Title: First-Order Logic
AIMA: Chapter 8 (Sections 8.1 and 8.2)
Section 8.3, discussed briefly, is also required reading

Introduction to Artificial Intelligence
CSCE 476-876, Spring 2016
URL: www.cse.unl.edu/~choueiry/S16-476-876

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Outline

- First-order logic:
  - basic elements
  - syntax
  - semantics
- Examples
Pros and cons of propositional logic

- Propositional logic is declarative: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information
  (unlike most data structures and databases)
- Propositional logic is compositional:
  meaning of $B_{1,1} \land P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is context-independent
  (unlike natural language, where meaning depends on context)
- But...
  Propositional logic has very limited expressive power
  E.g., cannot say “pits cause breezes in adjacent squares” except by writing one sentence for each square
Propositional Logic

- is simple

- illustrates important points:
  - model, inference, validity, satisfiability, ..

- is restrictive: world is a set of facts

- lacks expressiveness:
  \[ \rightarrow \text{In PL, world contains facts} \]

First-Order Logic

- more symbols (objects, properties, relations)

- more connectives (quantifier)
First Order Logic

→ FOL provides more "primitives" to express knowledge:
  — objects (identity & properties)
  — relations among objects (including functions)

**Objects:** people, houses, numbers, Einstein, Huskers, event,..
**Properties:** smart, nice, large, intelligent, loved, occurred,..
**Relations:** brother-of, bigger-than, part-of, occurred-after,..
**Functions:** father-of, best-friend, double-of,..

**Examples:** (objects? function? relation? property?)
  — one plus two equals four
  — squares neighboring the wumpus are smelly [sic]
Logic

Attributes: mathematicians, philosophers and AI people

Advantages:
— allows to represent the world and reason about it
— expresses anything that can be programmed

Non-committal to:
— symbols could be objects or relations
  (e.g., King(Gustave), King(Sweden, Gustave), Merciless(King))
— classes, categories, time, events, uncertainty

.. but amenable to extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

→ Some people think FOL *is* the language of AI
  true/false? donno :—( but it will remain around for some time..
Types of logic

Logics are characterized by what they commit to as “primitives”

**Ontological commitment**: what exists—facts? objects? time? beliefs?

**Epistemological commitment**: what states of knowledge?

<table>
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<tr>
<th>Language</th>
<th>Ontological Commitment (What exists in the world)</th>
<th>Epistemological Commitment (What an agent believes about facts)</th>
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<td>Propositional logic</td>
<td>facts</td>
<td>true/false/unknown</td>
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Higher-Order Logic: views relations and functions of FOL as objects
Syntax of FOL: words and grammar

The words: symbols

- Constant symbols stand for objects: QueenMary, 2, UNL, etc.
- Variable symbols stand for objects: $x$, $y$, etc.
- Predicate symbols stand for relations: Odd, Even, Brother, Sibling, etc.
- Function symbols stand for functions (viz. relation)
  Father-of, Square-root, LeftLeg, etc.
- Quantifiers $\forall$, $\exists$
- Connectives: $\land$, $\lor$, $\neg$, $\Rightarrow$, $\Leftrightarrow$
- (Sometimes) equality $=$

Predicates and functions can have any arity (number of arguments)
Basic elements in FOL (i.e., the grammar)

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
  - atomic sentences
  - complex sentences
- Quantifiers:
  - Universal quantifier
  - Existential quantifier
Term

logical expression that refers to an object

— built with: constant symbols, variables, function symbols

\[
\text{Term} \quad = \quad \text{function}(\text{term}_1, \ldots, \text{term}_n)
\]

or constant or variable

— ground term: term with no variable
Atomic sentences

state facts
built with terms and predicate symbols

Atomic sentence $= \text{predicate}(\text{term}_1, \ldots, \text{term}_n)$
or $\text{term}_1 = \text{term}_2$

Examples:
Brother (Richard, John)
Married (FatherOf(Richard), MotherOf(John))
Complex Sentences

built with atomic sentences and logical connectives

\( \neg S \)
\( S_1 \land S_2 \)
\( S_1 \lor S_2 \)
\( S_1 \Rightarrow S_2 \)
\( S_1 \Leftrightarrow S_2 \)

Examples:
Sibling(KingJohn, Richard) \( \Rightarrow \) Sibling(Richard, KingJohn)
\( > (1, 2) \lor \leq (1, 2) \)
\( > (1, 2) \land \neg > (1, 2) \)
Truth in first-order logic: Semantic

Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for

constant symbols $\rightarrow$ objects

predicate symbols $\rightarrow$ relations

function symbols $\rightarrow$ functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$
Model in FOL: example

The **domain** of a model is the set of objects it contains: five objects

**Intended interpretation:** Richard refers Richard the Lion Heart, John refers to Evil King John, Brother refers to brotherhood relation, etc.
Models for FOL: Lots!

We can enumerate the models for a given KB vocabulary:

For each number of domain elements $n$ from 1 to $\infty$
   For each $k$-ary predicate $P_k$ in the vocabulary
      For each possible $k$-ary relation on $n$ objects
         For each constant symbol $C$ in the vocabulary
            For each choice of referent for $C$ from $n$ objects . . .

Computing entailment by enumerating models is not going to be easy!

There are many possible interpretations, also some model domain are not bounded

$\rightarrow$ Checking entailment by enumerating is not an option
Quantifiers

allow to make statements about entire collections of objects

- universal quantifier: make statements about everything
- existential quantifier: make statements about some things
Universal quantification

\[ \forall \langle \text{variables} \rangle \langle \text{sentence} \rangle \]

\textbf{Example:} all dogs like bones \( \forall x \text{Dog}(x) \Rightarrow \text{LikeBones}(x) \)
x = Indy is a dog \quad x = \text{Indiana Jones is a person}

\( \forall x \ P \) is equivalent to the \underline{conjunction} of \underline{instantiations} of \( P \)

\[ \text{Dog}(\text{Indy}) \Rightarrow \text{LikeBones}(\text{Indy}) \]
\[ \land \quad \text{Dog}(\text{Rebel}) \Rightarrow \text{LikeBones}(\text{Rebel}) \]
\[ \land \quad \text{Dog}(\text{KingJohn}) \Rightarrow \text{LikeBones}(\text{KingJohn}) \]
\[ \land \quad \ldots \]

\textbf{Typically:} \( \Rightarrow \) is the main connective with \( \forall \)

\textbf{Common mistake:} using \( \land \) as the main connective with \( \forall \)

\textbf{Example:} \( \forall x \ \text{Dog}(x) \land \text{LikeBones}(x) \)

all objects in the world are dogs, and all like bones
Existential quantification

\[ \exists \langle \text{variables} \rangle \ \langle \text{sentence} \rangle \]

**Example:** some student will talk at the TechFair
\[ \exists \ x \ Student(x) \land \ TalksAtTechFair(x) \]

Pat, Leslie, Chris are students
\[ \exists \ x \ P \quad \text{is equivalent to the disjunction of instantiations of } P \]

\[
\begin{align*}
\text{Student}(Pat) & \land \text{TalksAtTechFair}(Pat) \\
\lor \quad & \text{Student}(Leslie) \land \text{TalksAtTechFair}(Leslie) \\
\lor \quad & \text{Student}(Chris) \land \text{TalksAtTechFair}(Chris) \\
\lor \quad & \ldots
\end{align*}
\]

**Typically:** \( \land \) is the main connective with \( \exists \)

**Common mistake:** using \( \Rightarrow \) as the main connective with \( \exists \)
\[ \exists \ x \ Student(x) \ \Rightarrow \ \text{TalksAtTechFair}(x) \]
is true if there is anyone who is not Student
Properties of quantifiers (I)

\( \forall x \forall y \) is the same as \( \forall y \forall x \)

\( \exists x \exists y \) is the same as \( \exists y \exists x \)

\( \exists x \forall y \) is not the same as \( \forall y \exists x \)

\( \exists x \forall y \text{ Loves}(x, y) \)

“There is a person who loves everyone in the world”

\( \forall y \exists x \text{ Loves}(x, y) \)

“Everyone in the world is loved by at least one person”

**Quantifier duality:** each can be expressed using the other

\( \forall x \text{ Likes}(x, \text{IceCream}) \quad \neg \exists x \neg \text{Likes}(x, \text{IceCream}) \)

\( \exists x \text{ Likes}(x, \text{Broccoli}) \quad \neg \forall x \neg \text{Likes}(x, \text{Broccoli}) \)

**Parsimony principal:** \( \forall, \neg, \) and \( \Rightarrow \) are sufficient
Properties of quantifiers (II)

Nested quantifier:
\[ \forall x (\exists y (P(x, y))) : \]
every object in the world has a particular property, which is the property to be related to some object by the relation \( P \)

\[ \exists x (\forall y (P(x, y))) : \]
there is some object in the world that has a particular property, which is the property to be related to every object by the relation \( P \)

Lexical scoping: \[ \forall x \left[ Cat(x) \lor \exists x \text{Brother}(Richard, x) \right] \]

Well-formed formulas (WFF): (kind of correct spelling)
every variable must be introduced by a quantifier
\[ \forall x P(y) \] is not a WFF
Examples

Brothers are siblings
.

“Sibling” is symmetric
.

One’s mother is one’s female parent
.

A first cousin is a child of a parent’s sibling
Examples

\[ \forall x, y \ \text{Brother}(x, y) \implies \text{Sibling}(x, y) \]

\[ \forall x, y \ \text{Sibling}(x, y) \implies \text{Sibling}(y, x) \]

\[ \forall x, y \ \text{Mother}(x, y) \implies (\text{Female}(x) \land \text{Parent}(x, y)) \]

\[ \forall x, y \ \text{FirstCousin}(x, y) \iff \\
\exists a, b \ \text{Parent}(a, x) \land \text{Sibling}(a, b) \land \text{Parent}(b, y) \]
Tricky example

Someone is loved by everyone
\[ \exists x \, \forall y \, Loves(y, x) \]

Someone with red-hair is loved by everyone
\[ \exists x \, \forall y \, Redhair(x) \land Loves(y, x) \]

Alternatively:
\[ \exists x \, Person(x) \land Redhair(x) \land (\forall y \, Person(y) \Rightarrow Loves(y, x)) \]
**Equality**

\( \text{term}_1 = \text{term}_2 \) is true under a given interpretation if and only if \( \text{term}_1 \) and \( \text{term}_2 \) refer to the same object.

**Examples**

- Father(John) = Henry
- \( 1 = 2 \) is satisfiable
- \( 2 = 2 \) is valid
- Useful to distinguish two objects:
  - Definition of (full) *Sibling* in terms of *Parent*:
    \[ \forall x, y \ Sibling(x, y) \iff \neg (x = y) \land \exists m, f \neg (m = f) \land Parent(m, x) \land Parent(f, x) \land Parent(m, y) \land Parent(f, y) \]
  - Spot has at least two sisters: ...

AIMA, Exercise 8.4. Write: “All Germans speak the same languages,” where *Speaks*(\( x, l \)) means that person \( x \) speaks language \( l \).
Knowledge representation (KR)

**Domain:** a section of the world about which we wish to express some knowledge

**Example:** Family relations (kinship):
- **Objects:** people
- **Properties:** gender, married, divorced, single, widowed
- **Relations:** parenthood, brotherhood, marriage..

**Unary predicates:** Male, Female

**Binary relations:** Parent, Sibling, Brother, Child, etc.

**Functions:** Mother, Father

\[ \forall m, c, \text{Mother}(c) = m \iff \text{Female}(m) \land \text{Parent}(m, c) \]
In Logic (informally)

- Basic facts: **axioms**
- Derived facts: **theorems**

**Independent axiom**

an axiom that cannot be derived from the rest

→ Goal of mathematicians: find the minimal set of independent axioms

In AI

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK
Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at \( t = 5 \):

\[ \text{Tell}(KB, \text{Percept}([\text{Smell, Breeze, None}], 5)) \]
\[ \text{Ask}(KB, \exists a \text{Action}(a, 5)) \]

I.e., does the KB entail any particular actions at \( t = 5 \)?

Answer: Yes, \( \{ a/\text{Shoot} \} \) ← substitution (binding list)

Given a sentence \( S \) and a substitution \( \sigma \),
\( S\sigma \) denotes the result of plugging \( \sigma \) into \( S \); e.g.,
\[ S = \text{Smarter}(x, y) \]
\[ \sigma = \{ x/\text{Hillary}, y/\text{Bill} \} \]
\[ S\sigma = \text{Smarter}(\text{Hillary}, \text{Bill}) \]

\( \text{Ask}(KB, S) \) returns some/all \( \sigma \) such that \( KB \models S\sigma \)
Prepare for next lecture: AIMA, Exercise 8.24, page 319

Takes(x, c, s): student x takes course c in semester s

Passes(x, c, s): student x passes course c in semester s

Score(x, c, s): the score obtained by student x in course c in semester s

x > y: x is greater than y

F and G: specific French and Greek courses

Buys(x, y, z): x buys y from z

Sells(x, y, z): x sells y from z

Shaves(x, y): person x shaves person y

Born(x, c): person x is born in country c

Parent(x, y): person x is parent of person y

Citizen(x, c, r): person x is citizen of country c for reason r

Resident(x, c): person x is resident of country c of person y

Fools(x, y, t): person x fools person y at time t

Student (x), Person(x), Man(x), Barber(x), Expensive(x), Agent(x), Insured(x), Smart(x), Politician(x),
**AI Limerick**

If your thesis is utterly vacuous
Use first-order predicate calculus
With sufficient formality
The sheerest banality
Will be hailed by the critics: "Miraculous!"

*Henry Kautz*

_In Canadian Artificial Intelligence, September 1986_

_head of AI at AT&T Labs-Research_

_Program co-chair of AAAI-2000_

_Professor at University of Washington, Seattle_

_Founding Director of Institute for Data Science and Professor at University of Rochester_