Recitation 4

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- 1.7:23) Show that at least 10 of any 64 days chosen must fall on the same day of the week.
  - We proceed with a proof by contradiction.
  - First, for the purposes of contradiction, we assume that it is not the case that "at least 10 days of any 64 days chosen fall on the same day of the week."
  - So "less than 10 days of any 64 days chosen fall on the same day of the week." Assume, it is 9 days. Then we have "9 days of any 64 days chosen fall on the same day of the week."
  - There are 7 days in a week. If we choose 9 days out of every day of the week, then we have chosen $9\times$ 7days $= 63$ days, exactly.
  - However, we need to choose 64 days. Therefore, there is one extra day that needs to be chosen. Whichever day we choose, this choice will bump up the 'count' of chosen days to 10, which contradicts the statement "9 days of any 64 days chosen fall on the same day of the week."
  - Therefore, at least 10 of any 64 days chosen must fall on the same day of the week.
- Now a similar example: let's look at problem 1.7:27. Show that, given a positive integer n, then n is odd if and only if (iff) $5n + 6$ is odd.
  - First we prove the $\to$ direction.
    1. Let n be a positive integer that is odd. So n can be written $2k + 1$ for some integer $k \geq 0$. So $5n + 6 = 5 \times (2k + 1) + 6$.
    2. This is $10k + 5 + 6 = 10k + 11$
    3. This can again be rewritten $2(5k + 5) + 1$
    4. We can then substitute b for $5k + 5$, since we know $5k + 5$ is still a positive integer.
    5. So $5n + 6 = 2b + 1$...clearly $2b + 1$ has the form of an odd number.
    6. So $5n + 6$ is odd $\square$

- Now let's prove the $\leftarrow$ direction.
  1. For the purposes of a contrapositive, let's assume that if n is even, then $5n+6$ is even.
  2. So, let $n = 2k$ for some positive integer k *by definition* (note this is $\neg p$.
  3. Then $(5 \times 2k) + 6 = 10k + 6$.
  4. But we can see that this is even, and can be rewritten $2(5k + 3) = 2(b)$ for some positive integer b.
  5. So $5n + 6 = 2b$ $(\neg q)$.
  6. So if n is even $5n + 6$ is even.
  7. By contraposition, if $5n + 6$ is odd, n is odd. $\square$

- 1.6:15a) Given

  1. All students in this class understand logic.
  2. Xavier is a student in this class.

  Prove that, Xavier understands logic.

  - Define your predicates:
    * $Q(x)$: x is in the class
    * $P(x)$: x understands logic
  - Universe of Discourse: All students
  - Theory:
    1. $\forall x P(x) \rightarrow Q(x)$
    2. $P(Xavier)$
  - We want to prove : $Q(Xavier)$
  - Proof:

	Step	**Reason**
1.	$\forall x P(x) \rightarrow Q(x)$	Premise
2.	$P(Xavier)$	Premise
3.	$P(Xavier) \rightarrow Q(Xavier)$	Universal instantiation from (1)
4.	$Q(Xavier)$	Modus ponens from (2) and (3)

- Now problem 1.6:19 Determine whether the following argument is valid. If so, what rule of inference is being used, if not what logical error occurs:
  b) If n is a real number with $n \geq 3$, then $n^2 \geq 9$. Suppose $n^2 \leq 9$ then $n \leq 3$.

  1. Is this true? Yes
  2. Is this a valid argument? Yes, using the rule of inference *modus tollence*.

  c) If n is a real number with $n \geq 2$, then $n^2 \geq 4$. Suppose $n \leq 2$ then $n^2 \leq 4$.

1. Is this a valid argument? No this is denying the hypothesis

- What rules of inference are used in Lewis Carroll's poem from example 26 in 1.4.
  - 1. All lions are fierce.
    2. Some lions do not drink coffee
    3. Some fierce creatures do not drink coffee.
  - By the second premise, we have that there is a lion that does not drink coffee, let that lion be *Leo*.
  - Using *simplification* we can tell that Leo is a lion.
  - Now using *modus ponens* on the first premise we know that Leo is fierce.
  - So Leo is fierce and does not drink coffee
  - Now using existential generalization, we can say that there exists a creature that is fierce, and does not drink coffee.