Watched Literals and Restarts in MiniSAT

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CSCE 235H Introduction to Discrete Structures (Honors)
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All questions: Piazza

Reminder: Unit Propagation

- Assignments may result in unit clauses

$$a \leftarrow \mathit{false} \text{ and } a \vee b \text{ yield } b$$

- Unit clauses immediately force an assignment

$$b \leftarrow \mathit{true}$$

- This can lead to a chain reaction as new assignments 'propagate' throughout the clauses

When Does Unit Propagation Fire Up?

- Consider the clause

$$(x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4)$$

  - If no variable is assigned, no unit propagation occurs
  - If all but two variables are assigned, unit propagation becomes possible

$$(0 \vee \neg x_2 \vee \neg 1 \vee x_4)$$

- Watched-literal mechanism
  - Watches 2 unassigned literals in every clause in 'preparation' of unit propagation

Watched Literals

- Technique for efficiently implementing unit propagation
- Only two literals per clause must be *watched* to determine when a clause becomes unit

$$(x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4)$$

- Each literal keeps a *watcher list* containing the clauses it is currently watched by.

Initializing Watched Literals (1)

- For every clause, select two literals to be watched.

$$c_1 = (x_1 \vee \underline{\neg x_2} \vee \neg x_3 \vee \underline{x_4})$$
$$c_2 = (\neg x_1 \vee \underline{x_3} \vee \underline{\neg x_4})$$
$$c_3 = \underline{(x_2} \vee \underline{x_4)}$$

Initializing Watched Literals (2)

- Every time a literal becomes watched, add the watching clause to the literal's watcher list

$$c_1 = (x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4)$$
$$c_2 = (\neg x_1 \vee x_3 \vee \neg x_4)$$
$$c_3 = (x_2 \vee x_4)$$

$$w(x_1) = \{\} \qquad w(\neg x_1) = \{\}$$
$$w(x_2) = \{c_3\} \qquad w(\neg x_2) = \{c_1\}$$
$$w(x_3) = \{c_2\} \qquad w(\neg x_3) = \{\}$$
$$w(x_4) = \{c_1, c_3\} \qquad w(\neg x_4) = \{c_2\}$$

Watched Literal Assignment (1)

- Assign

$$x_4 \leftarrow 0$$

- Check watcher list of newly false literal

$$w(x_4) = \{c_1, c_3\}$$

Watched Literal Assignment (2)

- If clause has become unit, propagate

$$c_3 = (x_2 \vee \mathbf{0} x_4)$$

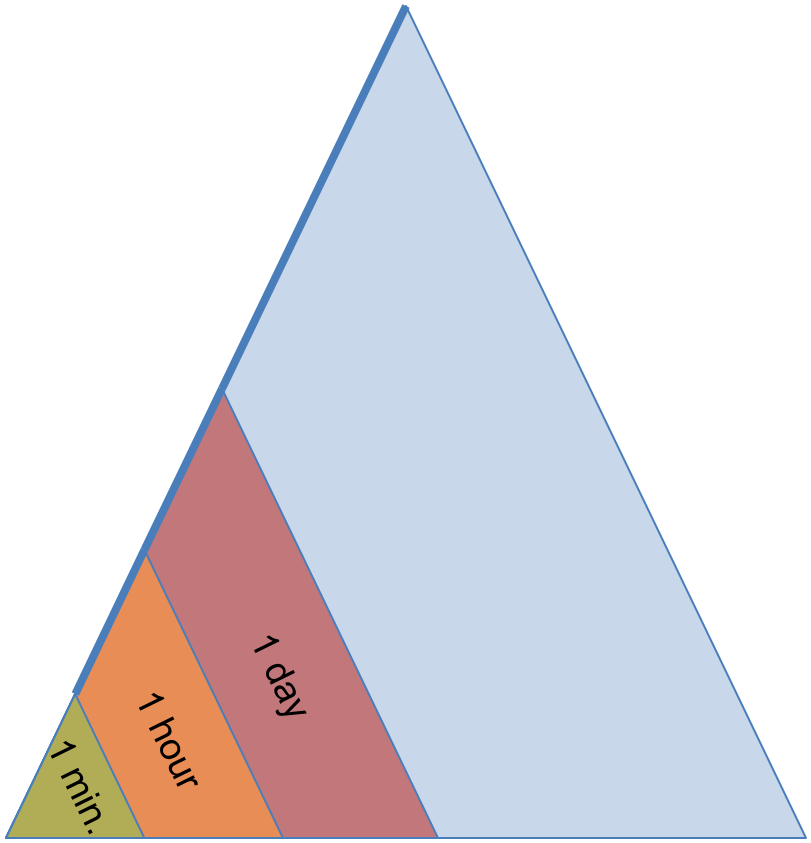
$$x_2 \leftarrow 1$$

- If not, select a new literal to watch

$$c_1 = (x_1 \vee \underline{\neg x_2} \vee \underline{\neg x_3} \vee \mathbf{0} x_4)$$

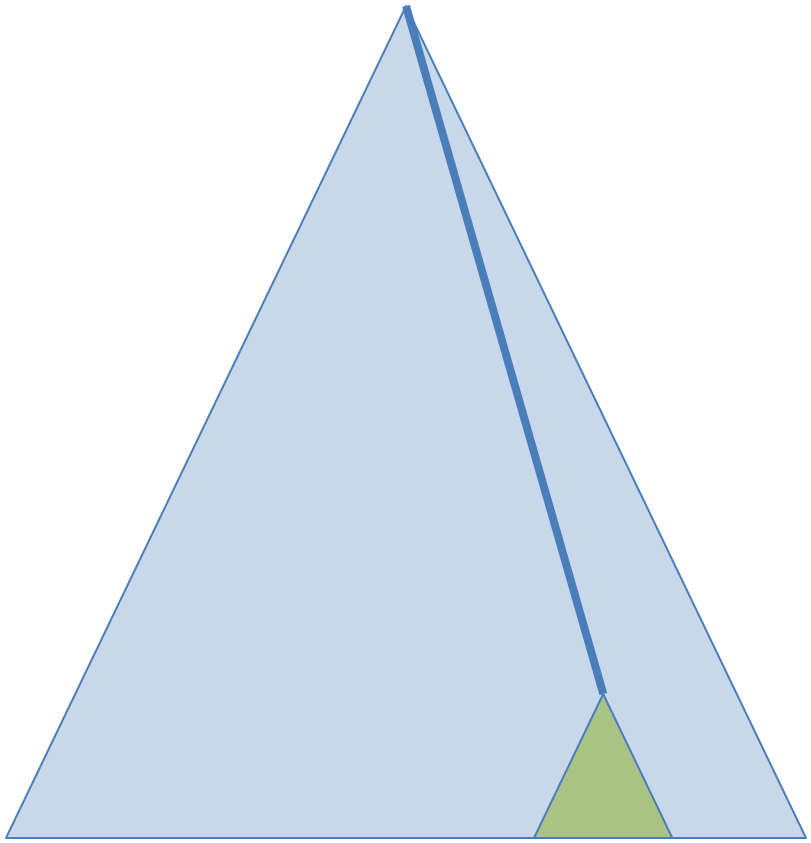
$$w(\neg x_3) = \{c_1\}$$

Danger of Search (1)



- If search space is very, very large
- We could be
  - searching and backtracking in one corner
  - and ignoring more promising part of the tree

Danger of Search (2)



- Solution
  - Occasionally, drop the search
  - Restart from somewhere else
  - Restart works well in practice

Restarts

- After searching for a specified amount of time (usually given by # conflicts) restart the search
- Undo all assignments
- Preserve:
  - learned clauses
  - variable activity values
- After performing a restart, the time until the next restart may change

Restart Sequences

- Geometric [Walsh '99]
  - 1, 2, 4, 8, 16, 32, 64, 128, …
  - Each successive search is given more time than the previous
- Luby universal strategy [Luby+, '93]
  - 1, 1, 2, 1, 1, 2, 4, 1, 1, 2, 1, 1, 2, 4, 8, ...
  - Optimal restart sequence if runtime distribution is unknown