Watched Literals and Restarts in MiniSAT
Reminder: Unit Propagation

- Assignments may result in unit clauses
  \[ a \leftarrow false \text{ and } a \lor b \text{ yield } b \]
- Unit clauses immediately force an assignment
  \[ b \leftarrow true \]
- This can lead to a chain reaction as new assignments ‘propagate’ throughout the clauses
When Does Unit Propagation Fire Up?

• Consider the clause

\[
(x_1 \lor \neg x_2 \lor \neg x_3 \lor x_4)
\]

  – If no variable is assigned, no unit propagation occurs
  – If all but two variables are assigned, unit propagation becomes possible

\[
(0 \lor \neg x_2 \lor \neg 1 \lor x_4)
\]

• Watched-literal mechanism

  – Watches 2 unassigned literals in every clause in ‘preparation’ of unit propagation
Watched Literals

• Technique for efficiently implementing unit propagation

• Only two literals per clause must be watched to determine when a clause becomes unit

\[(x_1 \lor \neg x_2 \lor \neg x_3 \lor x_4)\]

• Each literal keeps a watcher list containing the clauses it is currently watched by.
Initializing Watched Literals (1)

• For every clause, select two literals to be watched.

\[ c_1 = (x_1 \lor \neg x_2 \lor \neg x_3 \lor x_4) \]
\[ c_2 = (\neg x_1 \lor x_3 \lor \neg x_4) \]
\[ c_3 = (x_2 \lor x_4) \]
Initializing Watched Literals (2)

- Every time a literal becomes watched, add the watching clause to the literal’s watcher list

\[
\begin{align*}
  c_1 &= (x_1 \lor \neg x_2 \lor \neg x_3 \lor x_4) \\
  c_2 &= (\neg x_1 \lor x_3 \lor \neg x_4) \\
  c_3 &= (x_2 \lor x_4)
\end{align*}
\]

\[
\begin{align*}
  w(x_1) &= \{\} & w(\neg x_1) &= \{\} \\
  w(x_2) &= \{c_3\} & w(\neg x_2) &= \{c_1\} \\
  w(x_3) &= \{c_2\} & w(\neg x_3) &= \{\} \\
  w(x_4) &= \{c_1, c_3\} & w(\neg x_4) &= \{c_2\}
\end{align*}
\]
Watched Literal Assignment (1)

• Assign

\[ x_4 \leftarrow 0 \]

• Check watcher list of newly false literal

\[ w(x_4) = \{ c_1, c_3 \} \]
Watched Literal Assignment (2)

• If clause has become unit, propagate

\[ c_3 = (x_2 \lor \bar{0}_4) \]

\[ x_2 \leftarrow 1 \]

• If not, select a new literal to watch

\[ c_1 = (x_1 \lor \bar{x}_2 \lor \bar{x}_3 \lor \bar{0}_4) \]

\[ w(\neg x_3) = \{ c_1 \} \]
Danger of Search (1)

- If search space is very, very large
- We could be
  - searching and backtracking in one corner
  - and ignoring more promising part of the tree
Danger of Search (2)

- Solution
  - Occasionally, drop the search
  - Restart from somewhere else
  - Restart works well in practice
Restarts

• After searching for a specified amount of time (usually given by # conflicts) restart the search
• Undo all assignments
• Preserve:
  – learned clauses
  – variable activity values
• After performing a restart, the time until the next restart may change
Restart Sequences

• Geometric \[\text{[Walsh \textquotesingle} 99\text{]}\]
  – 1, 2, 4, 8, 16, 32, 64, 128, ...
  – Each successive search is given more time than the previous

• Luby universal strategy \[\text{[Luby+}, \text{\textquotesingle} 93\text{]}\]
  – 1, 1, 2, 1, 1, 2, 4, 1, 1, 2, 1, 1, 2, 4, 8, ...
  – Optimal restart sequence if runtime distribution is unknown