# Clause Learning and Intelligent Backtracking in MiniSAT

CSCE 235H Introduction to Discrete Structures (Honors)
Spring 2019

URL: cse.unl.edu/~cse235h

All questions: Piazza

#### Clause Learning

- At every conflict, determine the cause of the conflict
- Create a new clause to prevent the conflict from being reached in the future
- Tools
  - Implication graph to determine cause of conflicts
  - Added clause is a "learned" no-good

#### Implication Graph (1)

- Nodes correspond to assignments
- Nodes with no incoming edges are decision variables (assignments)
- Nodes with incoming edges were assigned through propagation

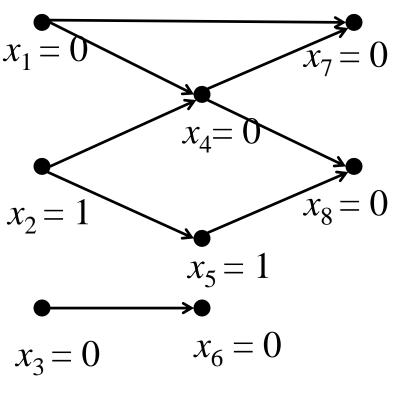
$$x_{1} \leftarrow 0 \qquad (x_{1} \vee \neg x_{2} \vee \neg x_{4}) \wedge$$

$$x_{2} \leftarrow 1 \qquad (x_{4} \vee \neg x_{5} \vee \neg x_{8}) \wedge$$

$$x_{3} \leftarrow 0 \qquad (x_{1} \vee x_{4} \vee \neg x_{7}) \wedge$$

$$(x_{3} \vee \neg x_{6}) \wedge$$

$$(\neg x_{2} \vee x_{5})$$



### Implication Graph (2)

 A node and its immediate predecessors correspond to the propagating clause

$$x_1 = 0$$

$$x_2 = 1$$

$$x_3 = 0$$

$$x_4 = 0$$

$$x_8 = 0$$

$$(x_1 \vee \neg x_2 \vee \neg x_4)$$

Note: transitive links are required to show which clause caused propagation

#### **Trail**

- Series of assignments made up to current point in search
- Broken up by 'decision levels'
- Each decision level includes propagations

Decision level	Assignment
1	$x_1 = 0$
2	$x_2 = 1$
	$x_4 = 0$
	$x_5 = 1$
	$x_7 = 0$
	$x_8 = 0$
3	$x_3 = 0$
	$x_6 = 0$

## Clause Learning Example (1)

DL	Assignment	$(x_5 \lor x_6 \lor x_{10} \lor \neg x_{12}) \land$
1	$x_1 = 0$	$(\neg x_3 \lor x_5 \lor \neg x_{11} \lor x_{12}) \land$
2	$x_2 = 1$	$(\neg x_3 \lor x_6 \lor \neg x_{10} \lor \neg x_{11}) \land$
	$x_3 = 1$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
3	$x_4 = 0$	$(\neg x_7 \lor x_9) \land$
	$x_5 = 0$	$(\neg x_7 \lor \neg x_8) \land$
	$x_6 = 0$	$(x_4 \vee \neg x_5) \wedge$
4	$x_7 = 1$	$(x_4 \vee \neg x_6) \wedge$
	$x_8 = 0$	$(x_{10} \vee x_{11}) \wedge$
	$x_9 = 1$	$- (\neg x_{10} \lor x_{11})$
5	$x_{10} = 0$	( 10 11)
	$x_{11} = 1$	
235H	$x_{12} = 0$	Clause Learning and Intelligent

Backtracking

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# Clause Learning Example (2)

DL	Assignment	$x_9 = 1$
1	$x_1 = 0$	$x_7 = 1$
2	$x_2 = 1$	<b>4</b>
	$x_3 = 1$	$x_8 = 0$
3	$x_4 = 0$	$x_1 - 0$
	$x_5 = 0$	$x_3 = 1$ Conflic
	$x_6 = 0$	$x_2 = 1$
4	$x_7 = 1$	$x_5 = 0$
	$x_8 = 0$	$x_4 = 0$
	$x_9 = 1$	$x_c = 0$
5	$x_{10} = 0$	$x_{10} = 0$ $x_{12} = 0$
	$x_{11} = 1$	
E 235H	$x_{12} = 0$	Clause Learning and Intelligent $x_{11} = 1$ 7 Backtracking

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### Clause Learning Example (3)

**Decision variables** 

Conflict caused by assignment

 $(\neg x_1 \land x_2 \land \neg x_4 \land \neg x_{10})$ 

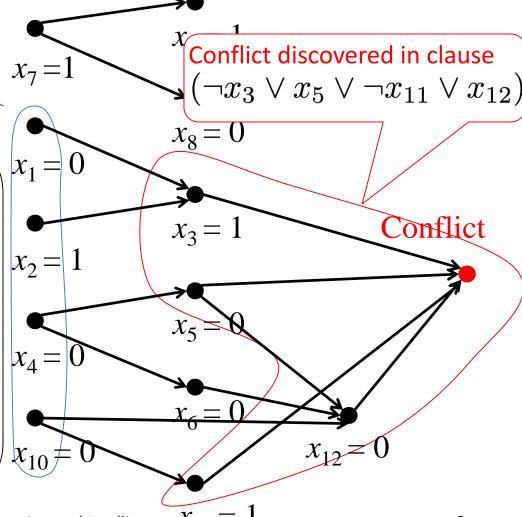
So, we negate it

$$\neg(\neg x_1 \land x_2 \land \neg x_4 \land \neg x_{10})$$

.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$$

to the formula to prevent this conflict in the future



Clause Learning and Intelligent  $x_{11} = 1$ Backtracking

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### Intelligent Backtracking

- When reaching a conflict, we
  - Consider conflicted clauses
  - Draw the implication graph
  - Identify the decision variables
  - Generate the learned no-good
  - Add learned clause to the formula
- Undo assignments until the learned clause becomes a unit clause

### Intelligent Backtracking Example (1)

**Backtracking** 

		_	
	$\mathbf{DL}$	Assignment	$(x_5 \lor x_6 \lor x_{10} \lor$
	1	$x_1 = 0$	$(\neg x_3 \lor x_5 \lor \neg x_{11}$
	2	$x_2 = 1$	$(\neg x_3 \lor x_6 \lor \neg x_{10})$
		$x_3 = 1$	$(x_1 \vee \neg x_2 \vee x_3) \wedge \\$
	3	$x_4 = 0$	$(\neg x_7 \lor x_9) \land$
		$x_5 = 0$	$(\neg x_7 \lor \neg x_8) \land$
		$x_6 = 0$	$(x_4 \vee \neg x_5) \wedge$
	4	$x_7 = 1$	$(x_4 \vee \neg x_6) \wedge$
		$x_8 = 0$	$(x_{10} \lor x_{11}) \land$
		$x_9 = 1$	$- (\neg x_{10} \lor x_{11}) \land$
	5	$x_{10} = 0$	$(x_1 \vee \neg x_2 \vee x_4)$
		$x_{11} = 1$	
CSCE 2	235H	$x_{12} = 0$	Clause Learning and Intelligent

$$(x_{5} \lor x_{6} \lor x_{10} \lor \neg x_{12}) \land \\ (\neg x_{3} \lor x_{5} \lor \neg x_{11} \lor x_{12}) \land \\ (\neg x_{3} \lor x_{6} \lor \neg x_{10} \lor \neg x_{11}) \land \\ (x_{1} \lor \neg x_{2} \lor x_{3}) \land \\ (\neg x_{7} \lor x_{9}) \land \\ (\neg x_{7} \lor \neg x_{8}) \land \\ (x_{4} \lor \neg x_{5}) \land \\ (x_{4} \lor \neg x_{6}) \land \\ (x_{10} \lor x_{11}) \land \\ (\neg x_{10} \lor x_{11}) \land \\ (x_{1} \lor \neg x_{2} \lor x_{4} \lor x_{10})$$

# Intelligent Backtracking Example (2)

_	DL	Assgn	DL	Assgn	$x_9 = 1$
	1	$x_1 = 0$	1	$x_1 = 0$	$x_7 = 1$
_	2	$x_2 = 1$	2	$x_2 = 1$	
_		$x_3 = 1$		$x_3 = 1$	$x_1 = 0$ $x_8 = 0$
	3	$x_4 = 0$	3	$x_4 = 0$	
		$x_5 = 0$		$x_5 = 0$	$x_3 = 1$
_		$x_6 = 0$		$x_6 = 0$	$x_2 = 1$
	4	$x_7 = 1$	4	$x_7 = 1$	$x_5 = 0$
		$x_8 = 0$		$x_8 = 0$	$x_4 = 0$
_		$x_9 = 1$		$x_9 = 1$	$x_6 = 0$
	5	$x_{10} = 0$		$x_{10} = 1$	
		$x_{11} = 1$			$x_{10} = 1$
	CSCE	235H = 0		Clau	se Learning and Intelligent Backtracking

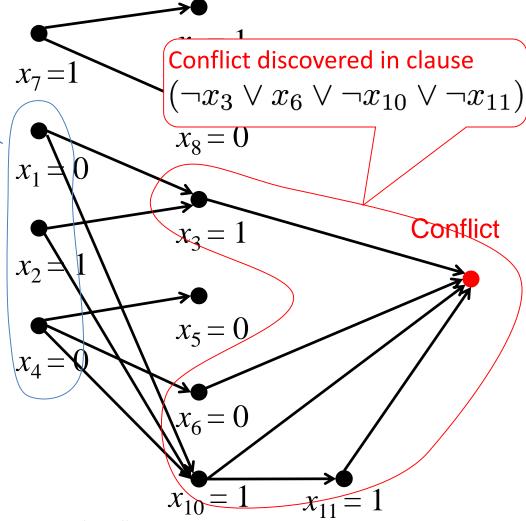
# Intelligent Backtracking Example (3)

Conflict caused by these decision variables

.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4)$$

to the formula to prevent this conflict in the future



# Intelligent Backtracking Example (4)

	DL	Assgn	$(x_5 \lor x_6 \lor x_{10} \lor \neg x_{12}) \land$
	1	$x_1 = 0$	$(\neg x_3 \lor x_5 \lor \neg x_{11} \lor x_{12}) \land$
	2	$x_2 = 1$	$(\neg x_3 \lor x_6 \lor \neg x_{10} \lor \neg x_{11}) \land$
		$x_3 = 1$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
	3	$x_4 = 0$	$(\neg x_7 \lor x_9) \land$
		$x_5 = 0$	$(\neg x_7 \lor \neg x_8) \land$
		$x_6 = 0$	$(x_4 \vee \neg x_5) \wedge$
	4	$x_7 = 1$	$(x_4 \vee \neg x_6) \wedge$
		$x_8 = 0$	$(x_{10} \lor x_{11}) \land$
		$x_9 = 1$	$(\neg x_{10} \lor x_{11}) \land$
		$x_{10} = 1$	$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10}) \wedge$
			$(x_1 \vee \neg x_2 \vee x_4)$
5H			Clause Learning and Intelligent
		•	Backtracking

### Intelligent Backtracking Example (5)

DL	Assgn	DL	Assgn	$(x_5 \lor x_6 \lor x_{10} \lor \neg x_{12}) \land \\$
1	$x_1 = 0$	1	$x_1 = 0$	$(\neg x_3 \lor x_5 \lor \neg x_{11} \lor x_{12}) \land$
2	$x_2 = 1$	2	$x_2 = 1$	$(\neg x_3 \lor x_6 \lor \neg x_{10} \lor \neg x_{11}) \land$
	$x_3 = 1$		$x_3 = 1$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
3	$x_4 = 0$	3	$x_4 = 1$	$(\neg x_7 \lor x_9) \land$
	$x_5 = 0$			$(\neg x_7 \lor \neg x_8) \land$
	$x_6 = 0$			$(x_4 \vee \neg x_5) \wedge$
4	$x_7 = 1$			$(x_4 \vee \neg x_6) \wedge$
	$x_8 = 0$			$(x_{10} \vee x_{11}) \wedge$
	$x_9 = 1$			$(\neg x_{10} \lor x_{11}) \land$
	$x_{10} = 1$			$(x_1 \lor \neg x_2 \lor x_4 \lor x_{10}) \land$
				$(x_1 \vee \neg x_2 \vee x_4)$
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#### Summary

- Search
  - Assign variable, Propagate
  - Detect conflict? Intelligent backtracking
- Intelligent backtracking
  - Identify decision variables source of conflict
  - Add no-good clause so conflict cannot arise in the future
  - Backtrack the deepest variables in the learned clause
  - Flip assignment of deepest variable in learnt clause
  - Proceed