

Title: Solving Problems by Searching
AIMA: Chapter 3 (Sections 3.4)

Introduction to Artificial Intelligence
CSCE 476-876, Spring 2015

URL: www.cse.unl.edu/~cse476 **URL:**
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```
function GENERAL-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

Essence of search: which node to expand first?
→ search strategy

A strategy is defined by picking the *order of node expansion*

Types of Search

Uninformed: use only information available in problem definition

Heuristic: exploits some knowledge of the domain

Uninformed search strategies

1. Breadth-first search
2. Uniform-cost search
3. Depth-first search
4. Depth-limited search
5. Iterative deepening depth-first search
6. Bidirectional search

Search strategies

Criteria for evaluating search:

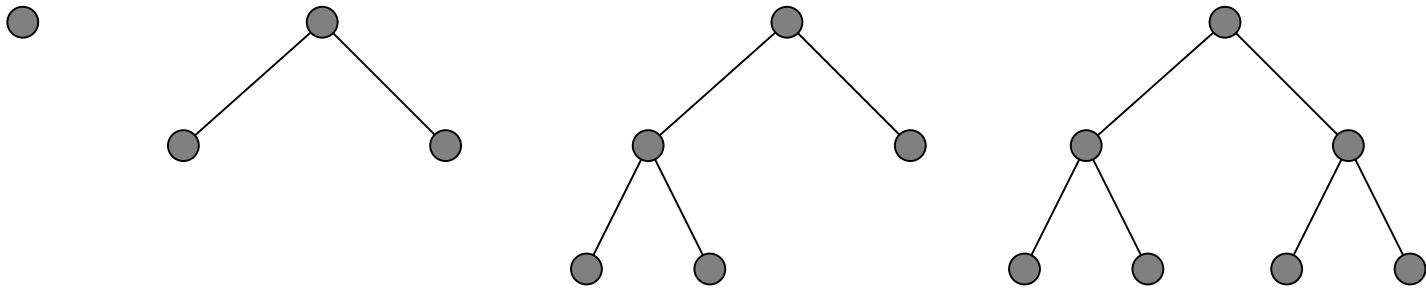
1. Completeness: does it always find a solution if one exists?
2. Time complexity: number of nodes generated/expanded
3. Space complexity: maximum number of nodes in memory
4. Optimality: does it always find a least-cost solution?

Time/space complexity measured in terms of:

- b : maximum branching factor of the search tree
- d : depth of the least-cost solution
- m : maximum depth of the search space (may be ∞)

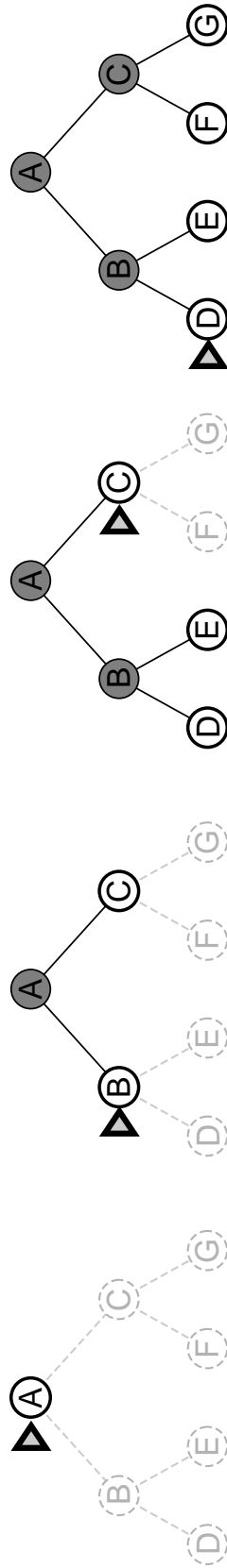
Breadth-first search (I)

- Expand root node
- Expand all children of root
- Expand *each* child of root
- Expand successors of each child of root, etc.



- Expands nodes at depth d before nodes at depth $d + 1$
- Systematically considers all paths length 1, then length 2, etc.
- Implement: put successors at end of queue.. FIFO

Breadth-first search (2)



Breadth-first search (3)

→ One solution?

→ Many solutions? Finds shallowest goal first

1. Complete? Yes, if b is finite
2. Optimal? provided cost increases monotonically with depth, not in general (e.g., actions have same cost)
3. Time? $1 + b + b^2 + b^3 + \dots + b^d + b(b^d - 1) = O(b^{d+1})$

$$O(b^{d+1}) \left\{ \begin{array}{l} \text{branching factor } b \\ \text{depth } d \end{array} \right.$$

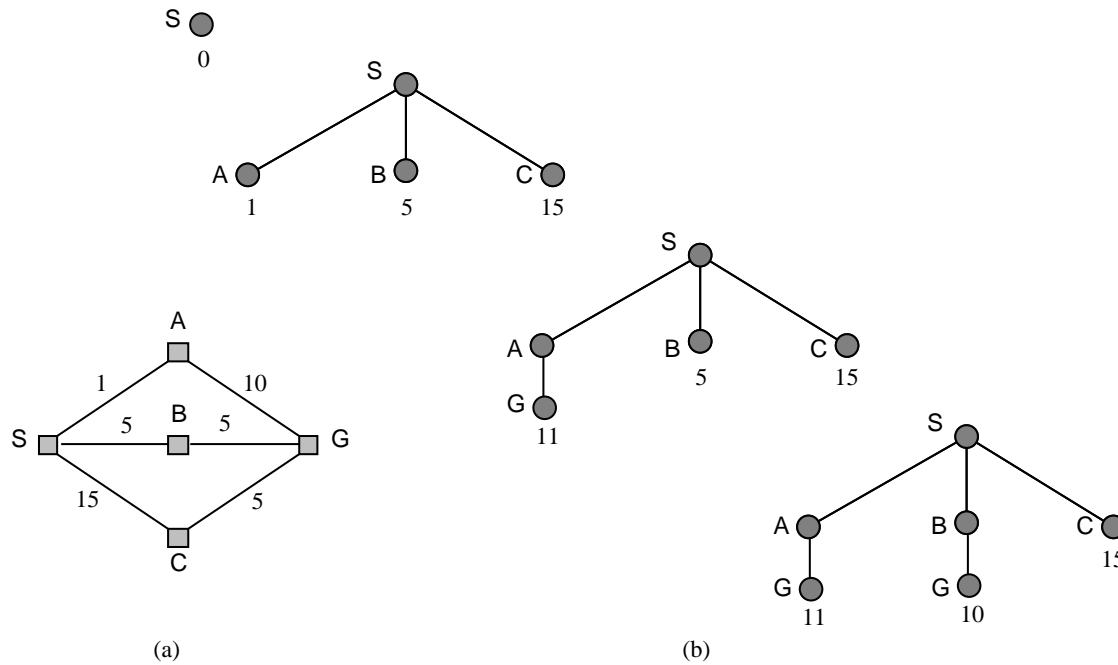
4. Space? same, $O(b^{d+1})$, keeps every node in memory, big problem

can easily generate nodes at 10MB/sec so 24hrs = 860GB

Uniform-cost search (I)

- Breadth-first does not consider path cost $g(x)$
- Uniform-cost expands first lowest-cost node on the fringe
- Implement: sort queue in decreasing cost order

When $g(x) = \text{Depth}(x) \rightarrow \text{Breadth-first} \equiv \text{Uniform-cost}$



Uniform-cost search (2)

1. Complete?

Yes, if cost $\geq \epsilon$

2. Optimal?

If the cost is a monotonically increasing function

When cost is added up along path, an operator's cost?

3. Time?

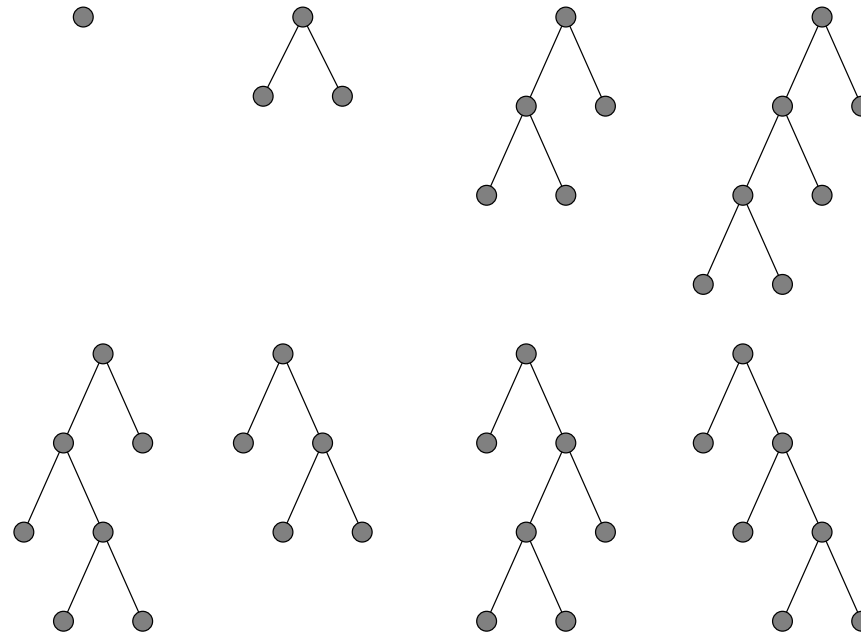
of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
where C^* is the cost of the optimal solution

4. Space?

of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

Depth-first search (I)

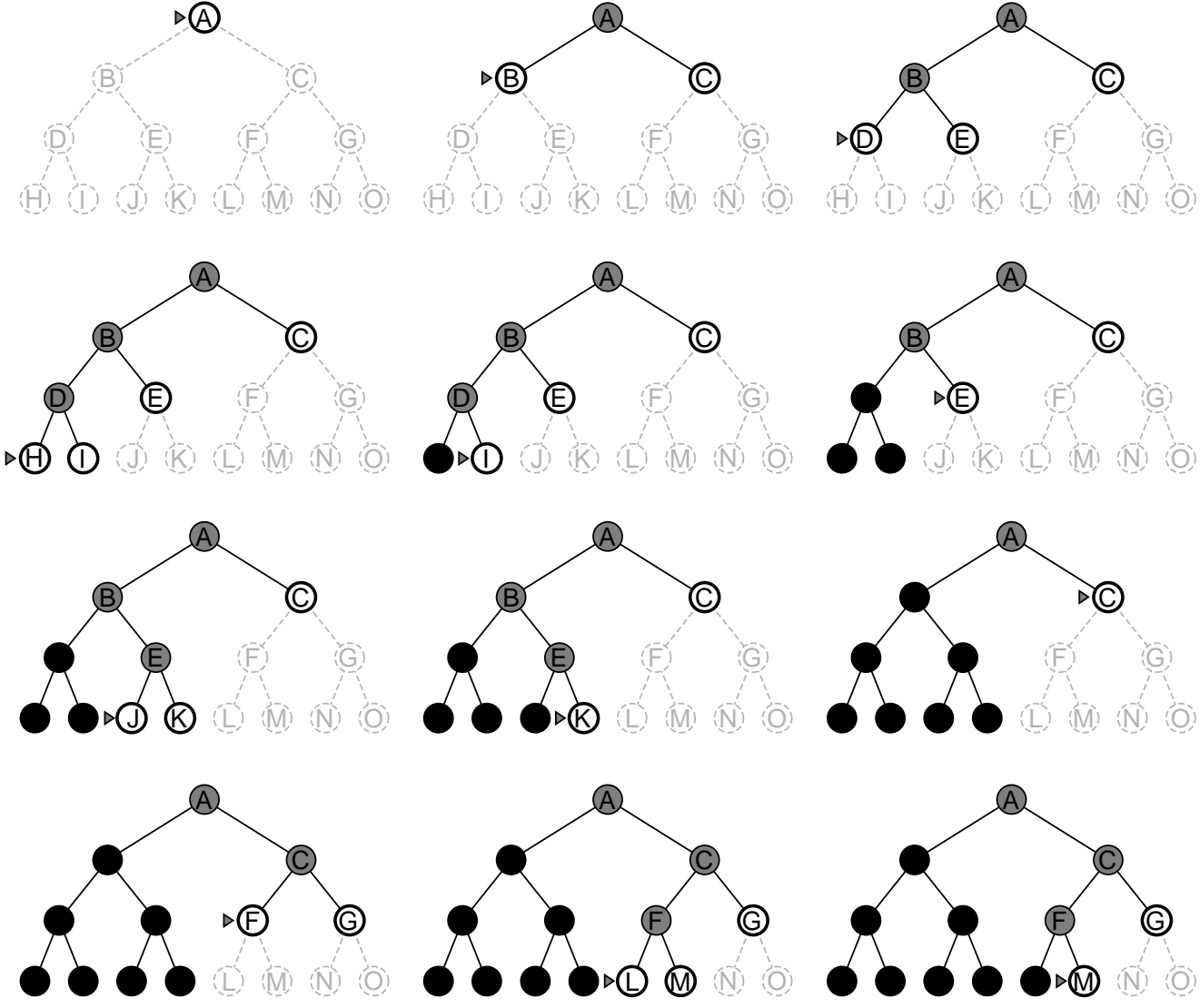
- Expands nodes at deepest level in tree
- When dead-end, goes back to shallower levels
- Implement: put successors at front of queue.. LIFO



- Little memory: path and unexpanded nodes

For b : branching factor, m : maximum depth, space

Depth-first search (2)



Depth-first search (3)

Time complexity:

We may need to expand all paths, $O(b^m)$

When there are many solutions, DFS may be quicker than BFS

When m is big, much larger than d , ∞ (deep, loops), .. troubles

→ Major drawback of DFS: going deep where there is no solution..

Properties:

1. Complete? Not in infinite spaces, complete in finite spaces

2. Optimal?

3. Time? $O(b^m)$

Woow..

terrible if m is much larger than d , but if solutions are dense,
may be much faster than breadth-first

4. Space? $O(bm)$, linear!

Woow..

Depth-limited search (I)

→ DFS is going too deep, put a threshold on depth!

For instance, 20 cities on map for Romania, any node deeper than 19 is cycling. Don't expand deeper!

→ Implement: nodes at depth l have no successor

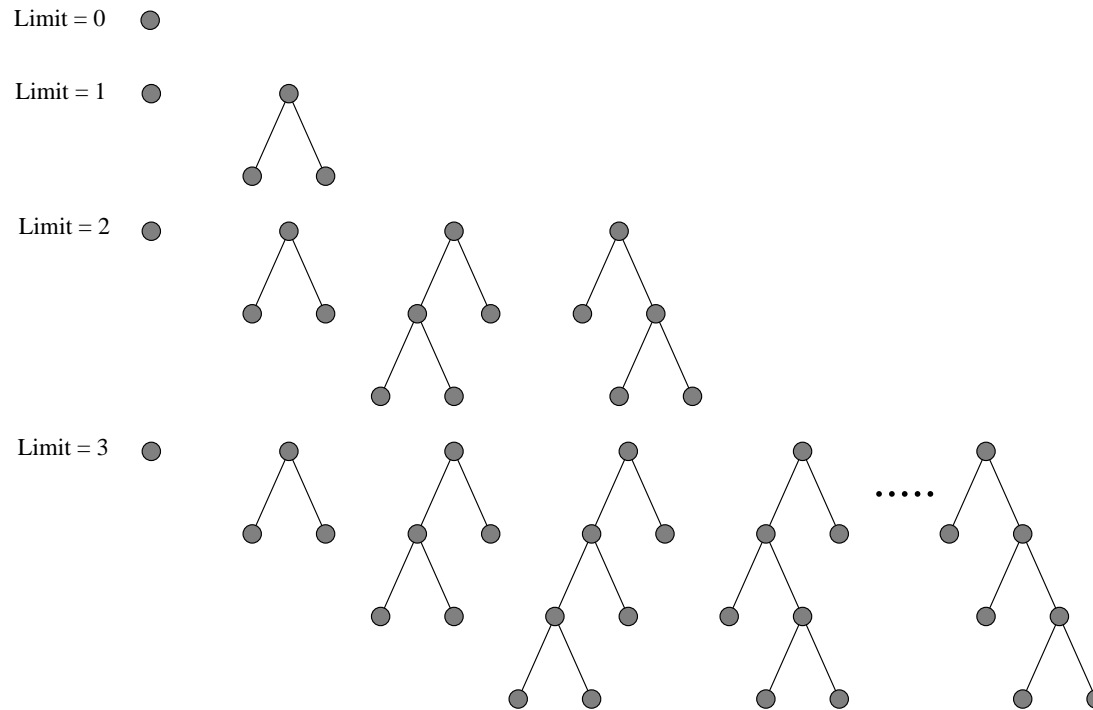
Properties:

1. Complete?
2. Optimal?
3. Time? (given l depth limit)
4. Space? (given l depth limit)

Problem: how to choose l ?

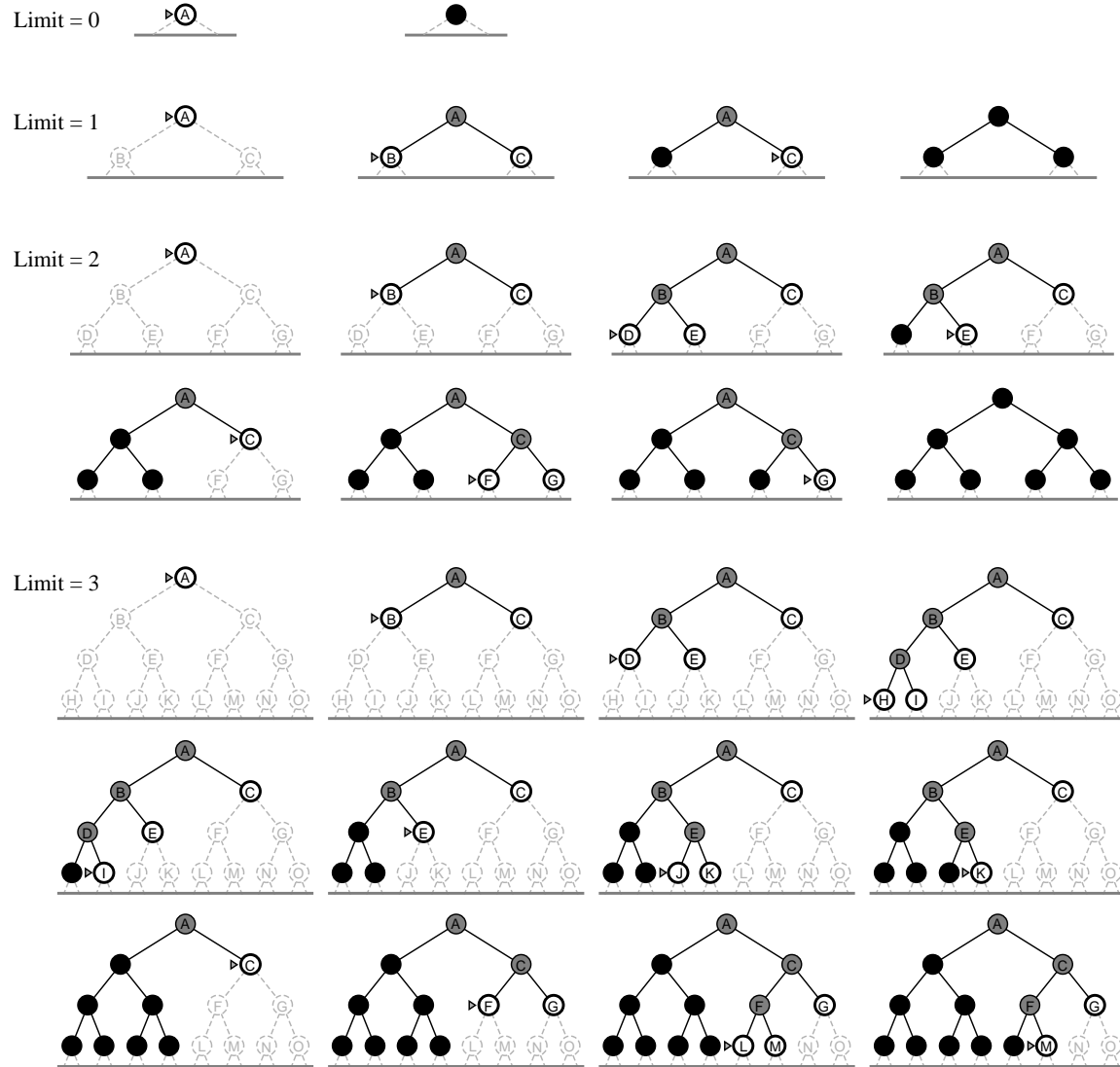
Iterative-deepening search (I)

- DLS with depth = 0
- DLS with depth = 1
- DLS with depth = 2
- DLS with depth = 3...



→ Combines benefits of DFS and BFS

Iterative-deepening search (2)



Iterative-deepening search (3)

→ combines benefits of DFS and BFS

Properties:

1. Time? $(d + 1).b^0 + (d).b + (d - 1).b^2 + \dots + 1.b^d = O(b^d)$
2. Space? $O(bd)$, like DFS
3. Complete? like BFS
4. Optimal? like BFS (if step cost = 1)

Iterative-deepening search (4)

→ Some nodes are expanded several times, wasteful?

$$N(\text{BFS}) = b + b^2 + b^3 + \dots + b^d + (b^{d+1} - b)$$

$$N(\text{IDS}) = (d)b + (d-1)b^2 + \dots + (1)b^d$$

Numerical comparison for $b = 10$ and $d = 5$:

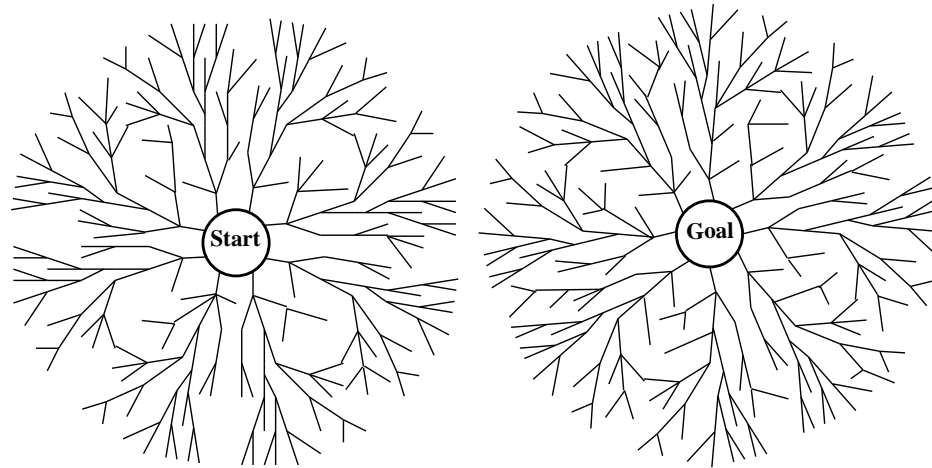
$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$

$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

→ IDS is preferred when search space is large and depth unknown

Bidirectional search (I)

→ Given initial state and the goal state, start search from both ends and meet in the middle



→ Assume same b branching factor, \exists solution at depth d , time:
 $O(2b^{d/2}) = O(b^{d/2})$

$b = 10, d = 6, \text{DFS} = 1,111,111$ nodes, $\text{BDS} = 2,222$ nodes!

Bidirectional search (2)

In practice :—(

- Need to define predecessor operators to search backwards
If operator are invertible, no problem
- What if \exists many goals (set state)?
do as for multiple-state search
- need to check the 2 fringes to see how they match
need to check whether any node in one space appears in the
other space (use hashing)
need to keep all nodes in a half in memory $O(b^{d/2})$
- What kind of search in each half space?

Summary

Criterion	Breadth- First	Uniform- Cost	Depth- First	Depth- Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes*	No	No	Yes

b branching factor

d solution depth

m maximum depth of tree

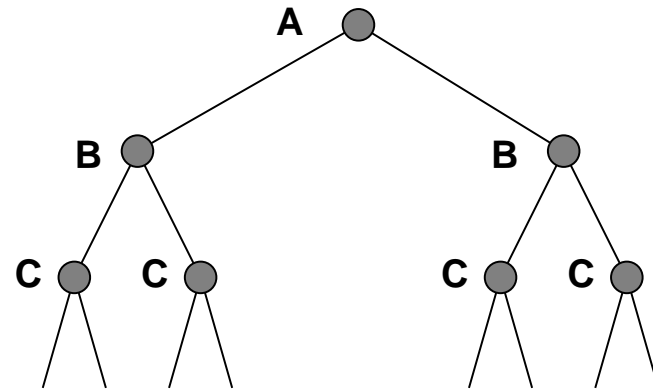
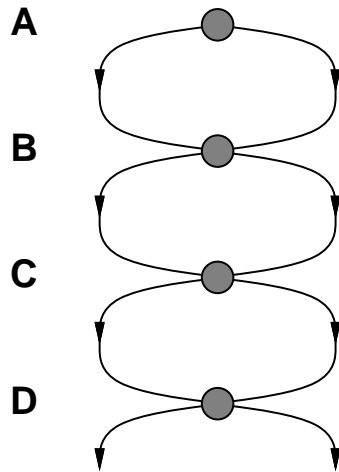
l depth limit

Loops: Avoid repeated states (I)

Avoid expanding states that have already been visited

Valid for both infinite and finite trees

Example: $\left\{ \begin{array}{l} m \text{ maximum depth} \\ m + 1 \text{ states} \\ 2^m \text{ possible branches (paths)} \end{array} \right.$

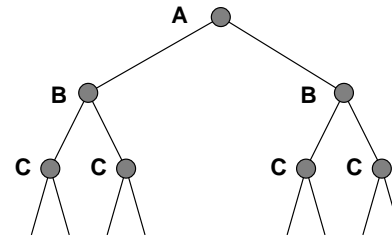
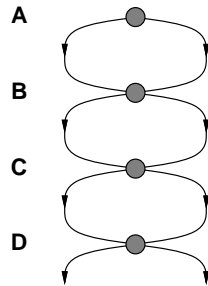


Loops: (2)

Keep nodes in two lists: $\left\{ \begin{array}{l} \text{Open list: Fringe} \\ \text{Closed list: Leaf and expanded nodes} \end{array} \right.$

Discard a current node that matches a node in the closed list

Tree-Search \longrightarrow Graph-Search



Issues:

1. Implementation: hash table, access is constant time
Trade-off cost of storing+checking vs. cost of searching
2. Losing optimality
when new path is cheaper/shorter of the one stored
3. DFS and IDS now require exponential storage

Summary

Path: sequence of actions leading from one state to another

Partial solution: a path from an initial state to another state

Search: develop a sets of partial solutions

- Search tree & its components (node, root, leaves, fringe)
- Data structure for a search node
- Search space vs. state space
- Node expansion, queue order
- Search types: uninformed vs. heuristic
- 6 uninformed search strategies
- 4 criteria for evaluating & comparing search strategies