

# Watched Literals and Restarts in MiniSAT

CSCE 235H Introduction to Discrete Structures (Honors)

Spring 2020

URL: [cse.unl.edu/~cse235h](http://cse.unl.edu/~cse235h)

All questions: Piazza

# Reminder: Unit Propagation

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- Assignments may result in unit clauses  
 $a \leftarrow false$  and  $a \vee b$  yield  $b$
- Unit clauses immediately force an assignment  
 $b \leftarrow true$
- This can lead to a chain reaction as new assignments ‘propagate’ throughout the clauses

# When Does Unit Propagation Fire Up?

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- Consider the clause

$$(x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4)$$

- If no variable is assigned, no unit propagation occurs
- If all but two variables are assigned, unit propagation becomes possible

$$(0 \vee \neg x_2 \vee \neg 1 \vee x_4)$$

- Watched-literal mechanism
  - Watches 2 unassigned literals in every clause in ‘preparation’ of unit propagation

# Watched Literals

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- Technique for efficiently implementing unit propagation
- Only two literals per clause must be *watched* to determine when a clause becomes unit

$$(x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4)$$

- Each literal keeps a *watcher list* containing the clauses it is currently watched by.

# Initializing Watched Literals (1)

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- For every clause, select two literals to be watched.

$$c_1 = (x_1 \vee \underline{\neg x_2} \vee \neg x_3 \vee \underline{x_4})$$

$$c_2 = (\neg x_1 \vee \underline{x_3} \vee \underline{\neg x_4})$$

$$c_3 = (\underline{x_2} \vee \underline{x_4})$$

# Initializing Watched Literals (2)

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- Every time a literal becomes watched, add the watching clause to the literal's watcher list

$$\begin{aligned}c_1 &= (x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4) \\c_2 &= (\neg x_1 \vee x_3 \vee \neg x_4) \\c_3 &= (x_2 \vee x_4)\end{aligned}$$

$$\begin{aligned}w(x_1) &= \{\} & w(\neg x_1) &= \{\} \\w(x_2) &= \{c_3\} & w(\neg x_2) &= \{c_1\} \\w(x_3) &= \{c_2\} & w(\neg x_3) &= \{\} \\w(x_4) &= \{c_1, c_3\} & w(\neg x_4) &= \{c_2\}\end{aligned}$$

# Watched Literal Assignment (1)

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- Assign

$$x_4 \leftarrow 0$$

- Check watcher list of newly false literal

$$w(x_4) = \{c_1, c_3\}$$

# Watched Literal Assignment (2)

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- If clause has become unit, propagate

$$c_3 = (x_2 \vee \mathbf{0}_4)$$

$$x_2 \leftarrow 1$$

- If not, select a new literal to watch

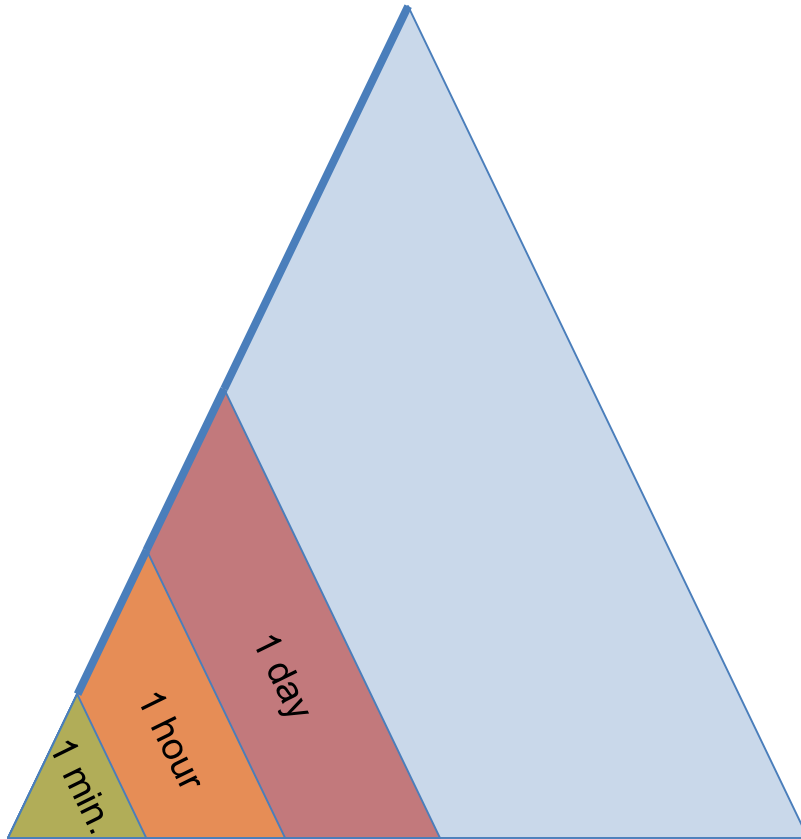
$$c_1 = (x_1 \vee \underline{\neg x_2} \vee \underline{\neg x_3} \vee \mathbf{0}_4)$$

$$w(\neg x_3) = \{c_1\}$$



# Danger of Search (1)

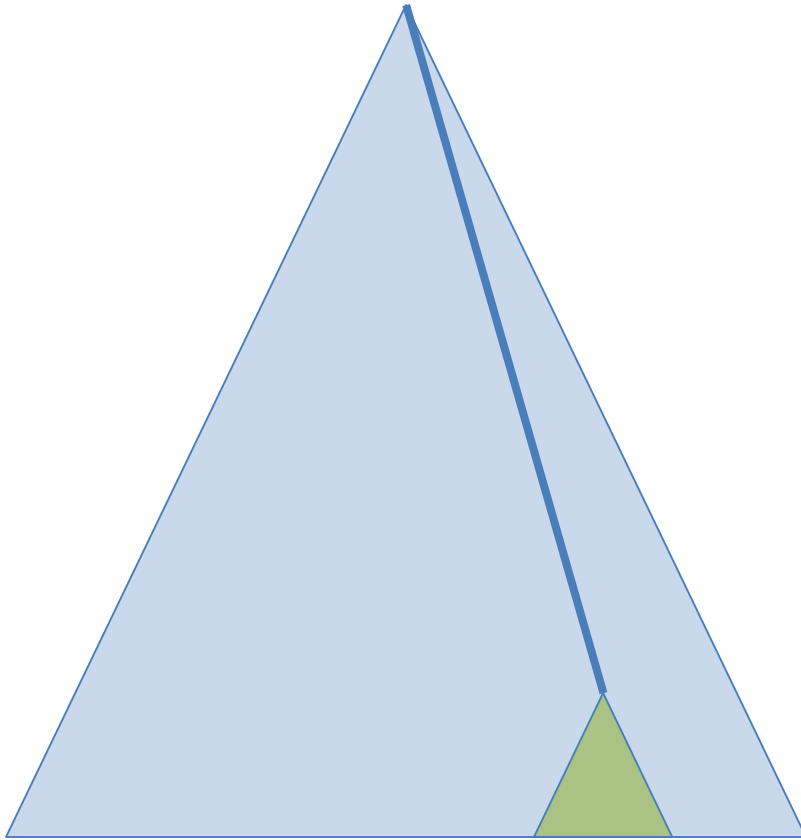
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- If search space is very, very large
- We could be
  - searching and backtracking in one corner
  - and ignoring more promising part of the tree

# Danger of Search (2)

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- Solution
  - Occasionally, drop the search
  - Restart from somewhere else
  - Restart works well in practice

# Restarts

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- After searching for a specified amount of time (usually given by # conflicts) restart the search
- Undo all assignments
- Preserve:
  - learned clauses
  - variable activity values
- After performing a restart, the time until the next restart may change

# Restart Sequences

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- Geometric [Walsh '99]
  - 1, 2, 4, 8, 16, 32, 64, 128, ...
  - Each successive search is given more time than the previous
- Luby universal strategy [Luby+, '93]
  - 1, 1, 2, 1, 1, 2, 4, 1, 1, 2, 1, 1, 2, 4, 8, ...
  - Optimal restart sequence if runtime distribution is unknown