

Watched Literals and Restarts in MiniSAT

CSCE 235H Introduction to Discrete Structures (Honors)

Spring 2019

URL: cse.unl.edu/~cse235h

All questions: Piazza

Reminder: Unit Propagation

- Assignments may result in unit clauses
 $a \leftarrow false$ and $a \vee b$ yield b
- Unit clauses immediately force an assignment
 $b \leftarrow true$
- This can lead to a chain reaction as new assignments ‘propagate’ throughout the clauses

When Does Unit Propagation Fire Up?

- Consider the clause

$$(x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4)$$

- If no variable is assigned, no unit propagation occurs
- If all but two variables are assigned, unit propagation becomes possible

$$(0 \vee \neg x_2 \vee \neg 1 \vee x_4)$$

- Watched-literal mechanism
 - Watches 2 unassigned literals in every clause in ‘preparation’ of unit propagation

Watched Literals

- Technique for efficiently implementing unit propagation
- Only two literals per clause must be *watched* to determine when a clause becomes unit

$$(x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4)$$

- Each literal keeps a *watcher list* containing the clauses it is currently watched by.

Initializing Watched Literals (1)

- For every clause, select two literals to be watched.

$$c_1 = (x_1 \vee \underline{\neg x_2} \vee \neg x_3 \vee \underline{x_4})$$

$$c_2 = (\neg x_1 \vee \underline{x_3} \vee \underline{\neg x_4})$$

$$c_3 = (\underline{x_2} \vee \underline{x_4})$$

Initializing Watched Literals (2)

- Every time a literal becomes watched, add the watching clause to the literal's watcher list

$$\begin{aligned}c_1 &= (x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4) \\c_2 &= (\neg x_1 \vee x_3 \vee \neg x_4) \\c_3 &= (x_2 \vee x_4)\end{aligned}$$

$$\begin{aligned}w(x_1) &= \{\} & w(\neg x_1) &= \{\} \\w(x_2) &= \{c_3\} & w(\neg x_2) &= \{c_1\} \\w(x_3) &= \{c_2\} & w(\neg x_3) &= \{\} \\w(x_4) &= \{c_1, c_3\} & w(\neg x_4) &= \{c_2\}\end{aligned}$$

Watched Literal Assignment (1)

- Assign

$$x_4 \leftarrow 0$$

- Check watcher list of newly false literal

$$w(x_4) = \{c_1, c_3\}$$

Watched Literal Assignment (2)

- If clause has become unit, propagate

$$c_3 = (x_2 \vee \mathbf{0}_4)$$

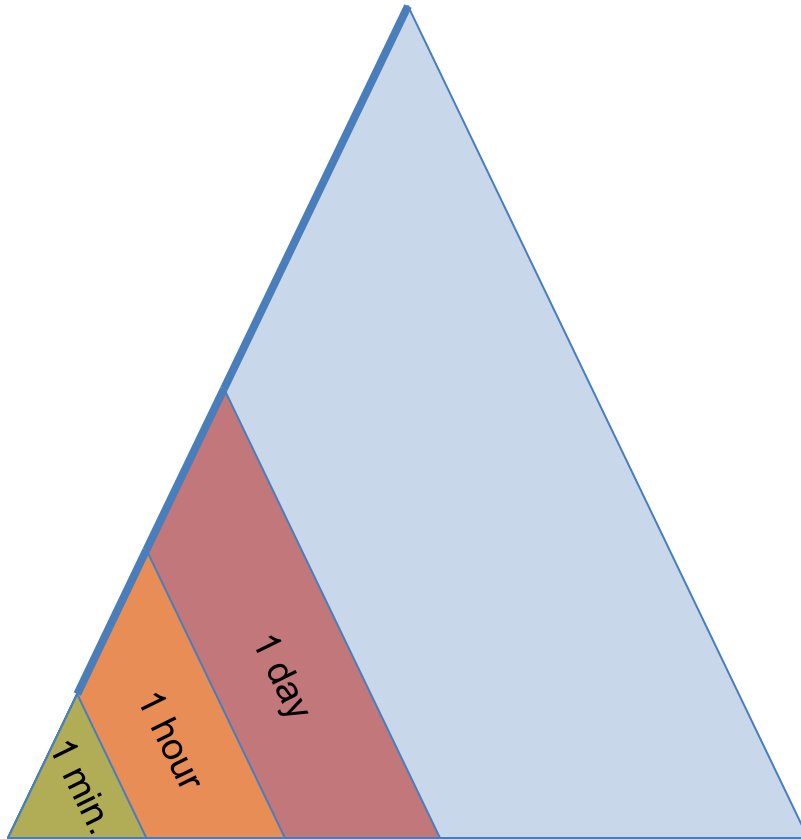
$$x_2 \leftarrow 1$$

- If not, select a new literal to watch

$$c_1 = (x_1 \vee \underline{\neg x_2} \vee \underline{\neg x_3} \vee \mathbf{0}_4)$$

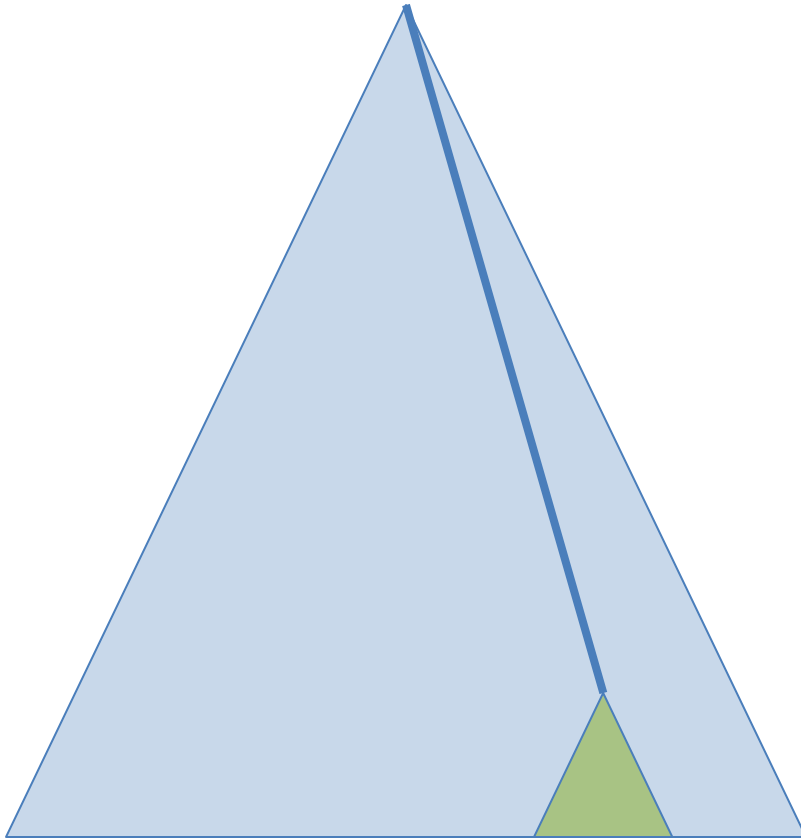
$$w(\neg x_3) = \{c_1\}$$

Danger of Search (1)



- If search space is very, very large
- We could be
 - searching and backtracking in one corner
 - and ignoring more promising part of the tree

Danger of Search (2)



- Solution
 - Occasionally, drop the search
 - Restart from somewhere else
 - Restart works well in practice

Restarts

- After searching for a specified amount of time (usually given by # conflicts) restart the search
- Undo all assignments
- Preserve:
 - learned clauses
 - variable activity values
- After performing a restart, the time until the next restart may change

Restart Sequences

- Geometric [Walsh '99]
 - 1, 2, 4, 8, 16, 32, 64, 128, ...
 - Each successive search is given more time than the previous
- Luby universal strategy [Luby+, '93]
 - 1, 1, 2, 1, 1, 2, 4, 1, 1, 2, 1, 1, 2, 4, 8, ...
 - Optimal restart sequence if runtime distribution is unknown