Recitation 5

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- To start, we'll look at some proofs. Consider problem 1.7.19:
  - Let $P(n)$: *If a and b are positive real numbers, then* $(a+b)^n \geq a^n + b^n$.
  - Prove that $P(1)$ is true.
    1. First, let a and b be positive real numbers
    2. $P(1) : (a+b)^1 \geq a^1 + b^1$.
    3. So $P(1) : a+b \geq a+b$.
    4. Well this is clearly true, since $a+b = a+b$.
  - What kind of proof did we use? This is a *direct* proof.
- Now a more difficult proof 1.7.29:
  - Prove or disprove:

$$\forall m, n \in \mathbb{Z},\ mn = 1 \Rightarrow (m = 1 \wedge n = 1) \vee (m = -1 \wedge n = -1)$$

    1. Case 1: Suppose $\mid m \mid > 1$. Regardless of what n is, $mn = 1$, so $n = \frac{1}{|m|}$. But we said that both m and n are integers. If $\mid m \mid > 1$, then n here cannot be an integer. So, contradiction.
    2. Case 2: Similar to case 1, but suppose $\mid n \mid > 1$. Completely symmetric, works out the same.
    3. Case 3: Suppose $m = 1$ and $n = -1$. Clearly then $mn = -1$. So, contradiction.
    4. Case 4: Suppose $m = -1$ and $n = 1$. Again clearly $mn = -1$. So, contradiction.
    5. Case 5: Suppose $m = 1$ and $n = 1$. Then $mn = 1$.
    6. Case 6: Suppose $m = 1$ and $n = 1$. Then $mn = 1$.
  - What techniques did we use? We used proofs by *cases* and by *contradiction*
- 1.8:Example 10: Show that there is a positive integer that can be written as the sum of cubes of positive integers, in two different ways:

  - Here, we are going to use a *constructive existence* proof.
  - $1729 = 10^3 + 9^3 = 12^3 + 1^3$.
  - So, 1729 can be written of the sum of cubes of positive integers in two different ways.
  - Therefore, there is a positive integer that can be written as the sum of cubes of positive integers, in two different ways.
  - These types of proof are basically finding an example.
  - Unfortunately, other than using your intuition, the only way to go about it is brute force.

- 1.8:Example 20: Can we tile a standard chess board with opposite corners removed using dominos (e.g., upper left, lower right removed)?
  - Suppose we can.
  - First, note that a standard chessboard can be covered by dominos, each having one white and one black square.
  - We know that the standard chess board has 64 square. Removing 2 gives 64-2 = 62 squares.
  - Tiling with dominos uses $62/2 = 31$ dominos.
  - These dominos each have one white and one black square (like chessboards have white and black).
  - So, using the tiles we have 31 white and 31 black squares.
  - However, when we remove two opposite corner squares, either 32 of the remaining squares are white and 30 are black, or 32 are black and 30 are white.
  - But, we said we had 31 of each color. Thus, we have a contradiction.
  - Therefore, we cannot tile a standard chessboard with two opposite corners removed, with dominos.

- Now moving on to sets, 2.1.11: Determine whether true or false:

  a. $x \in \{x\}$ is true.
  b. $\{x\} \subseteq \{x\}$ is true.
  c. $\{x\} \in \{x\}$ is false.
  d. $\{x\} \in \{\{x\}\}$ is true.
  e. $\emptyset \subseteq \{x\}$ is true.
  f. $\emptyset \in \{x\}$ is false.

- 2.1.27(a) Let $A = \{a, b, c, d\}$, and $B = \{y, z\}$.

  a $A \times B$?

    1. $|A \times B|$? As in, how big will it be? $4 \times 2 = 8$
    2. $\{(a, y), (b, y), (c, y), (d, y), (a, z), (b, z), (c, z), (d, z)\}$

  – How about $\mathbb{Z} \times (\mathbb{Z}^+ \setminus 0)$

    1. What is its *cardinality*? ∞
    2. What are some of its elements? (1,2),(3,4),(-1,5).
    3. Is $(2, 0) \in \mathbb{Z} \times (\mathbb{Z}^+ \setminus 0)$? No
    4. Does this set remind you of anything? Could be used to represent the rational numbers, $\mathbb{Q}$.

- Computer the power set of the following set $S = \{a, b, \{c\}, \emptyset\}$

  – First, how many elements are in $\mathcal{P}(S)$? $2^4 = 16$.

  – $\{\emptyset,$
  $\{\emptyset\}, \{\{c\}\}, \{a\}, \{b\},$
  $\{a, b\}, \{a, \{c\}\}, \{b, \{c\}\}, \{a, \emptyset\}, \{b, \emptyset\}, \{\{c\}, \emptyset\},$
  $\{a, b, \{c\}\}, \{a, b, \emptyset\}, \{b, \{c\}, \emptyset\}, \{a, \{c\}, \emptyset\},$
  $\{a, b, \{c\}, \emptyset\}\}$