Recitation 2

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January 18, 2018

- Questions about Piazza, LaTeX or lecture?
- Questions on the homework?
- Consider Section 1.1, problem 15 on page 14 of your Rosen textbook

  1. First, we define the following *propositions*:
     - p: Grizzly bears have been seen in the area.
     - q: Hiking is safe on the trail.
     - r: Berries are ripe along the trail.
  2. Now, we convert the following English sentences into PL sentences using logical connectives:

     a : "Berries are ripe along the trail, but grizzly bears have not been seen in the area:" $r \wedge \neg p$

     b : "Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail:" $\neg p \wedge q \wedge r$

     c : "If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area:" $r \rightarrow (q \leftrightarrow \neg p)$

     d : "It is not safe to hike on the trail, but grizzly bears have not been seen in the area and berries along the trail are ripe:" $\neg q \wedge \neg p \wedge r$

     e : "For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area:" $(q \rightarrow (\neg r \wedge \neg p)) \wedge \neg((\neg r \wedge \neg p) \rightarrow q)$

     f : "Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail:" $(p \wedge r) \rightarrow \neg q$

- Consider Section 1.1, problem 35, part C. Construct the truth table for the following sentence:

$$(p \to q) \vee (\neg p \to q)$$

1. We proceed step by step following three stages:
   (a) $p \to q$
   (b) $\neg p \to q$
   (c) $(p \to q) \vee (\neg p \to q)$
2. Now we construct a table filling in the p an q values:

p	q	$p \to q$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
0	0	-	-	-
0	1	-	-	-
1	0	-	-	-
1	1	-	-	-

3. Now fill in the rest of the table:

p	q	$p \to q$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
0	0	1	0	1
0	1	1	1	1
1	0	0	1	1
1	1	1	1	1

- Consider now a harder problem, from Section 1.2, problem 35.

1. First, we select *atomic propositions*:
   - Let b be the butler
   - Let c be the cook
   - Let g be the gardener
   - Let h be the handyman

   We use 0 to indicate that the person is lying, and 1 to indicate that the person is telling the truth.
2. Next, we transcribe the English statements into statements in PL:
   - "If the butler is telling the truth then so is the cook." $b \to c$
   - "The cook and the gardener cannot both be telling the truth." $\neg(c \wedge g) \equiv \neg c \vee \neg g$
   - "The gardener and the handyman are not both lying." $\neg(\neg g \wedge \neg h) \equiv g \vee h$
   - "If the handyman is telling the truth, then the cook is lying." $h \to \neg c$

3. Now, we want to find a *model* for our *sentence*. Remember that a model is a truth assignment to each *term* in our sentence so that the conjunction of all clauses is satisfied (i.e., each clause must evaluate to true). Because we are looking for a *model*, we can stop looking at an assignment once we detect a clause that evaluates to false: indeed, the conjuction of clauses cannot be satisfied. A "-" is used in Tabl1 to denote that we interrupted the process before evaluating that *clause*.

Table 1: Turth table

b	c	g	h	$b \to c$	$\neg c \lor \neg g$	$g \lor h$	$h \to \neg c$
0	0	0	0	1	1	0	-
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	0	-
0	1	0	1	1	1	1	0
0	1	1	0	1	0	-	-
0	1	1	1	1	0	-	-
1	0	0	0	0	-	-	-
1	0	0	1	0	-	-	-
1	0	1	0	0	-	-	-
1	0	1	1	0	-	-	-
1	1	0	0	1	1	0	-
1	1	0	1	1	1	1	0
1	1	1	0	1	0	-	-
1	1	1	1	1	0	-	-

4. From Table 1, we see that the sentence has three models (see line 2, 3, and 4). Examining those model, we conclude that the butler and the cook are both lying, and that either the gardener, the handyman, or both are telling the truth.

- Consider Section 1.1 problem number 27 (b) on page 15 of the textbook:
  - "I come to class whenever there is going to be a quiz." Notice that this can be rephrased "If there is going to be a quiz, I come to class." $p \to q$
  - What is the inverse of this sentence, $\neg p \to \neg q$? "If there is not going to be a quiz, then I do not come to class."
  - What is the converse of this sentence, $q \to p$? "If I come to class, then there is a quiz."
  - What is the contrapositive of this sentence, $\neg q \to \neg p$? "If I do not come to class, then there will not be a quiz."

- Below, examine the truth table with the inverse, converse, and contrapositive of the implication $p \to q$.

Propositions p	q	Implication $p \to q$	Inverse $\neg p \to \neg q$	Converse $q \to p$	Contrapositive $\neg q \to \neg p$
0	0	1	1	1	1
0	1	1	0	0	1
1	0	0	1	1	0
1	1	1	1	1	1

- Now, consider a different kind of problem, Section 1.1 problem 17 (c) on page 14.
  - Let p stand for $1 + 1 = 3$
  - Let q stand for "Dogs can fly."
  - What is $p \to q$? $0 \to 0$, which is true.
- Finally, Section 1.1 number 39, pg 15: Construct the truth table for the following sentence in PL:

$$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$$

  - First, can you tell immediately how many rows our truth table will have? With n terms, we must have 2^n rows.

a	b	c	d	$(p \leftrightarrow q)$	$(r \leftrightarrow s)$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$
0	0	0	0	1	1	1
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	1	1
0	1	0	0	0	1	0
0	1	0	1	0	0	1
0	1	1	0	0	0	1
0	1	1	1	0	1	0
1	0	0	0	0	1	0
1	0	0	1	0	0	1
1	0	1	0	0	0	1
1	0	1	1	0	1	0
1	1	0	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	1	1	1