) \vee p(9, 5, 9) \vee p(9, 6, 9) \vee p(9, 7, 9) \vee p(9, 8, 9) \vee p(9, 9, 9))
\end{aligned}$$

Rule: Every cell must take a number in $\{1, 2, \dots, 9\}$

$$\begin{aligned}
 \bigwedge_{i=1}^9 \bigwedge_{j=1}^9 \bigvee_{n=1}^9 p(i, j, n) = & (p(1, 1, 1) \vee p(1, 1, 2) \vee \dots \vee p(1, 1, 9)) \wedge \\
 & (p(1, 2, 1) \vee p(1, 2, 2) \vee \dots \vee p(1, 2, 9)) \wedge \\
 & \dots \wedge \\
 & (p(i, j, 1) \vee p(i, j, 2) \vee \dots \vee p(i, j, 9)) \wedge \\
 & \dots \wedge \\
 & (p(9, 9, 1) \vee p(9, 9, 2) \vee \dots \vee p(9, 9, 9)) \wedge
 \end{aligned}$$

$\wedge(p(9, 1, 1) \vee p(9, 1, 2) \vee p(9, 1, 3) \vee p(9, 1, 4) \vee p(9, 1, 5) \vee p(9, 1, 6) \vee p(9, 1, 7) \vee p(9, 1, 8) \vee p(9, 1, 9))$
 $\wedge(p(9, 2, 1) \vee p(9, 2, 2) \vee p(9, 2, 3) \vee p(9, 2, 4) \vee p(9, 2, 5) \vee p(9, 2, 6) \vee p(9, 2, 7) \vee p(9, 2, 8) \vee p(9, 2, 9))$
 $\wedge(p(9, 3, 1) \vee p(9, 3, 2) \vee p(9, 3, 3) \vee p(9, 3, 4) \vee p(9, 3, 5) \vee p(9, 3, 6) \vee p(9, 3, 7) \vee p(9, 3, 8) \vee p(9, 3, 9))$
 $\wedge(p(9, 4, 1) \vee p(9, 4, 2) \vee p(9, 4, 3) \vee p(9, 4, 4) \vee p(9, 4, 5) \vee p(9, 4, 6) \vee p(9, 4, 7) \vee p(9, 4, 8) \vee p(9, 4, 9))$
 $\wedge(p(9, 5, 1) \vee p(9, 5, 2) \vee p(9, 5, 3) \vee p(9, 5, 4) \vee p(9, 5, 5) \vee p(9, 5, 6) \vee p(9, 5, 7) \vee p(9, 5, 8) \vee p(9, 5, 9))$
 $\wedge(p(9, 6, 1) \vee p(9, 6, 2) \vee p(9, 6, 3) \vee p(9, 6, 4) \vee p(9, 6, 5) \vee p(9, 6, 6) \vee p(9, 6, 7) \vee p(9, 6, 8) \vee p(9, 6, 9))$
 $\wedge(p(9, 7, 1) \vee p(9, 7, 2) \vee p(9, 7, 3) \vee p(9, 7, 4) \vee p(9, 7, 5) \vee p(9, 7, 6) \vee p(9, 7, 7) \vee p(9, 7, 8) \vee p(9, 7, 9))$
 $\wedge(p(9, 8, 1) \vee p(9, 8, 2) \vee p(9, 8, 3) \vee p(9, 8, 4) \vee p(9, 8, 5) \vee p(9, 8, 6) \vee p(9, 8, 7) \vee p(9, 8, 8) \vee p(9, 8, 9))$
 $\wedge(p(9, 9, 1) \vee p(9, 9, 2) \vee p(9, 9, 3) \vee p(9, 9, 4) \vee p(9, 9, 5) \vee p(9, 9, 6) \vee p(9, 9, 7) \vee p(9, 9, 8) \vee p(9, 9, 9))$