Computer Science & Engineering 235 – Discrete Mathematics
Logical Equivalences, Implications, Inferences, and Set Identities

Table 1: Logical Equivalences

	Logical Equivalences	
1.a.	$(p \vee 0) \equiv p$	Identity laws
1.b.	$(p \wedge 1) \equiv p$	
2.a.	$(p \vee 1) \equiv 1$	Domination laws
2.b.	$(p \wedge 0) \equiv 0$	
3.a.	$(p \vee p) \equiv p$	Idempotent laws
3.b.	$(p \wedge p) \equiv p$	
4.	$\neg(\neg p) \equiv p$	Double negation law
5.a.	$(p \vee q) \equiv (q \vee p)$	Commutative laws
5.b.	$(p \wedge q) \equiv (q \wedge p)$	
5.c.	$(p \leftrightarrow q) \equiv (q \leftrightarrow p)$	
6.a.	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative laws
6.b.	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	
7.a.	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive laws
7.b.	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	
8.a.	$\neg(p \vee q) \equiv (\neg p \wedge \neg q)$	DeMorgan's Laws
8.b.	$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$	
8.c.	$\neg(\neg p \vee \neg q) \equiv (p \wedge q)$	
8.d.	$\neg(\neg p \wedge \neg q) \equiv (p \vee q)$	
9.a.	$p \vee (p \wedge q) \equiv p$	Absorption laws
9.b.	$p \wedge (p \vee q) \equiv p$	
10.a.	$p \vee \neg p \equiv 1$	Negation laws
10.b.	$p \wedge \neg p \equiv 0$	
	Logical Equivalences Involving Conditional Statements	
11.a.	$(p \to q) \equiv (\neg p \vee q)$	Implication
11.b.	$(p \to q) \equiv \neg(p \wedge \neg q)$	
11.c.	$(p \vee q) \equiv (\neg p \to q)$	
11.d.	$(p \wedge q) \equiv \neg(p \to \neg q)$	
12.	$(p \to q) \equiv (\neg q \to \neg p)$	Contrapositive
13.a.	$[(p \to r) \wedge (q \to r)] \equiv [(p \vee q) \to r]$	
13.b.	$[(p \to q) \wedge (p \to r)] \equiv [p \to (q \wedge r)]$	
14.	$(p \leftrightarrow q) \equiv [(p \to q) \wedge (q \to p)]$	Equivalence
15.	$[(p \wedge q) \to r] \equiv [p \to (q \to r)]$	Exportation Law
16.	$(p \to q) \equiv [(p \wedge \neg q) \to c]$	Reductio ad Absurdum
17.a.	$(p \oplus q) \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	Exclusive Or
17.b.	$(p \oplus q) \equiv (p \vee q) \wedge (\neg p \vee \neg q)$	

Table 2: Rules of Inference

1.	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus Ponens
2.	$[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$	Modus Tollens
3.a.	$[p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Transitivity or hypothetical syllogism
3.b.	$[p \leftrightarrow q) \wedge (q \leftrightarrow r)] \rightarrow (p \leftrightarrow r)$	
4.	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive syllogism or unit resolution
5.	$p \rightarrow (p \vee q)$	Addition
6.	$(p \wedge q) \rightarrow p$	Simplification
7.	$[(p) \wedge (q)] \rightarrow (p \wedge q)$	Conjunction
8.	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	Resolution
9.	$[(p \rightarrow c)] \rightarrow \neg p$	Absurdity
10.	$p \rightarrow [q \rightarrow (p \wedge q)]$	
11.a.	$(p \rightarrow q) \rightarrow [(p \vee r) \rightarrow (q \vee r)]$	
11.b.	$(p \rightarrow q) \rightarrow [(p \wedge r) \rightarrow (q \wedge r)]$	
12.a.	$[(p \rightarrow q) \wedge (r \rightarrow s)] \rightarrow [(p \vee r) \rightarrow (q \vee s)]$	Constructive Dilemmas
12.b.	$[(p \rightarrow q) \wedge (r \rightarrow s)] \rightarrow [(p \wedge r) \rightarrow (q \wedge s)]$	
13.a.	$[(p \rightarrow q) \wedge (r \rightarrow s)] \rightarrow [(\neg q \vee \neg s) \rightarrow (\neg p \vee \neg r)]$	Destructive Dilemmas
13.b.	$[(p \rightarrow q) \wedge (r \rightarrow s)] \rightarrow [(\neg q \wedge \neg s) \rightarrow (\neg p \wedge \neg r)]$	

Table 3: Set Identities

$A \cup \emptyset = A$	Identity laws
$A \cap U = A$	
$A \cup U = U$	Domination laws
$A \cap \emptyset = \emptyset$	
$A \cup A = A$	Idempotent laws
$A \cap A = A$	
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$	Commutative laws
$A \cap B = B \cap A$	
$A \cup (B \cup C) = (A \cup B) \cup C$	Associative laws
$A \cap (B \cap C) = (A \cap B) \cap C$	
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	
$\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$	
$A \cup (A \cap B) = A$	Absorption laws
$A \cap (A \cup B) = A$	
$A \cup \overline{A} = U$	Complement laws
$A \cap \overline{A} = \emptyset$	