

# Clause Learning and Intelligent Backtracking in MiniSAT

CSCE 235H Introduction to Discrete Structures (Honors)

Spring 2019

URL: [cse.unl.edu/~cse235h](http://cse.unl.edu/~cse235h)

All questions: Piazza

# Clause Learning

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- At every conflict, determine the cause of the conflict
- Create a new clause to prevent the conflict from being reached in the future
- Tools
  - Implication graph to determine cause of conflicts
  - Added clause is a “learnt” no-good

# Search and Unit Propagation

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$$x_1 \leftarrow 0$$

$$\begin{aligned} &(x_1 \vee \neg x_2 \vee \neg x_4) \wedge \\ &(x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ &(x_1 \vee x_4 \vee \neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \wedge \\ &(\neg x_2 \vee x_5) \end{aligned}$$

$$\begin{aligned} &(0 \vee \neg x_2 \vee \neg x_4) \wedge \\ &(x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ &(0 \vee x_4 \vee \neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \wedge \\ &(\neg x_2 \vee x_5) \end{aligned}$$

$$\begin{aligned} &(\neg x_2 \vee \neg x_4) \wedge \\ &(x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ &(x_4 \vee \neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \wedge \\ &(\neg x_2 \vee x_5) \end{aligned}$$

Decisions:

$$x_1 \leftarrow 0$$

Propagations:

# Search and Unit Propagation

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$$x_2 \leftarrow 1$$

$$\begin{aligned} &(\neg x_2 \vee \neg x_4) \wedge \\ &(x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ &(x_4 \vee \neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \wedge \\ &(\neg x_2 \vee x_5) \end{aligned}$$

$$\begin{aligned} &(0 \vee \neg x_4) \wedge \\ &(x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ &(x_4 \vee \neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \wedge \\ &(0 \vee x_5) \end{aligned}$$

$$\begin{aligned} &(\neg x_4) \wedge \\ &(x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ &(x_4 \vee \neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \wedge \\ &(x_5) \end{aligned}$$

Decisions:

$$x_1 \leftarrow 0$$

$$x_2 \leftarrow 1$$

Propagations:

# Search and Unit Propagation

$$x_4 \leftarrow 0, x_5 \leftarrow 1$$

$$\begin{aligned} &(\neg x_4) \wedge \\ &(x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ &(x_4 \vee \neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \wedge \\ &(x_5) \end{aligned}$$

$$\begin{aligned} &(1) \wedge \\ &(0 \vee 0 \vee \neg x_8) \wedge \\ &(0 \vee \neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \wedge \\ &(1) \end{aligned}$$

$$\begin{aligned} &(\neg x_8) \wedge \\ &(\neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \end{aligned}$$

Decisions:

$$x_1 \leftarrow 0$$

$$x_2 \leftarrow 1$$

Propagations:

$$x_4 \leftarrow 0$$

$$x_5 \leftarrow 1$$

# Search and Unit Propagation

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$$x_8 \leftarrow 0, x_7 \leftarrow 0$$

$$\begin{aligned} &(\neg x_8) \wedge \\ &(\neg x_7) \wedge \\ &(x_3 \vee \neg x_6) \end{aligned}$$

$$\begin{aligned} &(1) \wedge \\ &(1) \wedge \\ &(x_3 \vee \neg x_6) \end{aligned}$$

$$(x_3 \vee \neg x_6)$$

Decisions:

$$x_1 \leftarrow 0$$

$$x_2 \leftarrow 1$$

Propagations:

$$x_4 \leftarrow 0$$

$$x_5 \leftarrow 1$$

$$x_8 \leftarrow 0$$

$$x_7 \leftarrow 0$$

# Search and Unit Propagation

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$$x_3 \leftarrow 0$$

$$x_6 \leftarrow 0$$

$$(x_3 \vee \neg x_6)$$

$$(0 \vee \neg x_6)$$

$$(\neg x_6)$$

$$(1)$$

Decisions:

$$x_1 \leftarrow 0$$

$$x_2 \leftarrow 1$$

$$x_3 \leftarrow 0$$

Propagations:

$$x_4 \leftarrow 0$$

$$x_5 \leftarrow 1$$

$$x_8 \leftarrow 0$$

$$x_7 \leftarrow 0$$

$$x_6 \leftarrow 0$$

# Implication Graph (1)

- Nodes correspond to assignments
- Nodes with no incoming edges are **decision** variables (assignments)
- Nodes with incoming edges were assigned through **propagation**

$$\begin{aligned} & (x_1 \vee \neg x_2 \vee \neg x_4) \wedge \\ & (x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ & (x_1 \vee x_4 \vee \neg x_7) \wedge \\ & (x_3 \vee \neg x_6) \wedge \\ & (\neg x_2 \vee x_5) \end{aligned}$$

Decisions:

$$x_1 \leftarrow 0$$

$$x_2 \leftarrow 1$$

$$x_3 \leftarrow 0$$

Propagations:

$$x_4 \leftarrow 0$$

$$x_5 \leftarrow 1$$

$$x_8 \leftarrow 0$$

$$x_7 \leftarrow 0$$

$$x_6 \leftarrow 0$$



# Implication Graph (2)

$$\begin{aligned} & (x_1 \vee \neg x_2 \vee \neg x_4) \wedge \\ & (x_4 \vee \neg x_5 \vee \neg x_8) \wedge \\ & (x_1 \vee x_4 \vee \neg x_7) \wedge \\ & (x_3 \vee \neg x_6) \wedge \\ & (\neg x_2 \vee x_5) \end{aligned}$$

Decisions:      Propagations:

$$x_1 \leftarrow 0$$

$$x_4 \leftarrow 0$$

$$x_2 \leftarrow 1$$

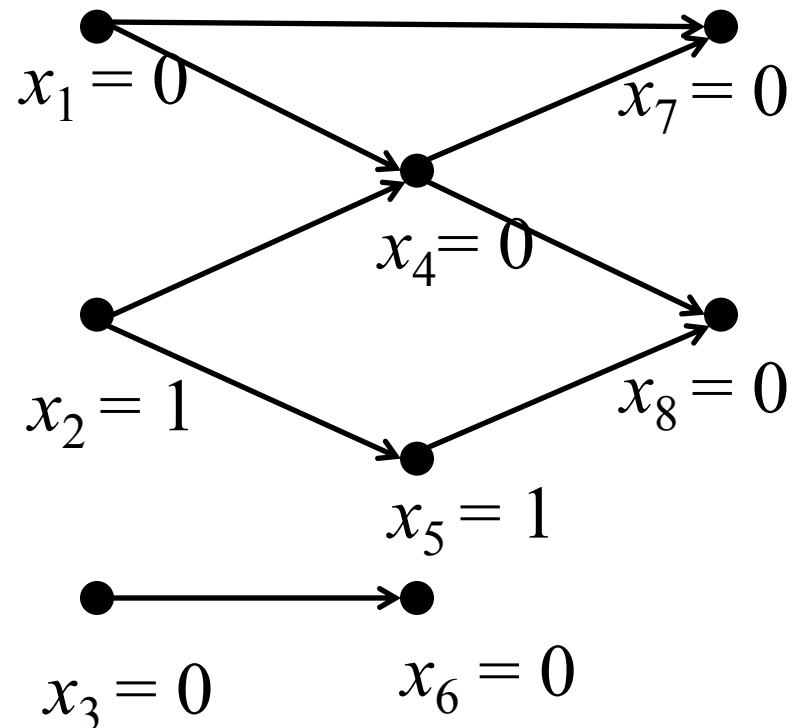
$$x_5 \leftarrow 1$$

$$x_3 \leftarrow 0$$

$$x_8 \leftarrow 0$$

$$x_7 \leftarrow 0$$

$$x_6 \leftarrow 0$$

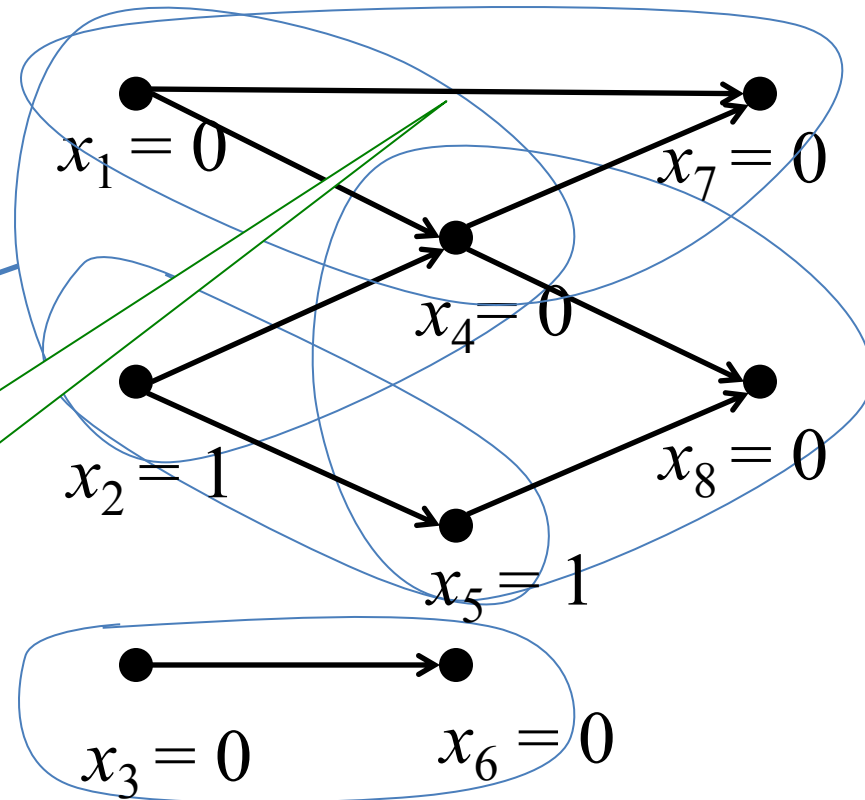


# Implication Graph (3)

- A node and its immediate predecessors correspond to the propagating clause

$$(x_1 \vee \neg x_2 \vee \neg x_4)$$

Note: transitive links are required to show which clause caused propagation



# Trail

- Series of assignments made up to current point in search
- Broken up by ‘decision levels’
- Each decision level includes propagations

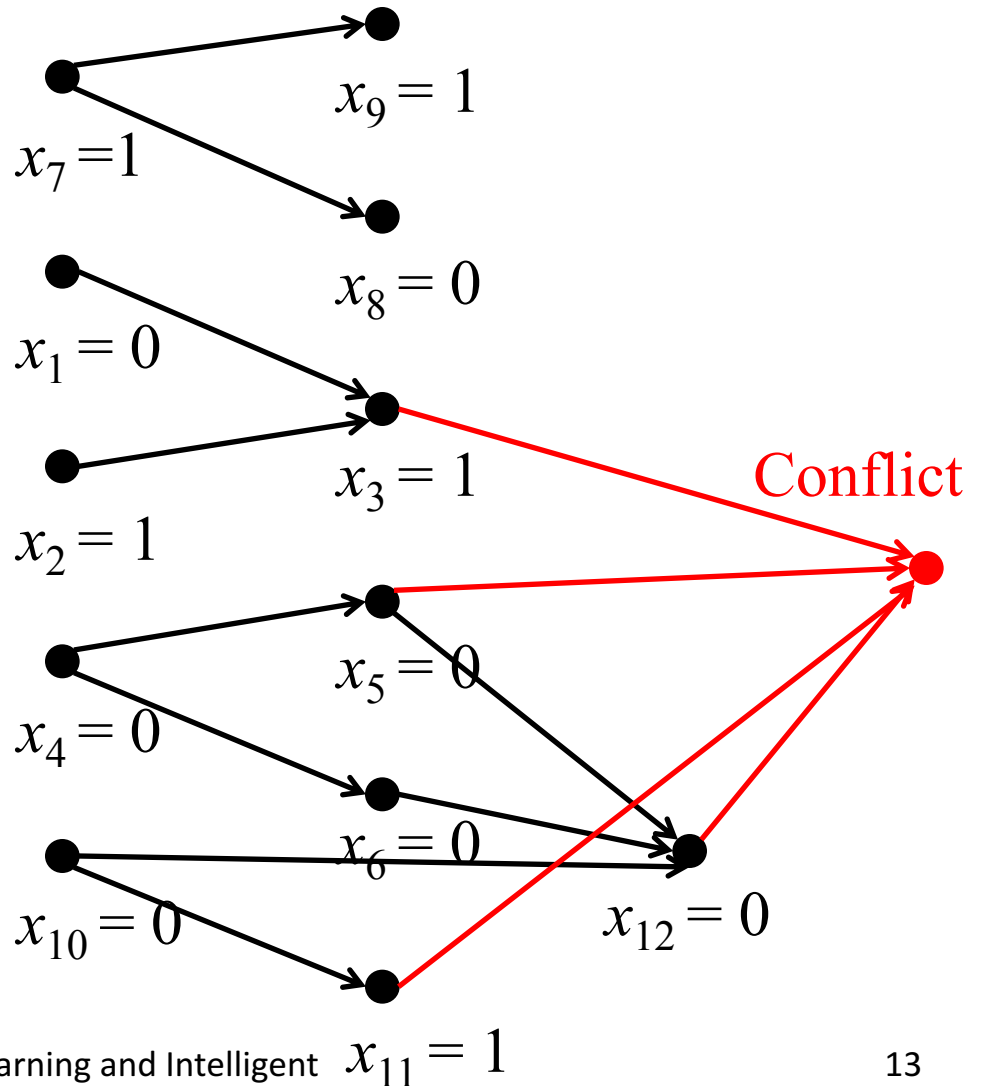
Decision level	Assignment
1	$x_1 = 0$
2	$x_2 = 1$ $x_4 = 0$ $x_5 = 1$ $x_7 = 0$ $x_8 = 0$
3	$x_3 = 0$ $x_6 = 0$

# Clause Learning Example (1)

DL	Assignment	
1	$x_1 = 0$	$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$
2	$x_2 = 1$	$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$
	$x_3 = 1$	$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$
3	$x_4 = 0$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
	$x_5 = 0$	$(\neg x_7 \vee x_9) \wedge$
	$x_6 = 0$	$(\neg x_7 \vee \neg x_8) \wedge$
4	$x_7 = 1$	$(x_4 \vee \neg x_5) \wedge$
	$x_8 = 0$	$(x_4 \vee \neg x_6) \wedge$
	$x_9 = 1$	$(x_{10} \vee x_{11}) \wedge$
5	$x_{10} = 0$	$(\neg x_{10} \vee x_{11})$
	$x_{11} = 1$	
	$x_{12} = 0$	

# Clause Learning Example (2)

DL	Assignment
1	$x_1 = 0$
2	$x_2 = 1$ $x_3 = 1$
3	$x_4 = 0$ $x_5 = 0$ $x_6 = 0$
4	$x_7 = 1$ $x_8 = 0$ $x_9 = 1$
5	$x_{10} = 0$ $x_{11} = 1$ $x_{12} = 0$



# Clause Learning Example (3)

Decision variables

Conflict caused by assignment

$$(\neg x_1 \wedge x_2 \wedge \neg x_4 \wedge \neg x_{10})$$

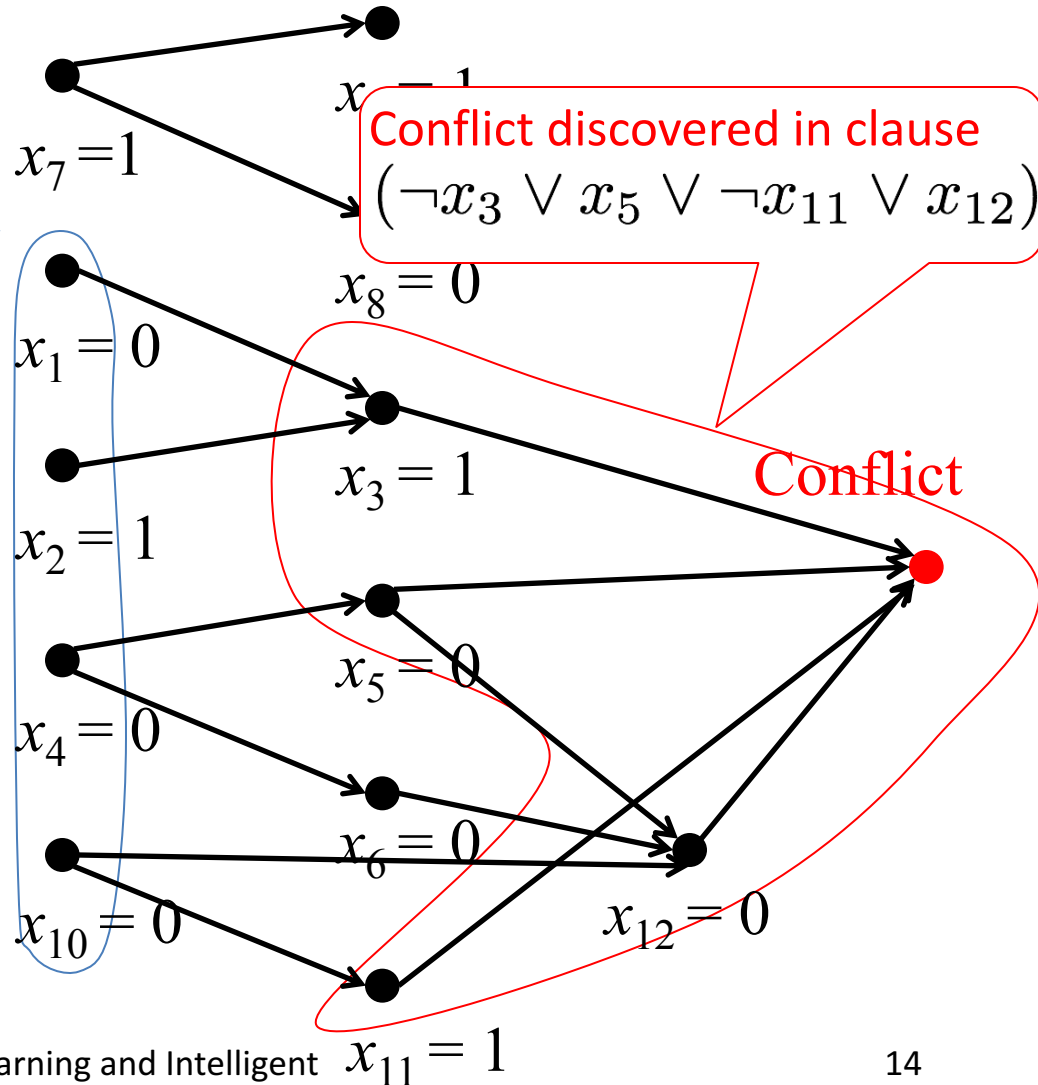
So, we negate it

$$\neg(\neg x_1 \wedge x_2 \wedge \neg x_4 \wedge \neg x_{10})$$

.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})$$

to the formula to prevent this conflict in the future



# Intelligent Backtracking

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- When reaching a conflict, we
  - Consider conflicted clauses
  - Draw the implication graph
  - Identify the decision variables
  - Generate the learnt no-good
  - Add learned clause to the formula
- Undo assignments until the learned clause becomes a unit clause

# Intelligent Backtracking Example (1)

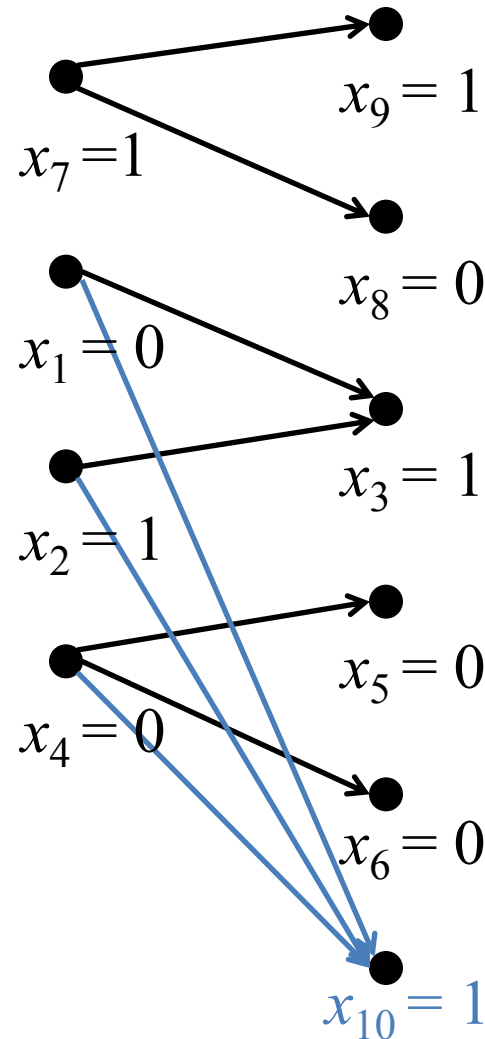
DL	Assignment	
1	$x_1 = 0$	$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$
2	$x_2 = 1$	$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$
	$x_3 = 1$	$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$
3	$x_4 = 0$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
	$x_5 = 0$	$(\neg x_7 \vee x_9) \wedge$
	$x_6 = 0$	$(\neg x_7 \vee \neg x_8) \wedge$
4	$x_7 = 1$	$(x_4 \vee \neg x_5) \wedge$
	$x_8 = 0$	$(x_4 \vee \neg x_6) \wedge$
	$x_9 = 1$	$(x_{10} \vee x_{11}) \wedge$
5	<b><math>x_{10} = 0</math></b>	$(\neg x_{10} \vee x_{11}) \wedge$
	$x_{11} = 1$	<b><math>(x_1 \vee \neg x_2 \vee x_4 \vee x_{10})</math></b>
	$x_{12} = 0$	

$x_{10}$  deepest  
decision variable



# Intelligent Backtracking Example (2)

DL	Assgn	DL	Assgn
1	$x_1 = 0$	1	$x_1 = 0$
2	$x_2 = 1$ $x_3 = 1$	2	$x_2 = 1$ $x_3 = 1$
3	$x_4 = 0$ $x_5 = 0$ $x_6 = 0$	3	$x_4 = 0$ $x_5 = 0$ $x_6 = 0$
4	$x_7 = 1$ $x_8 = 0$ $x_9 = 1$	4	$x_7 = 1$ $x_8 = 0$ $x_9 = 1$
5	$x_{10} = 0$ $x_{11} = 1$ $x_{12} = 0$		$x_{10} = 1$



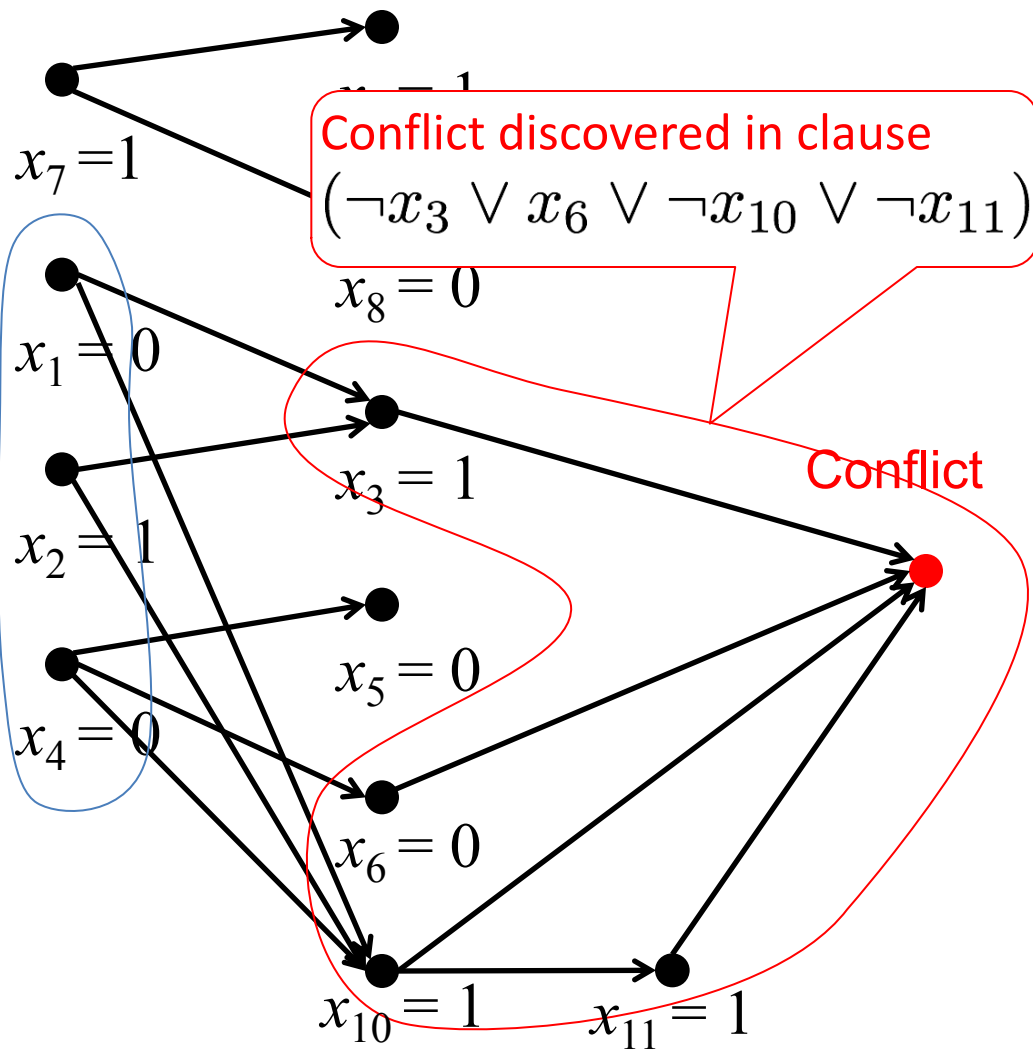
# Intelligent Backtracking Example (3)

Conflict caused by these decision variables

.. and add the clause

$$(x_1 \vee \neg x_2 \vee x_4)$$

to the formula to prevent this conflict in the future



# Intelligent Backtracking Example (4)

DL	Assgn	
1	$x_1 = 0$	$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$
2	$x_2 = 1$	$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$
	$x_3 = 1$	$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$
3	$x_4 = 0$	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
	$x_5 = 0$	$(\neg x_7 \vee x_9) \wedge$
	$x_6 = 0$	$(\neg x_7 \vee \neg x_8) \wedge$
4	$x_7 = 1$	$(x_4 \vee \neg x_5) \wedge$
	$x_8 = 0$	$(x_4 \vee \neg x_6) \wedge$
	$x_9 = 1$	$(x_{10} \vee x_{11}) \wedge$
	$x_{10} = 1$	$(\neg x_{10} \vee x_{11}) \wedge$
		$(x_1 \vee \neg x_2 \vee x_4 \vee x_{10}) \wedge$
		$(x_1 \vee \neg x_2 \vee x_4)$

# Intelligent Backtracking Example (5)

DL	Assgn	DL	Assgn	
1	$x_1 = 0$	1	$x_1 = 0$	$(x_5 \vee x_6 \vee x_{10} \vee \neg x_{12}) \wedge$
2	$x_2 = 1$	2	$x_2 = 1$	$(\neg x_3 \vee x_5 \vee \neg x_{11} \vee x_{12}) \wedge$
	$x_3 = 1$		$x_3 = 1$	$(\neg x_3 \vee x_6 \vee \neg x_{10} \vee \neg x_{11}) \wedge$
3	$x_4 = 0$	3	<b><math>x_4 = 1</math></b>	$(x_1 \vee \neg x_2 \vee x_3) \wedge$
	$x_5 = 0$			$(\neg x_7 \vee x_9) \wedge$
	$x_6 = 0$			$(\neg x_7 \vee \neg x_8) \wedge$
4	$x_7 = 1$			$(x_4 \vee \neg x_5) \wedge$
	$x_8 = 0$			$(x_4 \vee \neg x_6) \wedge$
	$x_9 = 1$			$(x_{10} \vee x_{11}) \wedge$
	$x_{10} = 1$			$(\neg x_{10} \vee x_{11}) \wedge$
				<b><math>(x_1 \vee \neg x_2 \vee x_4 \vee x_{10}) \wedge</math></b>
				<b><math>(x_1 \vee \neg x_2 \vee x_4)</math></b>

# Summary

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- Search
  - Assign variable, Propagate
  - Detect conflict? Intelligent backtracking
- Intelligent backtracking
  - Identify decision variables source of conflict
  - Add no-good clause so conflict cannot arise in the future
  - Backtrack the deepest variables in the learnt clause
  - Flip assignment of deepest variable in learnt clause
  - Proceed
- Do you see any problem in this strategy?